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ABSTRACT

This document contains Volumes 1 and 2 of the proceedings of the 23rd annual conference of the Mathematics Education Research Group of Australasia Incorporated (MERGA) held at Fremantle, Western Australia, July 5-9, 2000. Papers in Volume 1 include: (1) "Bridging Practices: Intertwining Content and Pedagogy in Teaching and Learning To Teach" (Deborah Loewenberg Ball); (2) "Looking Backwards, Looking Forwards, and Looking for Direction" (Andy Begg); (3) "Bridging the Gap: A Challenge for the Dual Community" (John A. Malone); (4) "Collaborative Problem Solving in Senior Secondary Mathematics Classrooms" (Merrilyn Goos); (5) "Changing the Professional Knowledge of Teachers" (Janette Bobis and Peter Gould); (6) "Teachers, Students and Research: One Possibility for Teaching Early Numeracy" (Philip Clarkson); (7) "Mapping Children's Stages of Arithmetical Solution Methods" (Peter Gould and Robert Wright); (8) "Reviewing Literature Relevant to a Systemic Early Numeracy Initiative: Bases of CMIT" (Robert Wright and Peter Gould); (9) "Emergence of Mathematical (In)Competence and Identity" (Pat Forster and Peter Taylor); (10) "Is There More to Numeracy Than Meets the Eye? Stories of Socialization and Subjectification in School Mathematics" (Mary Klein); (11) "Mental Effort and Errors in Bracket Expansion Tasks" (Paul Ayres); (12) "Developing Task Specific Criteria: A Preliminary Report" (Dawn Bartlett); (13) "Construction of a Numeration Model: A Theoretical Analysis" (Annette Baturo); (14) "Modes of Representation in Students' Explanations" (Brenda Bicknell); and (15) "What Is Taught versus What Is Learnt: The Case of Linear Measurement" (Philippa Bragg and Lynne Outhred). Papers in Volume 2 include: (1) "Classroom and Teacher Effects in Mathematics Achievement: Results from TIMSS" (Stephen Lamb and Sue Fullarton); (2) "3-Dimensional Geometry: Assessment of Students' Responses" (Chrisitine Lawrie and John Pegg); (3) "Mathematics and Gender: Beliefs They Are a-Changin'" (Gilah Leder and Helen Forgasz); (4) "Informal Assessment Questions Used by Secondary School Mathematics Teachers" (Sanka Liyanage, Kathryn Irwin, and Mike Thomas); (5) "Grade 1 Children in Problem-Posing

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Contexts: Problem Solving Prior to Completing the Task" (Tom Lowrie); (6) "Knowledge and Strategies Students Employ To Solve Open-Ended Problem-Solving Activities" (Tom Lowrie, Rod Francis, and Geoff Rogers); (7) "Mental Computation, Number Sense and General Mathematics Ability: Are They Linked?" (Alistair McIntosh and Shelley Dole); (8) "Designing Constructivist Computer Games for Teaching about Decimal Numbers" (Janine McIntosh, Kaye Stacey, Calvin Tromp, and Darren Lightfoot); (9) "Scaffolding: A Suitable Teaching Characteristic in One-to-One Teaching in Maths Recovery" (Bronwyn McMahon); (10) "Low Achieving Mathematics Students: Attitudinal and Achievement Changes as a Result of Using an Integrated Learning System" (Campbell McRobbie, Annette Baturo, and Tom Cooper); (11) "Teaching for Abstraction: Reconstructing Constructivism" (Michael Mitchelmore and Paul White); (12) "Graphical Representations of Statistical Associations by Upper Primary Students" (Jonathan Moritz); (13) "Reasoning and Expressing Probability in Students' Judgements of Coin Tossing" (Jonathan Moritz and Jane Watson); (14) "The Practicum as Context: Two Snapshots" (Judith Mousley and Georgina Herbert); and (15) "A Comparative Analysis of the 1996-1999 Calculus TEE Papers" (Ute Mueller and Patricia Forster). (ASK)

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Volume 1 [and] Volume 2.

Jack Bana and Anne Chapman, Editors

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Mathematics Education Research Group of Australasia Incorporated

Mathematics Education Beyond 2000

Proceedings of the Twenty-third Annual Conference of the
Mathematics Education Research Group of Australasia Incorporated
held at Fremantle, Western Australia, 5-9 July, 2000

Volume 1

Jack Bana

Anne Chapman

(Editors)

MERGA 23
fremantle 2000

Mathematics Education Research Group of Australasia Incorporated

Mathematics Education
Beyond 2000

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Preface

The Twenty-third Annual Conference of the Mathematics Education Research Group of Australasia (MERGA) was held in Fremantle, Western Australia, 5–9 July, 2000. The Conference attracted papers from researchers in Australia, Canada, Israel, New Zealand, Singapore, South Africa, and the United States of America. Contributions from a large number of established researchers—many with substantial international reputations—as well as a significant number of beginning researchers were included in the program.

The Proceedings of the Conference have been prepared after a refereeing process which conforms with current practices for ensuring that the quality of the individual papers is of a high standard, and constitutes scholarly research which is providing an addition to knowledge. The Proceedings are published by MERGA and are available for purchase from the MERGA Publications Officer, whose address is given elsewhere in this publication.

Research studies reported at the Conference encompassed a variety of mathematics education related topics and made use of a wide range of research methodologies and theoretical perspectives. Published presentations were organised into Keynote Papers, Symposia, Research Papers, and Abstracts of Short Communications. In accepting research papers for the Conference, every effort was made to be as inclusive as possible of the research needs and interests of members of the mathematics education research community.

Refereeing processes for the Conference were overseen by an Editorial Committee comprising the Co-editors Jack Bana (Edith Cowan University) and Anne Chapman (University of Western Australia); and David Haimes, John Malone, and Len Sparrow (Curtin University of Technology). MERGA members were invited to act as referees, and the responsibilities of this role was made clear to those who accepted. Several experts outside the MERGA membership were also invited to referee papers submitted.

Authors who wished to have a paper considered for the Conference were required to submit the full text of their paper to double-blind peer reviewing by two referees. As far as was possible the research interests of referees were taken into account when papers were allocated for review. Referees were asked to heed guidelines for published papers consistent with those set out in the call for papers and to provide their written comments in a standard format under the following suggested headings:

- Statement of problem/issues and discussion of its significance
- Literature review and theoretical framework
- Methodology and data analysis (where applicable)
- Discussion
- Conclusions and implications
- General comments on the paper

At least one of the Co-editors and at least one other member of the Editorial Committee read all referees' reports. Where only one of the two referees recommended acceptance, the paper was given to a third referee and in normal cases the majority decision prevailed. In a few cases several members of the Editorial Committee read all the reports and also the original paper before making a final decision.

Some research reports were submitted as Early Bird papers, which were refereed and accepted in the usual way. In cases where revision of the paper was recommended, authors were invited to resubmit, and the resubmitted articles were re-refereed in the usual way, often by the original referees.

Some research reports were submitted as entries for the Practical Implications Award. Refereeing of these papers was organised by the Vice-President (Research) of MERGA, and involved being reviewed by at least four referees chosen by the Vice-President (Research).

Similar criteria were used for choosing these referees, and for recommending whether a paper should be accepted for presentation as a research paper if it was not selected for the Award. Several research reports were accepted for presentation at the Conference in two Symposia. Abstracts for short communications were reviewed by the Editorial Committee to ensure their relevance to the Conference.

The purposes of adopting the methods indicated above were to facilitate high quality feedback to authors and achieve the best possible assurance that the Conference Proceedings conformed to MERGA research guidelines. It was especially hoped that beginning researchers would benefit from the feedback received.

On behalf of the Editorial Committee I wish to thank all those who generously agreed to act as referees for the research papers submitted. Their support has ensured that the quality of the Conference Proceedings is of a standard befitting the mathematics education research community.

Jack Bana
Editorial Committee, Convenor
Perth, Australia
June, 2000

Referees for MERGA23 Published Papers

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Gary	Asp	University of Melbourne
Paul	Ayres	University of New South Wales
Jack	Bana	Edith Cowan University
Mary	Barnes	University of Melbourne
Dawn	Bartlett	NSW Dept of Education
Bill	Barton	University of Auckland
Andy	Begg	University of Waikato
Fred	Biddulph	University of Waikato
Alan	Bishop	Monash University
Janette	Bobis	University of Sydney
Gard	Brekke	TFN Institute, Norway
Peter	Brinkworth	Flinders University
Paul	Brown	Curtin University of Technology
Roger	Brown	Swinburne University of Technology
Fiona	Budgen	Curtin University of Technology
Ann	Buzeika	Auckland College of Education
Robert	Cavanagh	Curtin University of Technology
Anne	Chapman	University of Western Australia
Mohan	Chinnappan	University of Wollongong
Philip	Clarkson	Australian Catholic University
Martin	Cooper	Curtin University of Technology
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Keynote Papers

MERGA 23
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Bridging Practices:
Intertwining Content and Pedagogy in Teaching and Learning to Teach[1]

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Teacher education throughout the 20th century has consistently been structured across a persistent divide between subject matter and pedagogy. The prevalent conceptualization and organization of teachers' learning tends to fragment practice and leave to individual teachers the challenge of integrating subject matter knowledge and pedagogy in the contexts of their work. We assume that the integration required to teach is simple and happens in the course of experience. In fact, however, this does not happen easily, and often does not happen at all. This paper takes up three problems that we must solve if we are to meet this challenge to prepare teachers who not only know content, but can make use of it to help all students learn. The first problem concerns identifying the content knowledge that matters for teaching; the second regards understanding how such knowledge needs to be held; and the third centres on what it takes to learn to use such knowledge in practice.

Chasms in Knowing and Learning Practice

At the turn of the 20th century, John Dewey (1904/1964) articulated a fundamental tension in the preparation of teachers: that of the "proper relationship" of theory and practice. At the turn of the 21st century, this tension endures. In fact, many of the same questions persist: On one hand, to what extent does teaching and learning to teach depend on the development of theoretical knowledge and knowledge of subject matter? On the other, to what extent does it rely on the development of pedagogical method?

Clearly, the answer must be that it depends on both. Yet, across the century, this tension has continued to simmer, with strong views on both sides of what is unfortunately often seen as a dichotomy. Policy makers debate whether teachers should major in education or in a discipline. Others argue that what matters is caring for students and having the skills to work effectively with diverse learners. Dewey's (1904/1964) conception of the relationship of subject matter knowledge and method was sophisticated and subtle, so much so that one hundred years later, his idea is still elusive. He wrote, "Scholastic knowledge is sometimes regarded as if it were something quite irrelevant to method. When this attitude is even unconsciously assumed, method becomes an external attachment to knowledge of subject matter" (p. 160).

Dewey (1904/1964) believed that good teachers were those who could recognize and create "genuine intellectual activity" in students, and he argued that methods of such activity were intimately tied into disciplines. Subjects, he believed, were the embodiments of the mind, the product of human curiosity, inquiry, and the search for truth. A mind "which is habituated to viewing subject-matter from the standpoint of the function of that subject matter in connection with the mental signs of intellectual activity when exhibited in the child of four, or the youth of sixteen" would, in his view, be prepared to hear and extend students' thinking. To do this, teachers would need to be able to study subject matter in ways that took it back to its "psychical roots" (p. 162).

Despite these prescient ideas, teacher education throughout the 20th century has consistently been structured across a persistent divide between subject matter and pedagogy.

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This divide has many faces. Sometimes it appears in institutional structures as the chasm between the arts and sciences and schools of education, or as the divide between universities and schools (Lagemann, 1996). Sometimes the divide appears in the prevailing curriculum of teacher education, separated into domains of knowledge: educational psychology, sociology of education, foundations, methods of teaching, and the academic disciplines corresponding to school subjects. These knowledge chunks are complemented by experience: supervised practice, student teaching and practice itself. In all of these, the gap between theory and practice fragments teacher education by fragmenting teaching.

In recent years, in yet another peculiar fragmentation, commitments to equity and concerns for diversity have often been seen as in tension with concerns for content preparation. Yet, understanding subject matter is essential to listening flexibly to others and hearing what they are saying or where they might be heading. Knowing content is also crucial to being inventive in creating worthwhile opportunities for learning that take learners' experiences, interests, and needs into account. Contending effectively with the resources and challenges of a diverse classroom requires a kind of responsibility to subject matter, without which, efforts to be responsive may distort students' opportunities to learn (Ball, 1995). Moreover, the creativity entailed in designing instruction in ways that are attentive to difference requires substantial proficiency with the material.

The overarching problem across these many examples is that the prevalent conceptualization and organization of teachers' learning tends to fragment practice and leave to individual teachers the challenge of integrating subject matter knowledge and pedagogy in the contexts of their work. We assume that the integration required to teach is simple and happens in the course of experience. In fact, however, this does not happen easily, and often does not happen at all.

A closer look at a sliver of the work of teaching serves to remind us of its demands. Consider a teacher examining and preparing to teach the following deceptively simple math problem (Gelfand & Shen, 1993),

> Write down a string of 8's. Insert some plus signs at various places so that the resulting sum is 1,000.

At first glance, this task may look trivial and uninteresting. One way of solving it entails simply adding 125 8's together. A closer look reveals that if several 8's are written together (i.e., 888 or 88), many more solutions are possible. Working on the problem a little further reveals interesting and provocative patterns in the solution set. Figuring out how to organize the solutions is itself an interesting component of the work; depending on how they are organized, different elements of the problem and its solutions are visible.

The teacher must contemplate: Would this be a good task for my students? What would it take to figure out the patterns and nuances? Is it worthwhile in terms of what students might learn? At the very least, it would be important to know what the problem is asking, whether it has one or many solutions, and how the solutions might be found. It seems obvious that the task entails some computation, for example, verifying any one solution, but what is the mathematical potential of the task? Are there important ideas or processes involved in the problem? What would it take to use this task well with students? It would help to know what might make the problem hard, and how kids might get stuck, and anticipate what the teacher might do if they did. Would students find this interesting? What might it take to hook them on it?

Perhaps, after looking at this problem, the teacher decides that it is interesting, but a little too difficult for her students. What would it take to make a mathematically similar problem that is a bit easier? At what grade levels would some mathematically equivalent but simpler version of this problem be accessible? How might one rescale the problem, for example, for third graders? First graders? Suppose, in contrast, that the teacher worries that this problem is too easy. What would it take to make a more challenging, but again, mathematically similar task? What happens to the problem if one replaces 1,000 with other numbers, or 8 with some other digit? How might one modify the problem so that there are no solutions or infinitely many solutions? Would this be fruitful?

This sort of analysis and preparation of a single math problem reveals how much core tasks of teaching involve significant mathematical reasoning in the context of practice. And these represent only a fraction of the work that a teacher would do to make productive use of this problem with students. When the teacher collects students' work and peruses it, the teacher may grade it, determine where her students are, or decide whether to go further. When teachers hold class discussions, they make decisions about which (and whose) ideas to pick up and pursue and which (and whose) to let drop. The teacher formulates probes, pushes students, offers hints, and provides explanations. Students get stuck: What does one do to help them remobilize? None of these tasks of teaching is possible to do generically. No matter how committed one is to caring for students, to taking students' ideas seriously, to helping students develop robust understandings, none of these tasks of teaching is possible without making use in context of mathematical understanding and insight.

Herein lies a fundamental difficulty in learning to teach, for despite its centrality, usable content knowledge is not something teacher education, in the main, provides effectively. Although some teachers have important understandings of the content, they often do not know it in ways that help them hear students, select good tasks or help all their students learn. Not being able to do this undermines and makes hollow efforts to prepare high-quality teachers who can reach all students, teach in multicultural settings, and work in environments that make teaching and learning difficult. Despite frequently heard exhortations to teach all students, many teachers are unable to hear students flexibly, represent ideas in multiple ways, connect content to contexts effectively, and think about things in ways other than their own. For example, in their study of a middle school teacher's attempt to teach the concept of rate, Thompson and Thompson (1994) highlight the crucial role played by language. They vividly describe the situation of one teacher who, although he understood the concept of rate himself, was restricted in his capacity to express or discuss the ideas in everyday language. Satisfied with computational language for his own purposes, when these did not help students understand, he was not able to find other means of expressing key ideas. In addition, teachers may not be able to size up their textbooks and adapt them effectively; they may omit topics central to students' futures or make modifications that distort key ideas. They may substitute student interest for content integrity in making choices about subject matter. Knowing subject matter and being able to use it is at the heart of teaching all students.

A recent analysis provides a glimpse of the importance of the distinction between knowing how to do math and knowing it in ways that enable its use in practice. This distinction is key to understanding how mathematics knowledge matters in good teaching. In general, astonishingly little empirical evidence has exists to link teachers' content knowledge to their students' learning. One hypothesis has been that what is being measured as "content knowledge" (often teachers' course attainment) is a poor proxy for subject matter understanding. However, in an article describing their analysis of data from the National Education Longitudinal Study of 1988, Rowan Chiang, and Miller (1997) report strong positive correlations between teachers' responses to items designed to measure the use of mathematical knowledge in teaching and their students' performance (see Kennedy, Ball, & McDiarmid, 1993). This analysis provides some confirmation that understanding the use of mathematics in the work of teaching is a critical area ripe for further examination. It is not just what mathematics teachers know, but how they know it and what they are able to mobilize mathematically in the course of teaching.

An important challenge for teacher education at the turn of 21st century is to bridge the chasm identified by Dewey (1904/1964) almost 100 years ago. Our schools are more diverse than ever and we ask more of both teachers and students. Finding ways to integrate knowledge and practice is essential if we are to help teachers develop the resources they need for their work. This is a call to preservice teacher education, where academic studies dominate, as well as to professional development, where opportunities to study content are far more rare (Wilson, Theule-Lubienski, & Mattson, 1996). In neither setting is this divide being closed. What would it take to bring the study of content closer to practice and prepare teachers to know and be able to use subject matter knowledge effectively in their work as teachers?

Three problems stand out, problems that we must solve if we are to meet this challenge to prepare teachers who not only know content but can make use of it to help all students learn. The first problem concerns identifying the content knowledge that matters for teaching, the second regards understanding how such knowledge needs to be held, and the third centers on what it takes to learn to use such knowledge in practice.

First, we would need to re-examine what content knowledge matters for good teaching. Subject matter knowledge for teaching has too often been defined by the subject matter knowledge that students are to learn. Put simply, many assume that what teachers need to know is what they teach, along with a broad perspective on where their students are heading. Nothing is inherently wrong with this perspective. However, the lists of what teachers should know that are produced by analyzing the school curriculum are long as well as arbitrary and unsubstantiated. Little is known about how "knowing" the topics on these lists affects teachers' capabilities. The unexamined conviction that possessing such knowledge is what teachers need to know has blocked the inquiry needed to bring together subject matter and practice in ways that would enable teacher education to be more effective.

Instead of beginning solely with the curriculum, our understanding of the content knowledge needed in teaching must start with practice. We must understand better the work that teachers do and analyze the role played by content knowledge in that work. What are the recurrent core task domains of teachers' work? For example, the teacher examining the 8's problem would have to probe the task, consider how it might be done by particular students, decide how difficult it might be, and perhaps re-scale it to make an easier or more challenging version. In teaching it, the teacher would to have to listen carefully to what students said, interpret what they meant, ask them questions, give hints, observe.

To improve our sense of what content knowledge matters in teaching, we would need to identify core activities of teaching, such as figuring out what students know; choosing and managing representations of ideas; appraising, selecting, and modifying textbooks; and deciding among alternative courses of action, and analyze the subject matter knowledge and insight entailed in these activities. This approach, a kind of job analysis of classroom teaching focused on the actual work that teachers do, could provide a view of subject matter as it is used in practice.

Turning the usual approach on its head, this approach would uncover what teachers need to know and what they need to be sensitive to regarding content to teach well. This kind of analysis may bring some surprises. For example, in our recent work with Bass on mathematics teaching (Bass & Ball, in press), we expected to see that concepts such as place value and decimal notation would be central, and they have been, as have operations and methods of reasoning. However, beyond that, we have been struck by the unanticipated but recurrent prominence of certain mathematical notions. For instance, we have found that ideas about equivalence, similarity, and even isomorphism emerge across many instances of ordinary and extraordinary teaching and learning. We have also uncovered salient issues involving mathematical language: symbolic notation, mapping among representations, and definitions of terms. Similarly new notions are emerging from parallel work in the teaching and learning of history and science (Rose, 1999; Wilson, in press). Inquiries that begin with practice are revealing subject matter demands of teachers' work that are not seen when we begin with lists of content to be taught derived from the school curriculum. These content demands emerge from analyzing the sorts of challenges with which teachers must contend in the course of practice, as they mediate students' ideas, make choices about representations of content, modify curriculum materials, and the like.

A second problem we would have to solve concerns the assumption that someone who knows content for himself or herself is able to use that knowledge in teaching. This is the problem of how subject matter must be understood to be usable in teaching. Simply increasing teachers' opportunities to study mathematics, English, history, or physics – one easy response – will be insufficient to impact their capacities as teachers. We need to probe not only what teachers need to know but also what sort of content understanding and insight matters in practice.

Viewed from the perspective of practice, and the actual work of teaching, at least two aspects seem central. First is the capacity to deconstruct one's own knowledge into less polished and final form, where critical components are accessible and visible. This feature of teaching means that paradoxically, expert personal knowledge of subject matter is often ironically inadequate for teaching. Because teachers must be able to work with content for students in its growing, unfinished state, they must be able to do something perverse: work backward from mature and compressed understanding of the content to unpack its constituent elements (Cohen, in preparation). Knowing for teaching requires a transcendence of the tacit understanding that characterizes and is sufficient for personal knowledge and performance (Polanyi, 1958). Understanding why a 6-year-old might write "1005" for "one hundred five," and not reading it as a mistaken count, "one thousand five," requires the capacity to appreciate the elegance of the compressed notation system that adults use readily for numbers but which is not automatic for learners. After all, Roman numerals follow precisely the same structure as the young child's inclination, each element with its own notation, CV for "one hundred five," without the place-value core of our system. Being able to see and hear from someone else's perspective, to make sense of a student's apparent error or appreciate a student's unconventionally expressed insight requires this special capacity to unpack one's own highly compressed understandings that are the hallmark of expert knowledge. Even producing a comprehensible explanation depends on this capacity to unpack one's own knowledge, because an explanation works only if it is at a sufficient level of granularity, that is, if it includes in it the steps necessary for the reasoning to make sense for a particular learner or a whole class, based on what they currently know or do not know (Ball & Bass, in press).

Another aspect of being able to use the knowledge one has is the special sort of knowledge that Shulman and his colleagues (Shulman, 1986, 1987; Wilson, Shulman, & Richert, 1987) called "pedagogical content knowledge:" a special amalgam of knowledge that links content and pedagogy. Included here is knowledge of what is typically difficult for students, of representations most useful for teaching a specific idea or procedure, and of ways to develop a particular idea, for example, the ordering of decimals, or interpreting poetry. What are the advantages and disadvantages of particular metaphors or analogies? Where might they distort the subject matter? For example, both "take away" and "borrowing" create problems for students' understanding of subtraction. These problems cannot be discerned generically because they require a careful mapping of the metaphor against core aspects of the concept being learned and against how learners interpret the metaphor. Knowing that subtraction is a particularly difficult idea for students to master is not something that can be seen from knowing the "big ideas" of the discipline. This kind of knowledge is not something a mathematician would necessarily have, but neither would it be familiar to a high school social studies teacher. It is quite clearly mathematical, yet formulated around the need to make ideas accessible to others. Pedagogical content knowledge highlights the interplay of mathematics and pedagogy in teaching. Rooted in content knowledge, it comprises more than understanding the content oneself.

These two aspects of content knowledge help to illuminate the territory to which Dewey (1904/1964) called attention almost a century ago, bridging the divide between content and pedagogy. However, they do not sufficiently illumine where and how such knowledge is used in teaching. Teaching is a practice. It is, in Lampert's (1998) terms, "a thinking practice"; that is, it integrates reasoning and knowing with action. Our tendency to focus either on its cognitive demands (teachers' knowledge, reasoning, decision making, reflection) or on its actions (teacher behavior) is yet one more recent form of fragmentation in teacher education, and in particular, in our efforts to help teachers acquire usable content knowledge.

Hence, a third problem we would have to solve is how to create opportunities for learning subject matter that would enable teachers not only to know but to learn to use what they know in the varied contexts of practice. Even with more grounded analyses of what there is to know and a more finely tuned conception of the nature of the understanding needed to teach, simply teaching such content may not solve the problems of use. How do teachers use content understanding in the context of practice to carry out the core activities of their work? How can we design opportunities for learning that are aimed at helping teachers use subject matter knowledge to figure out what their students know, to pose questions, to evaluate and modify their textbooks wisely, to design instructional tasks, to manage class discussions, and to explain the curriculum to parents?

Some such work along these lines is already underway. One promising possibility is to design and explore opportunities to learn content that are situated in the contexts in which subject matter is used, a core activity of teaching. For example, some teacher educators use student work as a site to analyze and interpret what students know and are learning and, in so doing, work on the content itself.[1] Another example is the use of videotaping classroom lessons or cases of classroom episodes (Lampert & Ball, 1998; Stein, Smith, Henningsen, & Silver, 2000), from which, the moves made by the teacher could be analyzed to consider the impact on the course of the lesson, the trajectory of the class's work, and the opportunities for learning for particular students and for the group. In both instances (using student work, using videotapes or cases of classroom lessons), teachers or prospective teachers might engage in content-based design work, developing a possible next assignment in response to their analysis of students' work, or planning a next instructional segment based on analysis of the classroom episode. Each of these activities takes a task of teaching that entails content knowledge and creates a possible site for teachers' learning of content in the contexts where they will have to use it.

However, much more work is needed to contend with this endemic problem of use. Working in specific contexts might run the risk of limiting the generality of teachers' learning of content and their capacity to use it in a variety of contexts. How can teachers be prepared to sufficiently know content flexibly so that they are able to make use of content knowledge with a wide variety of students across a wide range of environments? How could teachers develop a sense of the trajectory of a topic over time or how to develop its intellectual core in students' minds and capacities so that they eventually reach mature and compressed understandings and skills?

Solving these three problems – what teachers need to know, how they have to know it, and helping them learn to use it, by grounding the problem of teachers' content preparation in practice--could help to close the gaps that have plagued progress in teacher education. However, we should realize the challenges that doing this would pose. After all, Dewey (1904/1964) thought his vision at the turn of the 20th century was imminently realizable. He thought that what he was describing was "nothing utopian ." He suggested that, "the present movement . . .for the improvement of range and quality of subject matter is steady and irresistible" (p. 170). Almost 100 years later, as we stare at university and college catalogs that divide methods courses from disciplinary studies from practice, or at professional development offerings that are devoid of content or chock full of activities for kids, we should understand that bridging these strangely divided practices will be no small feat.[1]

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[1] Several professional development curricula in mathematics are built on this idea. See, for example, Schifter, D.; Barnett, C.; Stein, Smith, Henningsen, & Silver (in press).

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Looking Backwards, Looking Forwards, and Looking for Direction

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This personal and reflective story presents some incidents from my professional life that have (or have not) influenced me as a mathematics educator. Looking backwards I see some mistakes I made, opportunities I missed, and lessons I learnt. Looking forwards I see areas within our discipline that seem worthy of exploration, boundaries that may need to be crossed, and initiatives that need to be taken. From these perspectives a sense of direction is emerging for me and conferring with colleagues will help me see this more clearly.

Introduction

Why tell you my story about where I have been, where I am going, and why—not so others might learn from my mistakes, nor because of the conference theme 'beyond 2000'. The reason is to get my work into perspective and review my direction. This is consistent with my views of conferences and research. I see the benefits of conferences coming from collegial discussions and from the preparation rather than the presentation of papers. In research I see us often looking outside ourselves for causes and solutions to problems rather than seeing ourselves as significant and inseparable from our problems.

One aspect of story-telling that intrigues me is how incidents are chosen as important. When researching myself, how do I reconstruct my reality, which incidents lie forgotten, and why do I choose or reject things to report on when life is complex and all incidents are intertwined. In acknowledging this I use the word story rather than autobiography.

A researcher always makes the research instruments explicit. This is my story, I am the research instrument, so I must expose myself. One influence stands out. Descartes contributed much to Western thought, but Damasio (1994) and others have questioned these accepted dichotomies, and I think this is important. These dichotomies include the separation of self from others, mind from body, and personal from professional and social. We seem to have extended these to include mathematics from other subjects, learning from teaching, schools from life, schooling from education, past from present, and work from play (apart from at conferences). Many of us who are males, also separate cognitive from non-cognitive knowing. This contrasts with Asian thinking where contradictory elements are juxtaposed and seen as complementary—I believe we need to consider incorporating such thinking into our work.

This paper looks backwards and forwards. The first is historical, the second involves my concerns within mathematics education, but the two directions are mutually dependent.

Part 1: Looking Backwards

Primary School

In 1945 I began school in a small country town. I was quite able and have happy memories of my time there. I remember three incidents about mathematics (or arithmetic as it was called). Firstly, age six, walking home and wondering why every second number was omitted from consecutive letter boxes, and later finding them on the other side of the road. Secondly, chanting tables, multiplication to 12 x 12, and addition, subtraction, and division tables. Thirdly, getting $99\frac{1}{2}\%$ and being told by my teacher that he didn't think anyone deserved 100% (but more on assessment later). My best memory of learning at this school was not in mathematics, it was in social studies (although it involved scale drawings, graphs and statistics). Each student selected a project topic and did their research. I was very satisfied when I handed in an exercise book full of work on Canada.

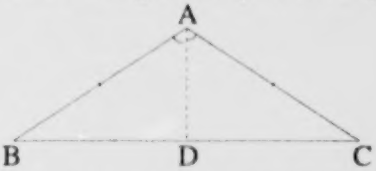
During these years I enjoyed family, I swam and I played games like monopoly and cards that reinforced my competitiveness, number skills, and capitalist upbringing.

High School

My memories of high school as a boarder at a single-sex, city school are mixed and generally pleasant, but I did resolve not to send my children to boarding school.

The first incident I remember was in year nine geometry. Each period started with a test on the previous theorem, then a new one was taught, and we did riders (examples) based on it. I remember one theorem about isosceles triangles, see Figure 1.

Theorem: If two sides of a triangle are equal, the angles opposite those sides are equal.



Given	Let ABC be any triangle, in which AB = AC		
Required	To prove	∠B = ∠C	
Construction	Draw AD to bisect ∠A, with D on BC		
Proof	In triangles BAD and CAD		
	Because	AB = AC (given)	
	and	AD = AD (common)	
	and	∠BAD = ∠CAD (construction)	
	therefore	ΔBAD ≡ ΔCAD (SAS)	
	thus	∠B = ∠C	Q.E.D

Figure 1. Isosceles triangle theorem.

For the test we knew that the diagram would be positioned and labelled differently to confuse pupils who had learnt the theorem by rote, see Figure 2.

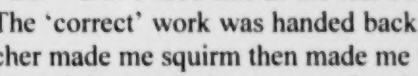


Figure 2. Alternative positioning and labelling.

A friend and I thought a shorter proof with no construction was possible. We thought (using Figure 1) that: AB = AC (given), AC = AB (given), BC = CB (common), thus ΔBAC ≡ ΔCAB (SSS), hence ∠B = ∠C. I used this in the test but my friend avoided the risk of an after-school detention. The 'correct' work was handed back, then the ones that earned a detention, then mine. The teacher made me squirm then made me feel great by saying that one person in the class was thinking and demonstrating my proof to the class.

The same teacher said we needed automatic recall with basic facts so we could do difficult problems without our logic being disrupted. Basic facts were extended to 20 x 20 multiplication, squares to 30 squared, and cubes to 12 cubed. At the end of year 9 he gave us most of the year 11 external examination and marked it. He assured us we could pass it, and told us not to think about year 11 but to set our sights on scholarship in year 13.

He thought mathematics was the most important subject. His mark schemes ensured that the top marks were spread over a broad range. This meant that when subject marks were aggregated mathematics results determined the top of the class. This was fine if one was good at mathematics, but if one was average one only attained 10%!

I learnt about assessment in other subjects too. I learnt to hate languages—translations were marked by deducting half a mark for each error from the 20% allowed for each translation in the test. Typically I had -3.5 and -4 added to the few marks I gained in the other sections. Meanwhile in science, with questions such as 'list the properties of oxygen' for 10 marks, I could list 24 properties at a half mark each and get 12 from a possible 10.

In five years at high school in pure and applied mathematics we were never given notes nor asked to write summaries. Teachers thought we learnt by doing mathematics, not by writing about it. When inspectors arrived in year 13 we were told to bring that years 'scribblers' to school, and I took 26 exercise books from my two mathematics subjects.

Our exercise books had always been called scribblers—not quad paper, not ruled, just plain paper. There were no marks for borders, crossed out work was acceptable, graphs were sketched not plotted, and figures represented, as far as possible, general triangles, circles, or whatever. We were expected to be legible, organised (equal signs under equal signs), use correct symbols ($=$, $\equiv$, $\therefore$, etc), and justify geometric statements we made.

University

University was different. Lecturers made little attempt to get to know students. Notes were dictated on material in our textbooks and we copied them and did not think. We had examples set and tutorials for help. It was fine for students from small schools who had learnt to work independently, and for those who had prepared for scholarship as no new topics were introduced. For many, disaster struck after year one because we did not know how to learn. Most scraped through year two, many failed at year three. I recall being able to follow lectures from line to line, but having little idea about the subject matter. I did not blame lecturers, I assumed I was not as bright as the students who passed with As or Bs.

There were lighter moments—one professor warned us about rigour, rigo(u)r, rigor mortis. There were exceptions too in terms of getting-to-know students—one lecturer learnt our names from enrolment photos, greeted us by name outside class, and asked individuals questions in lectures.

Teachers' College

I had a year at teachers' college preparing to teach and was introduced to 'new math' by a lecturer who spent many hours apologising for it. We 'did' education, but it had little connection with teaching or learning. Lesson planning and discussing teaching and management strategies were what we found useful. Our planning followed the emerging behaviourism and school 'sections' provided opportunities to try what we planned. The direct instruction that prevailed was hardly different from behaviourism and our teaching was generally acceptable. When pressured I reverted to how I had been taught, and coming from a school with an emphasis on 'doing' academic mathematics, this was not successful.

Teaching

My first teaching position was in the school that I had attended. I knew the staff, the routines, and the trouble spots. I taught mathematics and science and ran extra-curricular activities (swimming, lifesaving, gymnastics, table tennis and computing), and I learnt that respect earned from extra-curricular activities was of more benefit than that earned in class.

The school taught traditional mathematics but I was encouraged to try new maths and taught it for a year to an able year-ten class from a book by Papy (1963) written in French. I had a few language problems, I thought *cercle* meant circle, not circumference then found *disque* meant circle. This experience influenced my use of diagrams, in particular Papy-grams or arrow-graphs as they are generally known, and made me realize that the context for teaching mathematical concepts can be pure mathematics.

I taught applied mathematics (mechanics) and thought it useful in terms of 'modelling'. I particularly enjoyed this with a class who did five years of work in four years. I was humbled each term when marking their examination as I checked my mark scheme against the best student's script and saw how he did things much more efficiently than I did. After six years at this school I met the 'country service bar', so I looked for another position.

At this time in-service education was rarely offered by tertiary institutions. New mathematics was introduced by government advisers, by mathematics associations, by teachers reading overseas journals and textbooks, and by participation at teacher-refresher-courses that were organized by teachers, held in vacation time, and funded by government.

I continued part-time university study while teaching. I thought I had reached my limit with mathematics so started with education, then transferred to commerce. This reinforced my capitalist upbringing, and extended the contexts I used in class from the male 2Ms to 3Ms—machines, military, and money. An incident ten minutes after my final economics lecture is memorable; two of us were in the tavern talking. My friend asked if I enjoyed the lecture, I replied that I had and that many topics from the course seemed to click together. He looked at me quizzically. I asked him how was it for him? His response made me aware of our differences, he said the course had been great, but the final lecture had thrown many doors wide open! (He is now an economics professor in a prestigious US university.)

Teaching Again

To 'jump' the country service bar I successfully applied for the head of mathematics position in a Presbyterian boys school where I was to introduce modern mathematics. In this school the 'ethic of caring' that Noddings (1992) writes about was evident. The Principal knew all 600 boys in the school and their parents, and he taught for half the week. I was new, I was expected to make changes, the department was small, we worked well together, and in my four years I felt I was a colleague rather than an agent for change.

During this time our teacher union published a delightful book (Munro, 1969) that included a set of aims for education. These influenced my thinking, although possibly not my practice for some time. The aims were "the highest value is placed on the urge to enquire, self-respect, and a concern for others". These reflect the academic, personal, and social purposes of school. In hindsight I interpreted these too narrowly. I saw enquiry as cognitive enquiry about mathematics and not non-cognitive enquiry, or values, attitudes, beliefs, or emotions. I accepted self-respect but had not thought about self-knowledge. I interpreted a concern for others at a person-to-person level, but did not extend it to include a concern for culture or environment. I focussed on teaching and was concerned with self-respect and concern for others because they influenced conduct within my classroom.

Teaching Afresh

A new state coeducational high school was to open in 1973 beginning with only year 9 students. Having heard who the Principal was, I applied to be head of mathematics. Here I 'started' to learn about education, and to teach 'students' rather than 'mathematics'. A month before our first staff meeting we were told to prepare for it by reading "Teaching as a subversive activity" (Postman and Weingartner, 1971). Students had no uniforms—they were expected to develop responsibility for dress. Classes were not 'streamed'. Picking up rubbish was not a punishment as if we had pride in our school and environs then we would naturally keep our school tidy. Everyday at lunchtime the Principal delighted in walking around, chatting to students, and picking up rubbish with them. Each year many students had their names put into the 'honours' book but prizes were not part of the school's emerging traditions, all students were growing, and external rewards were irrelevant.

A regular task I had was collating cyclostyled chapters of a draft text for the students and I used this as a detention task for my unruly form class. One young woman always seemed to be in this detention. After a while I spoke to her about this. She said, "it's a game, if I behave badly you put me in detention, then I stay and help sort this paper. That's more pleasant than going home". When I invited her to stay and help without being in detention her behaviour improved. Later the Principal said that school detention was not working as it was always the same students in detention.

Teaching seven year-9 classes in one year nearly drove me crazy. After two months I moved different classes to different topics to keep sane. I had never had unstreamed classes and to cope I designed an individualised programme. I did not know how important social interaction was and only later restructured the course so groups progressed together.

School organisation had three foci—curriculum, guidance and administration. I was involved with curriculum and administration and came to appreciate the complexity and the interconnectedness of these. I valued the way the Principal delegated tasks, he did so completely and with trust. After four years in this school a state alternative high school was opening and my Principal encouraged me to apply for the Director's position.

Alternative Teaching

As Director/Principal of the alternative school I thought I had made it. Here was the opportunity and the expectation that one would be innovative. A school meeting, based on "Summerhill" (Neill, 1962), was established for decision-making. My role was to provide some input and ideas to meetings, and to implement their decisions (if they were legal). The staff including myself, students, and parents began to look at education more broadly, not just subjects. One result of this was that my awareness grew in terms of broader interpretations of the urge to enquire, concern for others, and self respect. The school had been conceived as a "school without walls", but most students wanted a place they could identify with and this was provided.

I had a supportive staff who shared the running of the school and I taught for 15 hours each week. Many students wanted traditional subjects, but in non-traditional ways, and we offered many non-academic learning activities. The school year was divided into six blocks. Students chose their courses each block and courses had to require as few pre-requisites as possible. As the maths teacher this meant ensuring that students could opt in and out, so I modified my individualised programme and encouraged students to work in groups. Assessment was not high stakes, we believed that students were responsible for their learning, so each six weeks they filled out their reports and staff countersigned them.

The school population was bimodal—some students and/or their families sought alternative education (whatever that meant), others were disenchanted with traditional schooling. Many parents expressed concern that their child might come to school but not attend class. I shared my feelings—I had two sons at the school, one did what parents hoped for, the other participated in numerous work-experience classes but avoided most formal classes in his three years at the school. I felt that formal academic classes were only one part of education. I saw schools as a safe and supportive space where students could learn by making mistakes and reflecting on them without fear of punishment.

There were no separate staff facilities and rules applied to both staff and students. The relationships between staff and students were close and we were all on first name terms. I heard of one ex-student who knew me as Andy and asked who Mr Begg was.

While the school was alternative, its value remained if it continued to experiment, push boundaries, and legitimise less radical innovation in other schools. Some other schools began doing some things we were doing (although not necessarily because we were doing these things), but others became more conservative and told parents 'if that's what you want, go to that other school down the road'. While I was at the school the politics of education had not swung to the right as it did later and when it did many of the 'liberal' activities were harder to achieve within the regulations. I enjoyed being at this school for nearly seven years, towards the end I felt we were not breaking new ground, I assumed my 'marginal utility' was decreasing and a new Director would bring in fresh ideas. Then, wondering about my future, I heeded a colleague's advice "to stay in education you need to be decontaminated" and sought a position in the Government's Curriculum Division.

Textbook Writing

While teaching I was involved in resource development, I had been with the NZ Mathematics Magazine from 1964 and in 1965 began co-authoring textbooks. At the time most schools taught arithmetic, algebra and geometry from separate textbooks, decimal currency was coming, and a need was seen for a new arithmetic book. My colleagues at my first school and I wrote this and continued with a trigonometry book. My next writing started because of my interest in statistics. I was asked to write chapters for two year-12 new mathematics books, and we went on and co-authored text/work books for slower students. The third phase was with authors that I had brought together. We co-authored a second generation of New Zealand texts for new mathematics, and to keep out of mischief, I published them—this is known in the trade as 'vanity press'.

At the time I believed that textbooks made a difference. I enjoyed the 'guru' status, the supplementary income, and the profile I gained from writing. I became aware of the awful 'power' of resources when students in class would not change a wrong answer in the back of the texts because 'the text book must be right', and when teachers wanted their planning done by the text rather than thinking through the issues for themselves.

Head Office

My next job was as a public servant/bureaucrat. I stopped writing textbooks because of the conflict of interest, but there was writing and editorial work to be done with curriculum documents and teachers guides. In 1984, my first year in this job, ICME was held at Adelaide and this started my overseas conference addiction.

The curriculum project I led was for years 11 to 13. It was planned to follow a 1–10 compulsory curriculum but to provide flexibility for alternative courses in the last three years of school. Over five years teachers thinking moved to a general acceptance of most of what the project team suggested, but a restructuring of the department, the closure of the curriculum development division, and the emergence of a new Ministry of Education with a different philosophy on curriculum overtook our initiative. Our curriculum was never implemented and some colleagues expected me to be disappointed—my feeling was that curriculum is a process not a product and that some people who would be involved in the next project had benefited from their participation in our one.

I was also involved with mathematics in the context of Maori education. This arose as part of the emphasis being put on Maori education and also my interest in ethnomathematics from ICME. My main role was getting funding so groups could meet but I took part in some of the discussions and learnt a lot from this participation.

An incident occurred while I was representing New Zealand in Japan at a three-week workshop. The sessions were in English so that English conventions would prevail and younger people could question and disagree with older participants. Outside the conference I hardly heard any English. Over the three weeks I noticed that my perceptions changed, I saw things differently and was seeing more and was surprised by this altered perception.

Another incident I remember involved a teacher writing letters to influence policy matters although he had only made a minimal submission when given a chance to comment. Identical letters were sent to the Prime Minister, the Minister for Education, the Director General of Education, the Head of the Curriculum Development Division, and to me as Mathematics Curriculum Officer. Within a week the five letters were on my desk awaiting replies. I knew the teacher so I succumbed to temptation. I replied to the first letter on behalf of the Prime Minister, xeroxed four copies, then with a pen crossed out 'Prime Minister' and wrote the other titles. I stapled the five together, put them in one envelope and posted it. Later I met the teacher, he saw the funny side of it, and we discussed the incident and other ways of working.

In hindsight, a concern with the curriculum division was its structure. It ensured a continuation of a subject-based approach in schools. Some work was done on a curriculum framework but the Western partitioning of knowledge by subjects was never questioned. This may disadvantage students from other cultures, and reinforce the false dichotomies set up by Descartes that I am uncomfortable with. I found the curriculum division personnel were like high school heads of departments, and colleagues at university—concerned about their subject and status, reluctant to cross boundaries, cooperate fully, or question assumptions or bosses. (I wonder what this says about our education system.)

University Teaching

In 1989 after the restructuring of our educational administration I was to be out of a job so I looked for a fresh challenge, and obtained my position at University. In this I am involved with graduate courses, research and supervision and not pre-service teacher education. I was appointed because of my background but had to upgrade my qualifications. I had perceived little take-up of innovations and curriculum during my time in head office so I chose professional development for my research topic. This made me aware of many misconceptions I had about teacher learning. Upgrading my qualifications also gave me an opportunity to design my first research project, and to become aware of how teachers feel when they return to study after some years.

Our centre was set up for education research in science, then expanded to include mathematics, technology and computing. Unfortunately there is little cross-fertilization between the disciplines and the centre is mainly an administrative unit with distinct subject foci. As I was teaching the first maths education masters papers in New Zealand it was important to network and I am thankful for the help I was given by colleagues from this association. This need also provided opportunities for me to satisfy my 'conference habit'.

Our research methods class and the approaches used in student research often remind me of the differences between my friend in year 3 economics and me. Many students think the course provides them with specific ways to do research, while I think it introduces methods that need modification for particular situations and it is intended to open doors for them. So far the ones that have been the least bound by the course are those from overseas who recognise cultural inadequacies in standard approaches.

An aspect of university study that concerns many students is that they think they are to learn what others think (and cite the sources of such ideas), but are not allowed to have their own opinions. This is unhealthy and unproductive. To encourage people to enquire and to foster their self respect, we need to pay attention to what they think, and my courses need to change so this happens more. An example of this from a school arose with two teachers from Fiji. Their problem and discussion went something like this—a mother has four children, they are all girls, she is expecting another, what is the probability that it will be a girl. One teacher said that her class had discussed this in biology and were confused. The other said that she knew that the answer I wanted was 1/2 but that the sex of a child is decided by God, not by probability. This reinforced for me that there are many viewpoints, that mathematics should be tentative when it presents a model, and we have no right to undermine models from biological or religious education.

An incident on sabbatical in Sweden at Luleå was important for me. I was researching science museums. I had been observing visitors at Teknikens Hus and was impressed by the quality of interaction with exhibits that was different from what I had seen at other interactive science and children's museums. I sensed engagement rather than the enjoyment and entertainment that I saw elsewhere. I was interviewing the Director and she told me that she had visited many other museums and had seen how they took a 'big idea' from science or mathematics and developed an exhibit to demonstrate it. She had not studied education but had heard the expression "start where the learner is". On this basis the exhibits at Luleå had been set up starting with familiar things that visitors knew, and then problematising the situations, and moving towards the underlying 'big ideas'.

At university we are getting more overseas/international students. The assumption seems to be made that if these people come to our universities then they want our degrees. However, fundamental to education is the notion that was reinforced for me at Luleå, one should start where the learner is. My concern is that we know very little about these students—their languages, beliefs cultures, attitudes and values, ways of thinking, and what they bring to education. Many are Asian and there are books that might help (for example Bond, 1996 and Watkins and Biggs, 1996), but how many of us have read them?

This increasing number of overseas students is partly due to an enticement policy to which the euphemism globalization is sometimes attached. Whether the real aim is international aid or whether it is pure greed is debatable but I wonder if it is not just a new form of colonization. I think that rather than working towards globalization we should celebrate cultural diversity. Diversity and differences seem to lead to new understandings, they make new syntheses possible, and they open the door for creative ways of seeing things. If we are to help people from other countries then we need to help them set up more tertiary institutions, or at least find their needs rather than satisfy our own.

After finishing my doctorate, I enrolled in an art class. At session one the tutor had us draw a boat, a house, and a person. He came round and gave us each a mark between 8 and 18. I scored 8. He then drew a series of pictures of boats, houses and people and labelled them all 8, 10, 12, … 18 and told us that these were typical of drawings by children of these ages. He suggested that we would have been these ages, when we had been told we could not draw. Eight weeks later, with 2 hours each week, we were all drawing pictures of each other and of other objects. The tutor started by getting us to draw what we saw instead of what we thought we saw. He had high expectations of us and we met his expectations. Reflecting about this class has been valuable. It was based on non-cognitive rather than cognitive knowing with many techniques intended to make one focus on the unfamiliar. Secondly, art is the construction of representations (models, metaphors?) of reality, and this is similar to mathematics. Thirdly, we became totally involved in the activities and time in class just whizzed by—I wonder why does this not happen in the same way when I am teaching mathematics? I am sure that I have lots to learn from how people engage with and learn other subjects and the separation of mathematics education from other forms of education may not be helping this learning process for me.

Missed Opportunities and Lessons Learnt

From my experiences in these work situations I have become aware of some of the mistakes I have made, or put less negatively, some of the opportunities I have missed. Before looking forward, I want to briefly summarise these:

- My jobs (and aspirations) have actions implied by their names— teach, write, develop the curriculum, lecture, profess. I have done these things rather than facilitate learning.
- I do not remember being taught how to learn, nor was this a focus in my teaching.
- I have not often used student-chosen extended projects for study in mathematics. (Nor one problem per period that Stigler (1994) has discussed from Japanese classrooms.)
- My emphasis in teaching and resource production was the subject not the student.
- I have focussed on subject specific outcomes rather than on students' broader concerns, and have opened doors to specific pathways, instead of to many opportunities.
- My teaching focussed on learning objectives rather than on learning activities and I lacked an understanding of how learning occurs co-incidentally.
- My expectations of students have often not been high enough.
- I accepted the need for assessment when it was not contributing positively to learning.
- I assumed that punishment had a place in education.
- I have not always had an adequate idea of what my students already knew, what their interests were, or what might be suitable contexts in which to put my teaching.
- I have questioned the partitioning of knowledge by subjects but I did not seek ways around this problem at the alternative school or in the curriculum division.
- I have not adequately dealt with the complexity and interconnectedness of knowledge across subject boundaries or to the other aspects of the lives of students.
- I have concentrated on operationalising the aims of mathematics education at the expense of the more general aims of education.
- I assumed that analytical and logical thinking are the most important ways of knowing.

I would not want anyone to get the wrong impression. In spite of these many missed opportunities (and these are only the ones that I have time to recount), I do look back on my work to date with satisfaction and a feeling of achievement. I am also very aware of some of the lessons I learnt, these include:

- Teaching (and supervision) involves learning by all parties involved.
- Respect for students should be automatic; respect for teachers needs to be earned.
- Authoritarian schools and classrooms do not help students learn self-discipline.
- To gain power, the best thing to do is to give it away.
- Students of all ages want to learn but many have been taught not to.
- Teachers make a difference, success makes even more, and teaching needs to change so all students experience meaningful success.
- Teaching is neither a necessary nor a sufficient condition for learning.
- Assessment and evaluation of students and teachers do not contribute to learning.
- Students of all ages need to be trusted. Learning to trust and to be trusted sometimes involves making mistakes. We all need a second chance, a third one, a fourth one, … .

- Students, parents, teachers, and bureaucrats all know what is best, but they can't agree. In this situation the views of the learner deserve to take priority, and the convictions of the teacher should take precedence over curriculum and/or regulations.

Part 2: Looking Forwards

Changing direction? Looking forwards is not dichotomous from looking backwards. My past, my missed opportunities, and my lessons learnt influence the things that now interest and concern me. In linking these I will use four interrelated foci implied by our association's name—mathematics, education, research, and community.

Mathematics

We often think of mathematics as hardly changing. In the six decades of my life there have been many changes in school mathematics, see Table1.

Table 1
Six Decades of Change in High School Mathematics in New Zealand

1940s	- leaving age raised and some high school education compulsory - mathematics becomes part of compulsory core in education - 'core' mathematics introduced for non-academic students
1950s	- change from Euclidean to coordinate geometry in senior mathematics - calculus introduced into school mathematics
1960s	- growth of mathematical associations (and mathematics teacher journals) - disappearance of 'core' mathematics and growing acceptance of 'maths for all' - introduction of 'new maths' (with an emphasis on sets, logic and structure) - start of computer clubs in schools teaching simple programming - introduction of transformation and vector geometry - decimal currency introduced - applied mathematics (mechanics and statistics) replacing mechanics - experimental approach advocated for senior statistics
1970s	- metric system replaced imperial units - more emphasis on practical work at all levels - increasing emphasis on calculators and then on computers - mechanics returned to physics - increasing acknowledgment of needs of less-able senior students
1980s	- computer studies replaces computing taught within mathematics - increasing emphasis on mathematics rather than arithmetic in primary schools - swing back from the excesses of 'new maths' - project work emphasised (and used to supplement external assessment) - trend to internationalisation (Cockcroft Report, 1982; ICME Adelaide, 1984)
1990s	- mathematics positioned within a curriculum framework - statistics included at all levels of schooling - emphasis on ethnomathematics and Pangarau (mathematics for Maori in their language) - increasing emphasis on processes (doing) as well as on content (knowing)

In considering the future we need to ask some questions. What mathematics should be taught, is less mathematics required as society relies more on technology, if mathematics is useful in contexts, should it be taught in these contexts? In answering these questions, who should have a say; whose mathematics is it; and should we listen to the voices of students, of our communities, or only of educators? Should we focus on Western mathematics? What is the future of mathematics in education and society, what sort of society do we want, and can we separate school from society? Groups such as Radical Statistics work for change in society, should we do the same? If we do not, will mathematics be the Latin of the 21st century, will it be important or impotent? The answers to these depend on our aims for teaching mathematics and our philosophies of education, but do we debate these enough? Scholarship is needed to move past the thinking of Descartes and others, to address basic issues about society and what we value, and to make our assumptions problematic.

Education

Historically psychology broke from philosophy, then education splintered from psychology, now mathematics education is separate. With the notion of 'making connections', is this useful? When I was at teachers' college the foci were mathematics and education, now they are more complex. Theories about knowledge, learning, teaching, curriculum, and assessment are not always consistent; lecturers espouse theories as if they are only relevant to their subjects; prospective teachers do not seem to construct generally applicable theories nor develop personal theories. "What's the Use of Theory" was asked by Thomas (1997), and deserves consideration. For me a theory is a model, it provides a way of looking at something, it is not reality, it is a metaphor, a representation or mapping of reality, and like all metaphors, it highlights some things and ignores others.

While wondering about the need for theories, I find myself wanting them. This may be because of my background and my lack of experience with experiential education and mindful awareness that has been written about in phenomenology (Merleau-Ponty, 1962), in education (Varela, Thompson and Rosch, 1991), and in mathematics education (Davis, 1996). Such ways of knowing were highlighted in the sixties when Eastern philosophies were gaining recognition. More recently Belenky, Clinchy, Goldberger, and Tarule (1986) stressed gender differences with separated and connected knowledge (that also emerge with some non-Western people), and Mason and Spence (1999) have discussed knowing-to as distinct from knowing -that, -how, and -why. I think that these notions need consideration.

The word education comes from the Latin *educere* which means 'to lead forth'. Do we know where we are leading our students? and are we leading them or telling them? We can only theorise about learning, teaching, curriculum, and assessment when we know what we (and the community and politicians) mean by education. We need to work towards what we believe in and be explicit in terms of our assumptions so those concerned understand where we are coming from even when they disagree with us.

When looking for theories we are influenced by the accessible Anglo-American ones, but there is much elsewhere. When behaviourism was 'in' we took little notice of Gestalt education which was written about in German. With constructivism Vygotsky's work was not translated for years. In the same way much has been thought about in the East that we ignore. As Engler (1984) wrote regarding Buddhist psychology, when talking about the "I":

> ... the self, as it has been understood from the perspective of newer developments in ego, object relations and self-psychology, is now seen to be exactly what Buddhist psychology and practice have always said it was: namely, a mental representation or construct, not an entity.

Related to ways of thinking is language. Looking at some differences between languages in the East and West highlights some of the things that reinforce the dualism of Descartes. Some comments from Kelman (1958) illustrate this:

> What characterises the East is the subjectifying attitude, the West, the objectifying one. Eastern cognition is interested in consciousness itself. Western cognition is interested in the objects of cognition. ...

> East and West differ in their attitudes towards time. ... The West attempts to define it explicitly, to oppose and dominate it, only to become its victim. The east shows pronounced discretion towards time.
>
> Our language is noun-oriented. We make propositions about things. Languages which are verb-oriented make propositions about events. They are more suitable for communicating immediacies. Our language, subject-predicate in form, creates a dualism ... Process languages facilitate experiencing. ...

The East has much to offer and in understanding their contributions, perhaps we will gain an understanding of our international students who come steeped in different traditions. There is considerable research on learning and teaching, but little on curriculum. Teachers see curriculum as beyond their control—nothing could be further from the truth. Curriculum depends on teachers. They implement it by modifying official versions to get teachable versions. They operationalise it by finding what works, they put ideas that advance it into school versions and into the next rounds of official curriculum writing. I would like curriculum reconceptualised as 'process' rather than as 'product', a shift from specific objectives to more general ideas and their interrelationships, and for flexibility to be built in so that innovation and local needs are encouraged. I would prefer official documents to be used for checking to ensure coverage rather than for planning. Associated with this is assessment—while learning and curriculum have moved past behaviourism, summative assessment has not. In some ways it is used for motivation with its emphasis on learning for qualifications rather than for delight. I feel that learning, teaching, curriculm and assessment need to be much more integrated.

The work on learning theories has been divergent. We have a range of constructivist theories according to one's emphasis on individual or social, and ones view of knowledge as being objectivist or relativist. If one considers enactivism (Davis, 1996; Sumara and Davis, 1997), which is an emerging theory that moves past constructivism and takes a more holistic view of knowledge, these differences can be extended to include cognitive and non-cognitive knowing. Some influences on enactivism are from:

- phenomenology (Merleau-Ponty, 1962);
- ideas about Cartesian dichotomies (Damasio, 1994);
- non-cognitive knowing (Maturana and Varela, 1987), which includes 'mindful awareness' from Theravadin Buddhism (Nhat Hanh, 1987), from Zen (Batchelor,1999) and from other Eastern thinkers (Krishnamurti, 1956);
- neural biological work which emphasises evolutionary or Darwinian notions (Edelman, 1987; Plotkin, 1998; Sacks, 1995; Varela, Thompson and Rosch, 1991);
- systems theory (Bertlanffy, 1968); and
- notions of embodiment (Varela, Thompson and Rosch, 1991; Lakoff and Johnson, 1999).

I can only give a hint of what enactivism means for me and will do this by providing some ideas from Dawson (1999), but I suggest that we need to familiarise ourselves with this theory. Enactivism involves becoming aware of what you are doing without judging it, and it moves *from a culture based on judgement to one based on possibility*. It requires us to see knowledge not as independent of individuals and their environments, or as something that can be tested/matched against external standards, but rather as embodied action with all of us being responsible for our actions. Enactivism and embodied action mean we maintain the *yin-yang* of self and world, and of ideas 'out there and 'in here'. From this perspective the aim in teaching is not to link learners' experiences to some external curriculum, *but to view the curriculum as being occasioned by the learners' experiences in their school environment*. Such ecological perspectives are located within a complex web of relations with all *decisions and actions being both constrained by and influencing all nodes of the web*. The enactivist classroom could be construed as a dynamic system with *the teacher listening not to check or* *model the students*, but *to participate* with them (Kieren, 1995). Whether one accepts enactivism is not the issue, consideration of alternatives shows how ideas are added to the field we started to explore two decades ago with constructivism.

I have three other concerns. With lecturing and supervision, little has changed; and some on-line alternatives seem no better. Secondly the affective domain—McLeod and Ortega (1993) noted the role of emotions in learning mathematics before Goleman (1996) wrote on emotional intelligence, but are we doing enough? I used to think that those of us drawn to mathematics rather than the humanities were the cold, rational and unemotional types, but looking around this room I do not have evidence for that hypothesis. Thirdly, having enjoyed working in an alternative school, I am heartened to see people addressing the challenges from critical mathematics education (Kietel, 1994), liberatory pedagogy in mathematics and teacher education. (Price and Ball, 1998), and alternative education (Uhrmacher, 1997). However, my concern is that most teachers and policy makers do not heed these challenges.

Research

In discussing research in mathematics education I will mention some concerns I have, these are from the past but need attention in the future.

Firstly where do we see research sitting with respect to theory and practice? I used to think of these aspects of our work as vertices of a triangle with the links between them being the sides of the triangle. The lengths of each side varied according to the relative importance I gave the links, but this image distanced the aspects from each other. I prefer sides of zero length and to think of the aspects as constantly interacting parts of a whole.

This leads to my second point, how broadly do we conceive of research? For some there is little acknowledgment of scholarship, and does scholarship include creative work? The New Zealand Qualifications Authority (1995) believes it is and that we also need to recognize consultancy and professional practice as often equivalent to research. I think we should take this more inclusive view of research in our discipline.

My third worry relates to scholarship. At conferences speakers talk of people like Gadamer, Heidegger, and Merleau-Ponty. Some attendees do not link these with our discipline but instead, they quickly back off. This seems to me to be an odd reaction when there are new ideas to engage with. Have we learnt the caution that perhaps we teach our students, are we afraid to take risks? How can we encourage others to inquire if we do not? In my case I know I am often unfamiliar with such people's work, but a conference of peers, like an alternative school, is a safe place to make mistakes. Why should we worry if our questions are naïve? They are often the questions others wanted to ask!

A fourth concern is to do with analysis and synthesis. In schools we analyse to get our objectives instead of synthesising for general ideas and their interrelationships. Research seems similar. Bishop (1998) made this point when commenting on research at an ICME conference. We seem to become so concerned about details that we ignore the total situation—we concentrate on immediate aims and forget about long-term goals. This emphasis on analysis may reflect Western ways of thinking and may make it harder for non-Western people to value our research.

My fifth notion is to do with language. Phrases like "the researcher found" are often used, but people do research. A person selects a topic, decides what data will be relevant, makes decisions about the sample, decides what will be asked or observed, selects from the data, and interprets it with this interpretation being based on prior experiences. Research is personal, we need to be honest and use "I" in our reports.

My sixth idea also relates to writing up research. The usual form of a thesis or report follows a traditional format. It was exciting for me to hear O'Reilly (1998) share with us how she had written her doctoral thesis using 'plateaus'. Later I read some of Deleuze and Guattari (1988) for a background on this term. Instead of focussing on findings, plateaus focus on the process and the changes that occur in the researcher's thinking while involved. I think this alternative is to be commended and we need to consider other possible formats.

My seventh concern is about what we measure and why. In economics the easy thing to measure is financial costs and we avoid social, personal and environmental costs that can not easily be put into dollar terms. The situation is similar in research. As Giorgi (1970) said, the problem is that for something to be measured only its tangible aspects can be apprehended, and, as a result, the indices associated with these aspects of the phenomenon become more important than the phenomenon. In the search for the measurable element one loses sight of the significance of the other aspects of the phenomenon. The non-measurable aspects tend to be discarded, and only the measurable elements are included.

My eighth worry is to do with time scales. I have been 'guilty' of collecting data and quickly interpreting it. The alternative is to take time living with it for a while, getting the feel of it, and interrogating it. Merleau-Ponty (1962) talks of the need to not merely describe what one perceives in a superficial way, but to become more mindfully aware of what one perceives. This for me relates to time for research and different ways of knowing.

My ninth point is why do research when we know that educational policies are not research-based? 'Policy' shares its origin with 'politic' and educational policies are political (and economic). Mason (1998) suggested that the fundamental purpose of research is the growth of the researcher. I believe it is this and more, research is finding out for ones self, which is the purpose of education. Our emphasis should be on this process rather than on outcomes. Currently primary school students do lots of (research) projects, high school students seem to do fewer, and undergraduates even fewer—they should all be doing lots.

Three final comments on research: I know we want independent researchers, but sink or swim is not the way to achieve this; we need to be more concerned about research supervision. Next, we have much in common with researchers in other disciplines and we need to have more contact with them. I would enjoy joint conferences where mathematics educators and people concerned with other subjects discuss research together. My last comment is a generalized finding from science to keep in mind, it is Newton's third law for researchers, "for every researcher there is an equal and opposite researcher".

Community

Group, Australasia, incorporated, and association imply collaboration, collegiality, and cooperation but these can not be assumed to be valuable. Collaboration can mean *working traitorously with the enemy*. Hargreaves (1994) talks of 'contrived collegiality'. Ball (1999) spoke of fabricating cooperation by putting more names on papers to increase research outputs for the bean counters (and using the salami methods for getting more papers off a project by cutting the slices very thinly). Politicians would argue for competition.

While we see ourselves as a community, we are also part of a larger community of educators, and of the society that we live in. Developing community requires us to listen to all the voices from this society because, although we might think that they are uninformed or conservative, it is their children's education that we are influencing.

Is the word community a verb or a noun? For me a noun suggests a set of people (peers, students, supervisors), but a verb implies an interacting between the people.

One final comment—incorporated means brought together into a larger body, and often this is to avoid (financial) risks. This suggests that we either fear taking risks, or want to take risks but avoid dire financial consequences. I hope we are in the second category.

Conclusion

Perhaps this sounds like a sermon with me telling you things you know already or would rather not know. This was intended. My professional responsibility is to keep stirring; although I wonder if this sermon will make any more impact than others you have heard. Our work often seems confined by boundaries and limits, but I believe we impose many of these on ourselves. I see taking risks not as putting our heads on the chopping block, but rather as flying free. I hope that in some small way I might have 'opened a door' or boosted someone's confidence to do something differently. Kia ora katoa.

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Bridging the Gap: A Challenge for the Dual Community

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The 'research-practice gap' has its roots in the concept of Technical Rationality – the 'Positivist epistemology of practice' (Schon, 1987). This is interpreted by some in mathematics education as the hierarchical division of labour, wherein it is the task of the tertiary-based mathematics educator to create the fundamental knowledge and theories, and the task of the mathematics teacher to apply that knowledge to practice. This paper considers the issues that arise from this demarcation, highlights the problems it creates, and suggests ways to alleviate these problems.

Four years ago, just around the time that I had been elected President of MERGA, I was privileged to listen to a talk by Tom Koballa, then President of NARST – the US-based National Association for Research in Science Teaching. He spoke about what he called the 'research-practice gap' – the discrepancy between what is known about teaching and learning, and what actually goes on in the science classroom or laboratory. Tom hadn't coined that term – its origin is debateable, but it is most often attributed to F. Weinert and his colleagues in a 1989 article in the *International Journal of Educational Research*. Undoubtedly other educators had considered this phenomenon earlier – David Schon in 1987, in his book *The Reflective Practitioner* for example. Even prior to that, as far back as 1974, Graham Nuthall when writing in the *New Zealand Journal of Educational Studies* had asked his readers a question that touched on this issue: "Is classroom interaction research worth the effort involved?". His advice to us at the time was that we should not answer the question too rapidly nor too glibly.

As I listened to Tom, I remember thinking to myself that we mathematics teachers in Australia were better off than our US-based science colleagues as far as putting the results of research into practice. I was aware of the excellent professional development opportunities available to Western Australia's mathematics teachers and those in other states too, and I was also aware that, in WA at least, a significant proportion of the mathematics teachers in the State attend the annual conference of the Mathematical Association where a considerable proportion of the discussion involves mathematics education research. I really did feel complacent about the situation at the time. I've thought about this issue as it applies to mathematics education since then, and it was at the centre of my thoughts throughout my term as MERGA President. I'd like to share a few of my ideas about this matter with you.

What does mathematics education research really achieve for us? Despite Graham Nuthall's advice, here are a few glib suggestions: It can bring us recognition from our colleagues. It can win us awards. It can even get us promoted. But apart from these self-serving gains, what does research really achieve for the teaching and learning of mathematics? Over the last 18 months I've taken the opportunity to ask this question of a number of secondary, pre-service and cooperating teachers with whom I've been working. There were many positive comments, but this is what some of them had to say:

> I guess that I've always assumed that the research part of teaching the maths curriculum has been done for me already by the curriculum people, and built into the syllabus and the guidelines they give us. I've never been conscious of research impacting on my day-to-day teaching. (Greg, Western Australia)

> Maths education research should be useful to me, but it really isn't. It looks at what teachers should do to make maths better, but ignores what's really going on in the classroom – irregular attendance, lack of motivation, no application among kids. (Cathy, New South Wales)

> Research results don't reach maths teachers, or it doesn't reach us in a form we can use. (Diana, Queensland)
>
> Maths education research isn't useful to everyday teaching. Most of it isn't practical – it doesn't translate easily into working ideas. (David, Tasmania)
>
> Maths education research is self-perpetuating, (and the) ill-founded study of very little that is applicable to 'real' teaching. (Margo, South Australia)

These responses surprised and disappointed me. I now believe that there are many mathematics teachers in Australia and around the world whose answers to the question of what mathematics education research really does for them would be in much the same vein. The literature supports my belief also:

> Research in mathematics education is of limited value unless it affects classroom practice and experience. (Hatch & Shiu, 1998, p. 297)
>
> Most educational research is perceived by (mathematics) teachers as irrelevant to their daily working lives. (Cochran-Smith & Lytle, 1992, p. 304)
>
> There is a growing concern that the voices of classroom teachers are absent from published accounts of educational research. (Patterson & Thomas, 1993, p. 1)
>
> There is a widespread view that educational research is of little relevance to practice. (National Board of Employment, Education & Training, 1992, p. 9)
>
> Teachers often see the proposals for change made by others as 'frivolous' when they do not actually affect their working constraints. (Bishop, 1998, p. 36)
>
> Researchers (need) to embed their work in a practical theory of pedagogy, one which takes account of the real world of teaching. (Chambers, 1992, p. 245)
>
> Research results are rarely functional in, or applicable to, real classrooms. (LeCompte & Preissle, 1993, p. 355)
>
> Firstly, teachers and researchers do not conceptualise teaching in the same way. They live in different intellectual worlds and so their meanings rarely connect. Secondly, the usual form of educational research - the psycho-statistical or agricultural-botany paradigm - has severe limitations as a method of construing and making sense of classroom reality. For these two reasons, teachers and others concerned with understanding classroom life have increasingly adopted different approaches to classroom research. (Hopkins, 1993, p. 40)

These eight quotations covering the last eight years mirror the discrepancy that Tom Koballa was at pains to describe – the 'research-practice gap'. Boostron, Jackson and Hanson provided a wonderful analysis of this phenomenon in a 1993 article in the *Teachers' College Record*. The 'Gap' has its roots in the concept of Technical Rationality which Donald Schon (1987) calls the 'Positivist epistemology of practice'. In mathematics education, technical rationality is seen in the hierarchical division of labour wherein it is the task of the tertiary-based mathematics educator to create the fundamental knowledge and theories, and it is the task of the mathematics teacher to apply that knowledge to practice. The mathematics education research community's historic adherence to the model of technical rationality has led to the situation where rigour and relevance are often at odds. As mathematics education researchers, we have tended to investigate problems framed by research-based theory that utilises a quantitative and/or qualitative methodology – the hallmark of technical rationality. Unfortunately, teachers see many of the problems that we investigate as irrelevant, impractical and out-of-touch with the actual classroom of today's schooling. Not that the actual research is considered unimportant or lacks rigour: Rather the research cannot be implemented as recommended because of the constraints of the classroom.

Virginia Richardson (1994) distinguishes between formal research and practical inquiry. The latter falls outside the model of technical rationality. According to Richardson (1994, p.5), 'practical inquiry (is) undertaken by teachers to improve their practice, while formal research (is) undertaken by researchers to ... contribute to an established and general knowledge base'. She contends that it is practical inquiry and not formal research that provides teachers with the knowledge that they need to address their immediate day-to-day classroom problems. Even as our research shifts more and more towards the hermeneutic purpose of understanding how mathematics teachers make sense of teaching and learning, we cannot but agree with Richardson (1994, p.6) when she observes that it is unlikely that we will be successful in developing a formal knowledge base that responds to the immediate day-to-day needs of mathematics teachers.

I can illustrate this point with an example. Some time ago I spoke to a pre-service teacher after having observed him teaching a mathematics lesson. I asked him about a number of students who had sat with their heads buried on their arms and otherwise disinterested throughout the lesson. He explained that that on one day those students would be totally engaged, while on another day they were as disinterested and as unmotivated as I had observed. Getting these students regularly on-task was among the teacher's priorities. My knowledge of mathematics education research, or for that matter educational research in general, was of little help in terms of offering advice to this teacher on how to get those students involved regularly for the entire class period.

If one acknowledges that our needs as researchers to address the more general conceptual questions of mathematics teaching and learning are not the same day-to-day needs of mathematics teachers, then one must also acknowledge the existence of two mathematics education research communities. As MERGA members, we are well acquainted with one of these communities, namely the one which engages in formal research on such topics as the discourse that mediates mathematics learning, the obstacles to mathematics education reform and the role of persuasion in mathematics teaching. This community shares the outcomes of its work at the annual MERGA meeting, among other places.

Such an association, however, does not necessarily represent the other research community that does not operate as part of a hierarchical structure. In that other community, teachers are the researchers, the recipients and the consumers of their own research. Their research activity is not defined in terms of rigorous methodologies and well-articulated theory. (The idea of evidence-based practice is one that informs many other professions – for example, medicine). Members of this other community exchange their research findings in the staffroom and corridors of their school between classes, often as anecdotes and stories, and they evaluate the quality of their research in a very pragmatic way – in terms of what works with their students. However, one does not have to be a primary, middle school or secondary mathematics teacher to be a member of this second research community. All of us who teach mathematics and mathematics method courses also engage in practical inquiry – we ask questions about our practice from our colleagues who offer suggestions for improvement. If the suggestions work, they become part of out teaching repertoire; if not, the suggestion is discarded or modified to make it work.

I should mention at this point that, quite recently, some questions have begun to be raised about the status of the knowledge produced and published through teacher research (Kilpatrick, 2000; Cochran-Smith & Lytle, 1999; Fenstermacher, 1994) and, in particular, whether that knowledge should be included in the same category as traditional academic research knowledge. The debate appears to hinge largely on the nature of the evidence offered in support of teachers' narratives that seem to constitute the major way of reporting teacher research nowadays. Jeremy Kilpatrick, while delivering a keynote address at a conference this year (Kilpatrick, 2000), stated that although 'teachers as researchers' has become an important mantra in many programs of teacher education in mathematics, such research has not had much impact on the larger community. I'd respond to this statement by saying that it is impossible to gauge how much research teachers have conducted on the teaching of mathematics, and I believe that there is ample evidence to demonstrate that teachers' research is having considerable impact locally, even if it doesn't have a high profile when viewed from outside the profession.

Jeremy's statement was made in the context of his somewhat controversial argument that the mathematics education community simply does not have good evidence to support its claims regarding the need for any sort of mathematics education reform, owing in large part to its move away from research that might have provided such evidence, namely 'well-controlled experimental and quasi-experimental research – the building blocks of scientific knowledge about teaching and learning' (Carnine & Gersten, 2000, pp.139-140). This is not the place to discuss the merits or otherwise of his argument – that we should shift back to more quantitative research-based paradigms – but it is interesting to consider the impact that such a move would have on the way that teachers view what we do, and how their reception of us into their classrooms might change as a result. In a period when it is becoming increasingly difficult to gain access into classrooms to conduct research, our position might well deteriorate further if we were to insist on, say, conducting long-term, large-scale formal experimentation. Reversing the shift in current methodological approaches this way would certainly highlight the gap between the evidence-based research of the teacher and the form of research they observed us implementing. Regardless of the research methodology we mathematics educators adopt however, formal guidelines for teacher research are being developed and promulgated by educational authorities in various English-speaking countries.

The Association for Science Education (ASE) in the UK recently set up a research group to promote teaching as 'a research-based profession', and it aims to 'promote an evidence-based culture'. In the first of a series of articles in *Education in Science* (September, 1999), Ratcliffe and Wellington looked at what they believed counted as science education research. They saw teachers becoming involved in conceptual research that built theory and cleared away the undergrowth so that the main ideas stood out. Such work might involve critiquing previous studies in a particular area. However empirical research is what we generally think of as research; it collects data on how and why something is happening, either by intervening as little as possible so that the situation can be described as it is (a case study), or by planning and implementing an intervention and describing its effect (an evaluation). Where teachers become engaged in the process of improving some aspect of teaching and learning through a cyclical process of questioning and making changes to their practice, the research is action research. The ASE urges teachers to become involved in any one of these activities – action research, evaluative research, case study research or survey research (using questionnaires).

With this science education initiative in mind, I was a little disappointed after browsing through the two-volume tome published in 1998 by Kluwer – *Mathematics Education as a Research Domain: A Search for Identity* edited by Anna Sierpinski and Jeremy Kilpatrick. While this work represents the outcome of an international study of the state of the field of mathematics education research - particularly as it relates to hard-core mathematics - the title of the work virtually proclaims the identity crisis I refer to in the title of this presentation. Of the 33 expert papers published in the two volumes, only four (including one by Alan Bishop) directly discusses the relationship between theory and practice (Alan makes the point in his chapter: "the lack of relationship between research and practice is well documented" (pg.35)). Gillian Hatch and Christine Shiu's article in this publication (pp.297-315) proposes some strong arguments for the multiple benefits of collaborative efforts, both to the teachers themselves and to the profession. Reviewers of this publication document a field in disarray, a "field whose high hopes for a science of mathematics education have been overwhelmed by complexity and drowned in a sea of competing theories" (Steen, 1999). Two questions that exemplify the dilemma, he believes, are:

- How relevant is basic, as opposed to applied research, for mathematics education? and
- Are qualitative, or quantitative methods, or a mixture of both, more appropriate for mathematics education research?

We may think that we know the answers to these questions, but do we really? Weinert and his colleagues (1995) believe that debate on the first of these questions is unproductive both theoretically and practically. On the one hand, it is difficult to differentiate pure or basic research from applied research; on the other hand, it has become clear that findings from both research prototypes have specific advantages and disadvantages with respect to their practical applications. The second debate has degenerated into what is almost a religious war for many, and is undoubtedly at the bottom of the 'math war' currently raging in the United States. Jeremy Kilpatrick's argument that qualitative research remains unacceptable to many 'hard-core' mathematics researchers and the public, and that a considerable number of the reforms in school mathematics that have upset these people are the direct result of qualitative research is a worrying accusation. On our part, we've come to believe that both qualitative and quantitative approaches are necessary and complementary components of any system of research in the mathematics, science and social sciences fields. It will be interesting to see how this issue develops over the next few years.

The research-practice gap is a related complexity. Consideration of the issues surrounding the gap has been given relatively little exposure in the literature, although there are notable exceptions - for example, its treatment in the *International Encyclopaedia of Teacher Education* (Anderson, 1995), and other references listed at the conclusion of this article. Carolyn Boulter from the University of Reading has examined this matter in the context of the effects of curriculum change on teachers in England and the fact that the professional development of teachers is becoming a central focus of government concern in the UK (Boulter, 2000). She suggests that not enough notice has been taken of the effectiveness of getting teachers in classrooms actively involved with a wide variety of research, and that developing teachers' research agendas can be a sustaining influence on their professional life. She believes that such agendas can be initiated during teacher training, and that they lead to gains in lifelong involvement with critical and reflective thinking about classroom practice. While many of us may not be involved in mathematics teacher preparation *per se*, the majority are involved in the professional development of teachers in some way, and this perhaps affords us with the opportunity to create a research agenda for working closely with the practitioner. We should be committed to research being an integral strand in professional development.

Carolyn Boulter's description of the reaction of the government in the UK is interesting, for I do not believe that State Education Departments and Ministries of Education in many other parts of the world see it as their job to encourage teachers and researchers to collaborate, preferring to leave this task to the teacher training institutions. This belief puts me in the same category as that West Australian teacher named Greg whom we heard at the start of this presentation. It seems that education authorities will utilise the results of research in, for instance, their curriculum development efforts certainly, but they are not in the business of actively encouraging teacher-researcher collaboration in their operations. To my knowledge there has only been one 'official' document published in the last ten years in Australia that emphasised the need for teachers to 'seek guidance for improving the mathematics curriculum, both its content and pedagogy', and which stressed that 'teachers, like other workers, need to be engaged in professional development throughout their careers if they are to keep abreast of developments'. This was the *National Statement on Mathematics for Australian Schools*, published by the Australian Education Council in 1990. In the United States, the push for such collaboration has, as we are all aware, been championed by the NCTM, particularly in its publication entitled *Professional Standards for Teaching Mathematics*, published in 1991.

So, as we begin working in this new millennium, it seems to me that it is up to us to acknowledge the existence of the two research communities, even though we like to link the two terms and speak of the *teacher – researcher* as though it describes a single entity. We must also take advantage of the strengths of both communities to meet the challenge of improving mathematics teaching and learning. Neither community can truly achieve its true potential without the other. How can we bring about this collaboration? Anderson (1995) lists the following four steps to be followed "if research is to be linked to improved practice". First, valid and reliable data on important characteristics of teachers and components of teaching must be collected. Second, the data must be interpreted properly, both in terms of what they mean (theoretical) and how they relate to effective teachers and teaching (practical). Third, the data must be integrated or synthesised across a set or series of studies addressing similar characteristics or components, initially looking for common findings and ultimately resolving contradictory ones. Finally, he says, 'ways must be found to translate research effectively into practice'. What a letdown that fourth step is, for it is precisely that information – those 'ways to translate research' – that we are seeking! Also somewhat disappointing is the NCTM's *Position Statement on Research* (NCTM, 1999). Following comments such as: 'If mathematics education research is to be responsive to questions regarding pedagogy and student learning, then collaboration between teachers and researchers is critical', and 'Teachers should meet with other teachers and researchers to discuss outcomes and observations....and to offer classroom data for analysis and interpretation', the Statement concludes with the advice: '....researchers must share their interpretive insights in ways that address teachers' daily concerns for classroom practice'. Unfortunately there are no suggestions about how this might be achieved effectively. Again, not very helpful to either teacher or researcher.

To its credit though, the NCTM has addressed this issue in a recent editorial by Judith Sowder in JRME (Jan, 2000) that asked the question: *how can we communicate mathematics education research beyond our own community to reach the broader audience?* Once again, a complete solution was not forthcoming, but one of the points made was that attention should be paid to the manner in which research is reported in journals such as JRME. The editorial suggested that the current form of reporting is often a turn-off for teachers. At the place where I work, we have been attempting to overcome this situation by preparing and distributing to every secondary school in Australia a user-friendly, four-page publication called *'What Research says to the Teacher'* that we hope goes some way to overcoming this particular problem.

In her keynote address at this conference, Deborah Ball has suggested another solution. Deborah has described to us how she and her colleague Magdalene Lampert have used the school context to situate their research question, namely: *How should prospective teachers come to know?* (Lampert & Ball, 1998). The question provides the background for their reporting on their extensive research in this area – research that has used the notion of constructing teacher knowledge *within* practice. These two researchers were not merely observers in the classroom – they helped plan, teach and reflect on the lessons they observed using a research-in-action model. They listened to students as they worked, and they were free to respond to students' needs for direction and assistance. Post-lesson sessions were devoted to analyses of the interactions that had taken place, group reflections and the planning of further instruction. Further learning activities were 'negotiated' between the students, the teachers and others.

My plan too would be to link research to improved practice. It would involve the notion of working more closely with teachers than we have before. There will be many in this audience who already do what I am about to say, but their efforts aren't shared by all of us – as the teachers' voices at the start of this presentation verify. To me, it seems that in order to take advantage of the strengths of both communities we must do at least four things: First, spend some proportion of our research activity on finding out the matters which are of major concern to the classroom teacher. Let's never be accused of researching irrelevant issues and being out-of-touch with the actual classroom. Undoubtedly, some of our efforts in the past have resulted in an indifferent and cynical reaction from our classroom colleagues. Let at least some of our research activity be directed towards solving the day-to-day problems within the classroom. Admittedly there are probably too few of us to go around – we cannot visit as many teachers as we would like and participate in their classroom research, and we do not have the time to devote to prolonged classroom action research. However we do need to become more 'user-friendly' for the practitioner, and one way we can do this, in addition to school-based work, is by making sure that we participate as often as we can at conferences and workshops involving mathematics teachers.

Next, double our research efforts to understand the practical inquiry undertaken by mathematics teachers. For example, over the past 10 years we have carried out significant research that has led to a better understanding of students' naïve and alternative conceptions in mathematics and how these conceptions affect mathematics learning. I would like to see a research program of the same magnitude that focuses on mathematics teachers' practical inquiry. This program would focus on how teachers develop knowledge of mathematics content, students' learning styles, lesson planning, instruction, assessment and reflection, and how they use this knowledge to solve their day-to-day problems. I believe that all teachers, not only those who teach mathematics and mathematics methods courses, would value the findings of such research. As we all know, one way to investigate the practical inquiry of mathematics teachers is through collaborative action research. However, while this strategy is used and is evident in many of the research studies reported during this conference, I feel that more can be made of this approach. Working together, MERGA members, other researchers and teachers can build mutual understandings and clarify complementary interests that can lead to learning and common actions. I see practical inquiry and formal research as two phases of a continuous collaborative action research cycle, where practical inquiry stimulates formal research, and formal research leads to practical inquiry. Work of this type is likely to encourage researchers and practitioners to grapple with issues of relevance and rigour.

Like Tom Koballa, I view collaborative action research as a win-win situation for researchers and teachers. When engaging in collaborative action research, teachers gain new insights into their practices and come to see themselves as researchers into those practices, while researchers grow in their understanding of the mathematics teaching-learning context and the day-to-day concerns of mathematics teachers. More importantly, both teachers and researchers come to respect and value each other's work and perspectives and, more often than not, revise their own assumptions about the relationship between research and practice.

The third thing I believe that we should do is this: we must rethink how mathematics education research is communicated to teachers. We must view ourselves not only as constructors of new knowledge, but also as the disseminators of the knowledge we construct. As disseminators, we must recognise that it is unlikely that the vast majority of our work will provide the answers to mathematics teachers' immediate day-to-day problems. More realistically, the results of our work may stimulate teachers to think about mathematics teaching and learning in new and useful ways (Fenstermacher & Richardson, 1993). Weinert and his colleagues (1989) suggest that because researchers tend to issue general statements which are difficult for practitioners to implement, and because practitioners tend to rely more on historical data (eg. traditional practices) rather than on current research evidence in informing their practice, the gap can only be closed if research findings were viewed as *one* source of background knowledge for the practitioner. In other words, researchers should be more interested in *informing* teachers than in influencing their practice. This was the thrust of the *'What Research says to the Teacher'* publication mentioned earlier.

From the perspective of many mathematics teachers, the product of mathematics education research is a body of prescriptive propositions about teaching and learning. Because many mathematics teachers tend not to participate in MERGA conferences, nor read JRME, MERJ or MERGA's new publication *Mathematics Teacher Education & Development*, they do not see mathematics education research as a process in which mathematicians grapple with uncertainties and display the art of inquiry akin to the uncertainties and art of practice. Also, inherent in the way in which mathematics education research is often presented to teachers is that hierarchy of status referred to earlier, where the knowledge constructed by the tertiary-based mathematics educator is elevated above the practical knowledge constructed by the teacher. Too many teachers interpret us as saying 'here's what research says you should do, so go out and do it!' We must do a better job of communicating with teachers about the nature of mathematics education research, and our communications with them must be on an equal footing. When discussing mathematics education research with teachers, we should speak about the uncertainty and challenges associated with our work and not just the prescriptive outcomes. Perhaps we need to get our hands chalkier, as did Deborah and Magdalene, and seek to contribute to the classroom action whenever it is appropriate and when we are invited. We probably need to communicate verbally more too. Deborah Schifter (2000) believes that mathematics education researchers are unused to addressing audiences outside their immediate field. Tightly focused on exchanges with peers, researchers share assumptions, language, references, goals, and concerns that make their discussions opaque to outsiders – including mathematics teachers.

My fourth suggestion for what we should do is to make students' and teachers' classroom experiences central to the university programs of pre-service teachers. I believe that it is in the teacher-preparation area where the seeds would be best planted to foster the realisation that, though the research-practice gap exists, it can be decreased significantly through the efforts of teachers and researchers working closely together. The inclusion of a small research project in the training program of a pre-service teacher would go a long way to fulfilling this aim. Those of us involved in teacher education have a big responsibility here to demonstrate to our charges not only *what* can be achieved, but also *how* it can be achieved. We need teachers who possess the desire to know how to access, interpret and conduct research. We are the ones to motivate them and to instil this desire and knowledge into them at the outset of their careers.

In summary, in this presentation I have focused on the plight of two mathematics education research communities, both striving to attain the same goal – to improve mathematics teaching and learning. If we are to succeed, it is imperative that we make use of the strength of both communities as we move ahead. As we attempt to bridge the research-practice gap, my suggestions can be summed up in two recommendations: the first is that we double our research efforts in order to understand the practical inquiry of mathematics teachers. Collaborative research is one way to do this, but certainly not the only way. My other recommendation is that we rethink how mathematics education research is transmitted to teachers. Successful performance here will, I believe, make an enormous difference in how mathematics education research is perceived and used to improve practice.

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Practical Implications Award Paper

MERGA 23
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Collaborative Problem Solving in Senior Secondary Mathematics Classrooms

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This paper reports on research into patterns of social interaction associated with metacognitive activity in senior secondary school mathematics classrooms. Unsuccessful collaborative problem solving sessions were characterised by students' poor metacognitive decisions exacerbated by lack of critical engagement with each other's thinking, while successful outcomes were favoured if students challenged and discarded unhelpful ideas and actively endorsed useful strategies. The findings have practical significance for implementation of new mathematics curriculum policies that emphasise problem solving, mathematical reasoning, and communication.

In a review of progress in mathematical problem solving research in the 25 years to 1994, Lester (1994) lamented that research interest in this area appears to be on the decline, even though there remain many unresolved issues that deserve continued attention. One such issue highlighted by Lester was the role of metacognition in problem solving – where metacognition refers to what students know about their own thought processes, and how they monitor and regulate their thinking while working on mathematical tasks. Although the importance of metacognition is now widely acknowledged, we still lack an adequate theoretical model for explaining the mechanisms of self-monitoring and self-regulation, and understand too little about how metacognition and other aspects of thinking mathematically cohere to give individuals their mathematical "point of view" (Schoenfeld, 1992).

Also lacking are clear guidelines for teachers on how to foster higher order reasoning and problem solving skills, despite some research evidence that students can be taught to think mathematically. For example, Schoenfeld's work with college students emphasises the benefits of small group problem solving in developing both metacognitive control skills and a sense of what the discipline of mathematics is about. Beyond this work, however, research on small group learning in mathematics has yielded few insights into how students think and learn while interacting with peers, since most studies have focused on narrow learning *outcomes*, such as memorisation of facts or computational skills, rather than learning *processes* associated with mathematical reasoning (Good, Mulryan, & McCaslin, 1992). Consequently, the potential for small group work to develop students' mathematical thinking and problem solving abilities has remained largely unexplored, along with related issues concerning the teacher's role in orchestrating students' discussion and social interactions.

The concerns outlined above are reflected in Lester's (1994) call for further research on the role of the teacher in a problem centred classroom, and on teaching and learning processes for small groups and whole classes as well as individuals. As suggested by Lester's research agenda, there are practical as well as theoretical reasons for studying mathematical thinking. Issues of practical interest arise from new curricular trends that emphasise problem solving, mathematical reasoning, and communication (Australian Education Council, 1991; National Council of Teachers of Mathematics, 1989). The impact of these trends is clearly seen in the Queensland Senior Mathematics Syllabuses (e.g., Board of Senior Secondary School Studies, 1992), where assessable objectives for students' learning now refer to Communication and Mathematical Applications as well as Mathematical Techniques. These significant curriculum reforms are intended to engage students more actively in the mathematical enterprise, yet are likely to require a major shift in the classroom practices of many mathematics teachers. How can changes to the social organisation of classrooms be justified when our understanding of the ways in which students' mathematical thinking is cultivated by these new forms of classroom interaction is far from complete? This is an important question that is neglected by curriculum documents promoting reform in mathematics education.

The challenges outlined above point to the need for further research on mathematical thinking and learning to be conducted in authentic classroom settings. This need was addressed by the study whose findings are presented here. The study investigated the characteristics of senior secondary students' metacognitive activity as they worked together on mathematical tasks, and considered how teachers can create a classroom culture of inquiry which promotes mathematical habits of mind (Goos, 2000). The specific research question addressed by this paper asks how metacognitive strategies are elicited and supported during collaborative peer interaction.

Collaborative Metacognitive Activity

Previous research on metacognitive development has used Vygotsky's notion of the zone of proximal development (ZPD) to explain how adults can scaffold learners' progress from assisted (other-regulated) to independent (self-regulated) performance (e.g., Bruner, 1985; Wertsch, 1985). It has also been suggested that interaction between peers with incomplete but roughly comparable expertise can create a collaborative ZPD in which students are able to coordinate their different perspectives in order to achieve progress (Forman & McPhail, 1993). However, the literature that deals specifically with improving metacognitive strategy use via peer interaction is limited, and has produced conflicting results (cf Artzt & Armour-Thomas, 1992; Forman & Cazden, 1985; Stacey, 1992). In particular, it is unwise to assume that students will interact spontaneously in productive ways, especially if they are not accustomed to working with peers as part of their regular classroom experience.

Although there is a large body of literature devoted to peer learning (see Good, Mulryan, & McCaslin for a review), not all forms of peer interaction can be classed as collaborative. For example, Damon and Phelps (1989) distinguish between various approaches to peer education according to the quality of engagement that is fostered. Thus they define "peer tutoring" as interaction in which students of unequal expertise are brought together so that one may instruct the other, and "cooperative learning" as an arrangement which allows teams of students to divide a task and master its separate parts. Damon and Phelps reserve the term "peer collaboration" for the interaction that occurs when students with similar levels of competence share their ideas in order to jointly solve a challenging problem. In this context of supportive communication and assistance students are encouraged to experiment with new ideas and critically re-examine their own assumptions – a form of interaction that seems to hold promise for improving students' metacognitive awareness and regulatory strategies.

Similarly, Granott (1993) maintains that highly collaborative interactions between peers of equal expertise are characterised by shared activity, a common goal, continuous communication, and co-construction of understanding. This view is consistent with the definition of collaboration offered by Teasley and Roschelle (1993), as "a coordinated, synchronous activity that is the result of a continued attempt to construct and maintain a shared conception of the problem" (p. 235).

From the above the distinguishing feature of peer collaboration can be defined as *mutuality* – a reciprocal process of exploring each other's reasoning and viewpoints in order to construct a shared understanding of the task. Because this kind of reasoned dialogue involves comparing one's own ideas with those of another person, collaborative interaction need not be based purely upon agreement and cooperation, but may also include disagreement and conflict. Thus the process of co-constructing understanding is more complex than simply reaching consensus on an agreed answer (Kruger, 1993). The purpose of this paper is to characterise mechanisms of peer collaboration that contribute to shared understanding through metacognitive activity in problem solving tasks.

The Study

The research study was conducted in senior secondary school classrooms over a three year period (1994-96). Five teachers and their mathematics classes, all in different schools, contributed to the study. As the emphasis was on interpreting learning in complex social settings rather than experimental manipulation and control of variables, research methods were consistent with a naturalistic inquiry approach (Lincoln & Guba, 1985), and included long term participant observation of classrooms (supplemented by audio and video recording), interviews with students and teachers, and survey questionnaires. Complementary perspectives provided by questionnaire and observational data resulted in one classroom being selected for intensive analysis over both years of the study.

Target students were chosen for observation on the basis of their metacognitive sophistication and preference for working collaboratively with peers, as judged from preliminary observations and questionnaire responses (refer to Goos, 1995, for questionnaire details). One lesson was observed each week, and target students were videotaped and audiotaped as they worked together.

Data Coding and Analysis

Selected portions of audio and videotapes were transcribed and subjected to two coding passes focusing on the conversational turns of all speakers (referred to here as Moves). Moves in the protocol were first coded to identify their metacognitive function. A coding scheme developed in an earlier study (Goos & Galbraith, 1996) was used to identify metacognitive acts where a *New Idea* was proposed or an *Assessment* of particular aspects of the solution (i.e., strategy, result, understanding) was made.

Conversational Moves were then coded a second time to identify their contribution to the collaborative structure of the interaction, indicated by the *transactive* quality of the dialogue. Transactive reasoning is defined as clarification, elaboration, justification, and critique of one's own or one's partner's reasoning (Kruger, 1993). Three types of transacts were coded: spontaneously produced transactive *statements* and *questions*, and passive *responses* to transactive questions. The orientation of each transact was also noted: operations on one's partner's ideas were labelled other-oriented, while reasoning directed at one's own ideas was coded as self-oriented. This procedure produced six transact codes: (three types) × (two orientations). While Teasley (1997) has argued that transactive coding of peer discussion is consistent with Vygotskian approaches to studying links between collaborative interaction and cognitive change, this approach does not do justice to the reciprocal nature of collaboration. The scheme was therefore modified by grouping the codes as follows:

- *Self-disclosure* – Self-oriented statements and responses that clarify, elaborate, evaluate, or justify one's own thinking.
- *Feedback Request* – Self-oriented questions that invite a partner to critique one's own thinking.
- *Other-monitoring* – Other-oriented statements, questions and responses that represent an attempt to understand a partner's thinking.

Successful Collaboration

Three transcripts were selected for detailed analysis to illustrate common features of collaborative metacognitive activity. The transcripts were drawn from different years of the study (1994, 1995, 1996), and involve three different groups of students. All record students' interactions in the early stages of learning a new mathematical concept or process (compound interest, projectile motion, and Hooke's Law), so that the tasks on which they worked were unfamiliar, challenging, and genuine "problems". The tasks were therefore likely to require metacognitive control of problem solving actions, and to elicit collaborative interaction.

Separate analyses of the metacognitive and transactive nature of the dialogues were undertaken for each transcript. In addition, collaborative metacognitive activity was identified in Moves double coded both as metacognitive acts and transacts. A summary of metacognitive transacts for all three problem solving protocols is provided in Table 1.

Table 1
Moves Double Coded as Metacognitive Acts and Transacts

	Transactive Structure (Frequencies)		
Metacognitive Function	Self-disclosure	Feedback Request	Other-monitoring
New Idea	14	2	4
Assessment–strategy	2	5	16
Assessment–result	–	3	–
Assessment–understanding	–	–	6
Total	16	10	26

These results indicate that joint metacognitive activity was characterised by:

- students clarifying, elaborating and justifying their New Ideas for the benefit of a partner *(Self-disclosure)*;
- students asking their peers for help in finding errors by inviting critique of strategies and results; and students seeking feedback on the New Ideas they proposed *(Feedback Request)*;
- students making an effort to understand their partners' thinking by offering critiques of their strategies, requesting explanations, or elaborating on and monitoring their understanding of partners' ideas *(Other-monitoring)*.

For the three transcripts (a total of 351 Moves), 26 of the metacognitive-transacts were self-oriented (Self-disclosure or Feedback Request) and 26 were other-oriented (Other-monitoring). Thus, when the students interacted with each other, their monitoring activity was directed at both their own thinking and the ideas of their peers.

Unsuccessful Collaboration

This microscopic analysis of problem solving transcripts provided a lens through which to view the peer interaction processes that establish a collaborative zone of proximal development. Nevertheless, it is necessary also to examine situations in which collaboration was metacognitively fruitless, and to identify reasons for this lack of success.

A framework for classifying types of metacognitive *failure* was created to guide selection and analysis of unsuccessful problem solving episodes. The metaphors of *blindness*, *vandalism*, and *mirage* were chosen to describe situations where students respectively overlooked errors, applied inappropriate conceptual structures to resolve an impasse, and reacted to difficulties that did not exist (see Goos, 1998, for details). A further three transcripts of students' dialogue were selected as before to illustrate each of these scenarios, with the analysis showing how poor metacognitive decisions contributed to problem solving failure. (The problem solving tasks dealt with combinatorics, motion of a body on an inclined plane, and area of the Koch snowflake). However, a more detailed analysis was called for if some of the subtle processes distinguishing successful from unsuccessful collaborative metacognitive activity were to be revealed.

The Role of Transactive Discussion

The first step was to calculate proportions of "success" and "failure" transcripts coded as having metacognitive function and/or transactive structure. This analysis showed that there was very little difference in the metacognitive proportions recorded in both groups of transcripts. However, a different picture emerged from the summary of transact proportions: first, there was a lower incidence of transactive discussion in unsuccessful problem solving sessions (17% of all Moves, compared to 26% in successful sessions); and second, a large part of this discrepancy was accounted for by the difference in the proportions of *non*-metacognitive transacts (4.7% in unsuccessful problem solving sessions, compared to 11% in successful sessions). It appears, then, that success is characterised not only by utterances which are simultaneously metacognitive and transactive (the double-coding criterion applied previously), but also by interactions involving purely transactive discussion. This finding suggests that the discussion *around*, and generated by, individual metacognitive acts is crucial to the success of the mathematical enterprise.

All transcripts were re-examined to identify metacognitive acts that either led to or were prompted by a transactive statement, question, or response. Moves so identified were thus connected to at least one transact, and were labelled *metacognitive nodes*. If the node was connected to more than one transact, then a *transactive cluster* was said to have formed around the node. For example, when working on a compound interest spreadsheet problem one student (Rob) challenged his partner (Belinda) to justify her proposal that the interest rate for each compounding period was 1.01:

52. B: Point zero one. *(New Idea)* It's not actually point zero one, it's (inaudible). *(Self-oriented transactive statement, clarification of the New Idea)*
53. R: How did you work that out? *(Other-oriented transactive question, seeking justification of New Idea in Move 52)*
54. S: You divided the –
55. B: See, compound interest, you've got to add one to it. *(Self-oriented transactive response to Move 53, justification of New Idea in Move 52)*
56. R: Is that it? That it, one? What for? *(Other-oriented transactive question, seeking further justification of Moves 52/55)*

Here, Move 52 is a metacognitive node because it prompts the transactive interchange, or cluster, comprising Moves 53, 55 and 56. This may be represented visually as in Figure 1. Note that circles superimposed on node symbols represent metacognitive transacts (e.g., Move 52), while arrows connecting metacognitive nodes with transacts, and particularly with transactive clusters, pinpoint instances of extended discussion of metacognitive ideas and assessments, and thus highlight the role of *non*-metacognitive transacts.

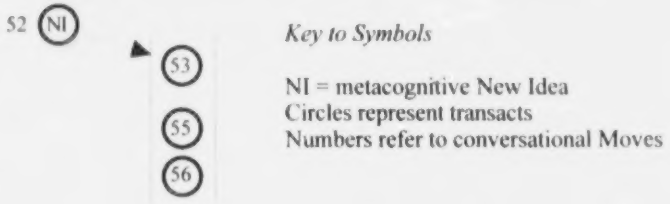
52
NI
53
55
56
Key to Symbols
NI = metacognitive New Idea
Circles represent transacts
Numbers refer to conversational Moves

Figure 1. Visual representation of metacognitive nodes and transactive clusters.

A complete visual representation of nodes and clusters was constructed for each transcript, and the numbers and proportions of nodes and clusters for successful and unsuccessful problem solving transcripts were recorded (see Table 2).

Table 2

Frequencies and Proportions of Metacognitive Nodes and Transactive Clusters for Successful and Unsuccessful Collaborative Metacognitive Activity

	Frequencies (Proportions)			
	Success Transcripts		Failure Transcripts	
Metacognitive nodes	26	(0.074)	11	(0.040)
Transactive clusters	10	(0.028)	3	(0.011)
Total Moves	351		277	

Successful collaboration was found to feature roughly twice the proportion of metacognitive nodes and transactive clusters as unsuccessful collaboration. In other words, transactive discussion of metacognitive New Ideas and Assessments appears to be a significant factor in successful collaborative problem solving. Closer inspection of the dialogues revealed that the interplay between transactive *challenges* and metacognitive decisions was significant in shaping problem solving outcomes, since challenges eliciting clarification and justification of strategies stimulated further monitoring that led to errors being noticed or fruitful strategies being endorsed. On the other hand, causes of metacognitive failures could be traced to the absence of such challenges. Being held accountable by peers for explaining "how" and "why" may have prompted students to explore an idea more thoroughly, or to step back from a task and recognise a mistake or anomaly.

Conclusions and Implications

This study set out to examine metacognition in the context of senior secondary school classrooms in order to develop a theoretical and practical rationale for the type of mathematics teaching envisioned by the NCTM *Standards* (National Council of Teachers of Mathematics, 1989) and the *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991). Because the study of metacognition has its roots in cognitive psychology, research in this area has tended to treat monitoring and regulation as individual, in-the-head processes that liken metacognitive control to internalised self-instruction. The present study has extended this view of metacognition as self-directed dialogue to include collaborative conversations that made the processes of monitoring and regulation overt. The notion of a collaborative zone of proximal development established through interaction between peers of comparable expertise has previously received little attention from researchers interested in relationships between small group processes and mathematical thinking.

Interestingly, the study of metacognitive failure afforded significant insights into possible mechanisms for successful collaboration. Previous research on metacognitive aspects of individual students' mathematical thinking has suggested that failure is virtually guaranteed by poor metacognitive control decisions (e.g., Schoenfeld, 1992), and there is some evidence that these decisions can be adversely affected by peer interactions in small group problem solving (e.g., Stacey, 1992). The results presented here extend and qualify these findings by highlighting the significance of transactive discussion in making good metacognitive decisions.

Working together on problems offered a realistic context for students to verbalise their ideas so that their implications could be considered and evaluated. Nevertheless, it may be helpful for teachers of younger or less able students to provide rubrics, or key phrases, to explicitly encourage them to engage with each other's thinking. Consider, for example, a pair of students working together on a problem (call them Student A and Student B). There are four ways in which Student A's thinking may become the subject of discussion:

1. Spontaneously, and initiated by the student (Self-disclosure, Student A operates on Student A's thinking: *Here is my idea*).
2. Spontaneously, and initiated by the partner (Other-monitoring, Student B operates on Student A's thinking: *Here is what I think of your idea*).
3. Through an invitation issued by the student. (Feedback Request, Student B is asked to operate on Student A's thinking: *What do you think of my idea?*).
4. Through a partner's challenge. (Other-monitoring, Student B asks Student A to operate on Student A's thinking: *What do you mean?*).

The italicised phrases provide a useful template for teachers to structure their students' mathematical discussions so that ideas are shared and evaluated.

Even with guidelines for discussion, critics of group work claim that students' thinking can become confused without the teacher's guidance, and this observation raises questions as to whether, when, and how the teacher should intervene to redirect students' efforts to more productive ends. Teachers face a number of dilemmas in tailoring assistance to meet students' specific needs. For example, their intervention may be misdirected and cause more confusion than clarification, or may deny students the opportunity to resolve their own difficulties. Decisions also need to be made about the timing of teacher interventions, and, indeed, whether to intervene at all. Such decisions become complicated when groups and individuals are working on different tasks in any given lesson, including tasks not sanctioned by the teacher. There are finely tuned appraisals to be made about the timing, amount, and type of assistance to provide, if a delicate balance between encouraging persistence and avoiding frustration is to be maintained. Further research is warranted on the development of mathematical thinking in secondary school classrooms in order to pursue these and other practical questions arising from this study.

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Symposium

Researching a Systemic Early Numeracy Initiative

MERGA 23
fremantle 2000

Changing the Professional Knowledge of Teachers

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Concept mapping exercises were used to determine whether changes to teacher knowledge had occurred as a result of their involvement in an early numeracy professional development program. Findings indicate that the most significant change to teacher knowledge occurred in relation to how children learn mathematics. While some change was evident in teachers' pedagogical knowledge, there was little change to teachers' knowledge of mathematics content.

A common assumption is that the more mathematical content knowledge a teacher knows, the more effective they will be in teaching it (Alexander, Rose, & Woodhead, 1992). While this assumption seems logical and is no doubt desirable, there is little research evidence to support this in practice (Rhine, 1998).

Recently, a study by Askew, Brown, Rhodes, Johnson and William (1997) sought to explore the knowledge, beliefs and practices of experienced primary school teachers. They found that what distinguished highly effective teachers of mathematics from other teachers was a particular set of understandings which underpinned a particular array of teaching practices. Askew et al. (1997, p. 3) concluded that "highly effective teachers of numeracy themselves had knowledge and awareness of conceptual connections between the areas which they taught in the primary mathematics curriculum", but that this was *not* associated with having high levels of formal education in mathematics. They also found that highly effective teachers were more likely than other teachers to have undertaken mathematics-specific professional development over an extended period of time. The researchers concluded that while having an extended knowledge base of mathematics is helpful, it is not necessarily enough to ensure effective teaching of mathematics. Furthermore, they concluded that what seemed to matter more was the nature of that knowledge—the more effective teachers had a better understanding of the interconnectedness of mathematics concepts and possessed a good understanding of children's mathematical thinking.

The study reported here is concerned with primary teachers' professional knowledge of mathematics. It is part of a larger study conducted on behalf of the NSW Department of Education and Training (DET) to evaluate its early numeracy program—Count Me In Too (Bobis, 2000). Investigations designed to investigate the impact of this program on teachers and children have been reported at previous MERGA conferences. Since many of these reports contain detailed descriptions of various aspects of the program (e.g., Bobis & Gould, 1998; Wright, 1998) they will not be repeated here. Instead, a brief introduction to the program is provided along with a rationale for the current study based on the program's main aim and specific findings of previous investigations.

Professional Development of Primary Teachers in Mathematics

Count Me In Too (CMIT) has been operating in NSW government schools since 1996. It is a classroom-based professional development program that has as its main aim: "for teachers to better understand children's mathematical strategies and their development from less sophisticated to more sophisticated strategies" (Stewart, Wright, & Gould, 1998, p. 557). Hence, there is a major focus on the development of children's mathematical thinking.

In 1996 an evaluation of the program focused on the impact it had on the professional development of teachers (Bobis & Gould, 1998). Open-ended questionnaires and structured interviews revealed that, generally, teachers considered that they had gained knowledge relating to mathematical content, teaching strategies and of children's mathematical thinking.

The investigation reported here intended to extend these findings by exploring the impact of Count Me In Too on the professional knowledge of teachers more explicitly. Rather than relying on teachers' self-reports of change, this study documented what, if any, knowledge changed and how it changed. More specifically, it was designed to address the following research questions:

1. Does Count Me In Too have an impact on the professional knowledge of teachers?
2. If so, how does the professional knowledge of teachers change as a result of their involvement in Count Me In Too?

The Study

To determine whether changes to teacher knowledge had occurred as a result of their involvement in CMIT, data from two research tools were combined and compared—concept mapping and semi-structured interviews. While the two data gathering strategies were complimentary, restrictions on length necessitate that this paper only deal with findings from the concept mapping exercise.

Concept Mapping

Concept mapping has been used to examine how individuals change and organise their knowledge (Jones & Vesilind, 1996). It has been shown to be a powerful and sound method for assessing conceptual change and allows researchers to see how knowledge is restructured over time (Markham, Mintzes, & Jones, 1994).

Researchers using concept mapping usually adopt a scoring scheme to assign a numerical value to each map (e.g., Chinnappan, Lawson, & Nason, 1999). Such scoring systems often rely on information being categorised as *examples, hierarchies, crosslinks, links, nodes* and *key nodes*. For an explanation of each category and an associated scoring scheme, the reader is directed to Jones and Vesilind (1996).

Such quantitative methods have not always been considered appropriate and some researchers have used only qualitative methods to analyse concept maps (e.g., Williams, 1998). For the purposes of this study, an analysis that relied solely on a quantitative or a qualitative analysis of the maps was considered inappropriate. Attempts to apply a scoring system similar to that outlined by Jones and Vesilind (1996) ignored the *type* of knowledge possessed by teachers. Since an aim of this study was to investigate the impact of CMIT on the professional knowledge of teachers it was more revealing to simply note the frequency with which each type of knowledge occurred. Hence, the basic units used in the quantitative analysis were *key nodes, links* and *crosslinks* associated with a *type* or *source* of knowledge.

Participants

Participants were drawn from the Sydney Metropolitan area. Twelve teachers from 4 primary schools participated in the investigation. While the final selection of teachers for inclusion in the study rested on their willingness to participate, a number of criteria were considered. Namely, the desire to include teachers of both genders, drawn from a variety of school districts and with a range of teaching experiences. Of the 12 teachers included in the study, three had been teaching for less than 8 years, six had been teaching between 12 and 20 years and the remainder had each taught for more than 20 years.

Procedure

Teachers were relieved from teaching for approximately 2-3 hours on each of two occasions—once at the start of the study just prior to their involvement in the CMIT project, and again approximately 4 months later. On the first occasion, teachers received standardised instructions on how to draw concept maps and were given an opportunity to draw a practice map about sample topics, such as sports. They were then asked to draw a concept map about the *knowledge they have to allow them to teach mathematics*. After a short break, each teacher was interviewed. The purpose of the interview was threefold:

1. It provided an opportunity for teachers to explain their concept maps to the interviewer;
2. It allowed the concept map to be used as a prompt to solicit teachers' knowledge about teaching mathematics to young children; and
3. It was used to gather information relating to the experiences and factors teachers perceive as being responsible for their existing knowledge of mathematics, the teaching of mathematics and how children learn mathematics.

During the initial interview, teachers were allowed to modify their map using a different coloured pen if they desired. Questions adapted from Jones & Vesilind (1996) were used to prompt discussion of each teacher's concept map. For a full list of questions and a discussion of the interview findings see Bobis (2000).

On the second occasion, teachers were asked to study their previous map and to decide if they would like to either draw a new map, modify and redraw the old map, or leave the old map as it was. After drawing their second map, teachers were interviewed for the second time. The major purpose of this interview was to allow teachers to explain the changes they had made to their original concept map and why.

Results and Discussion

This section presents and discusses the findings of the quantitative data derived from an analysis of the concept maps constructed by all 12 teachers. However, discussion will focus on one teacher—Theresa, a Year 3 teacher with 7 years teaching experience. Theresa's concept maps have been included to assist with an explanation of the results. These have been redrawn with a computer graphics program to allow them to be more easily read (see Figures 1 and 2).

Analysis of Concept Maps

Key nodes categorised as a type of knowledge were sub-divided to include the categories of content knowledge, pedagogical knowledge and knowledge of children's cognition. Examples of key nodes that were categorised as content knowledge from Figure 1 are "Topics", "Language", "Symbols" and "Problem Solving". The lines linking each of these key nodes to an example of content, such as "prime" and "subtraction", were categorised as node links. Hence, in Figure 1 (the concept map constructed by Theresa during the first round of interviews) there are 4 instances of key nodes pertaining to the teacher's content knowledge, 37 instances of links to more specific examples of content knowledge and 1 instance of a crosslink stemming from the key node "Problem Solving".

Of the 12 teachers participating in the study, 10 elected to modify their original concept map during the second concept mapping exercise and 2 teachers (including Theresa) decided to draw new maps. Table 1 summarises the mean number and standard deviation of key nodes, links and crosslinks for each category of knowledge for the whole teacher sample on the two occasions in which concept maps were constructed. Individual results for Theresa are presented in Table 2.

Table 1 highlights a number of features typical of the maps constructed during the first concept mapping session. These include:

1. The relatively high number of key nodes and node links associated with pedagogical knowledge;
2. The relatively low frequency of features associated with mathematics content knowledge, children's cognition and source of knowledge; and
3. The minimal level of integration between the different types of knowledge as evidenced by the low frequency of crosslinking.

A comparison of the means for the two mapping exercises reveals several trends in the concept map changes. For example, it is evident that there was:

1. A decrease in the frequency of key nodes and node links categorised as content knowledge;
2. An increase in the frequency of node links categorised as pedagogical knowledge and knowledge of children's cognition; and
3. A slight increase in the level of integration between the different types of knowledge as evidenced by the increase in the frequency of crosslinks.

It is evident from Table 1 that concept maps constructed prior to the commencement of CMIT contained more key nodes associated with teachers' pedagogical knowledge than any other type of knowledge. This category encompassed a wide range of knowledge regarding how teachers teach mathematics. For example, it included key nodes relating to teachers' knowledge of grouping children for instruction, their knowledge and use of resources and of various assessment strategies. Such key nodes were linked to numerous examples illustrating teachers' familiarity with the variations existing within each of these sub-categories. The large number of pedagogical knowledge node links can be explained in this way. For instance, the first concept map constructed by Theresa (see Figure 1) links 4 examples to the key node "equipment" and 5 examples to the key node "evaluating". Further characteristics of Theresa's concept maps are summarised in Table 2.

Table 1
Analysis of Concept Maps for Round 1 (R1) and Round 2 (R2). Standard deviations are in brackets (n=12)

Information	Mean Number of Key Nodes		Mean Number Of Node Links		Mean Number of Crosslinks	
Types of Knowledge	R1	R2	R1	R2	R1	R2
Content	1.7 (1.3)	0.9 (0.8)	6.8 (10.6)	4.5 (5.1)	0.2 (0.4)	0.5 (0.7)
Pedagogical	4.2 (2.4)	4.5 (2.4)	31.8 (18.3)	37.9 (20.7)	1.5 (2.0)	1.8 (2.6)
Children's Cognition	0.8 (0.6)	3.3 (5.8)	4.6 (4.1)	11.8 (5.0)	0.4 (0.7)	1.3 (2.0)
Source of Knowledge	1.3 (2.0)	1.5 (2.2)	7.3 (11.0)	8.3 (12.3)	0.7 (1.8)	0.7 (1.8)

Key nodes relating to teachers' content knowledge were less frequently represented on the concept maps than those relating to pedagogical knowledge. This was mainly due to the fact that teachers grouped their content knowledge under one key node. For example, one teacher used a single key node, titled "Topics", to encapsulate all mathematical content knowledge. Major content areas relating to the strands in the NSW Syllabus, *Mathematics K-6* (1989), were then linked to this single key node. On the other hand, the same teacher used 6 key nodes to map all the pedagogical knowledge she possessed.

Table 2.
Frequency of key nodes, node links and crosslinks on Theresa's maps for Round 1 and Round 2

Information	Number of Key Nodes		Number Of Node Links		Number of Crosslinks	
Types of Knowledge	R1	R2	R1	R2	R1	R2
Content	4	1	37	10	1	1
Pedagogical	3	2	25	20	2	2
Children's Cognition	0	3	1	19	1	6
Source of Knowledge	0	0	0	0	0	0

The initial concept map constructed by Theresa (Figure 1) is typical of those containing a large number of key nodes and node links categorised as content knowledge. It will be noticed from her concept map that she recorded numerous examples of content, thus making her tally of node links quite large compared to other teachers who gave few or no specific examples of content.

The overall low mean score for key nodes relating to the way children learn mathematics is explained by the fact that such knowledge was not represented in 4 of the 12 teachers' initial concept maps (see Table 1). This does not mean that these teachers knew nothing about children's cognition before undertaking CMIT. Instead, it may indicate that such knowledge was not of sufficient importance to these teachers to be given recognition as a key node at this point in time.

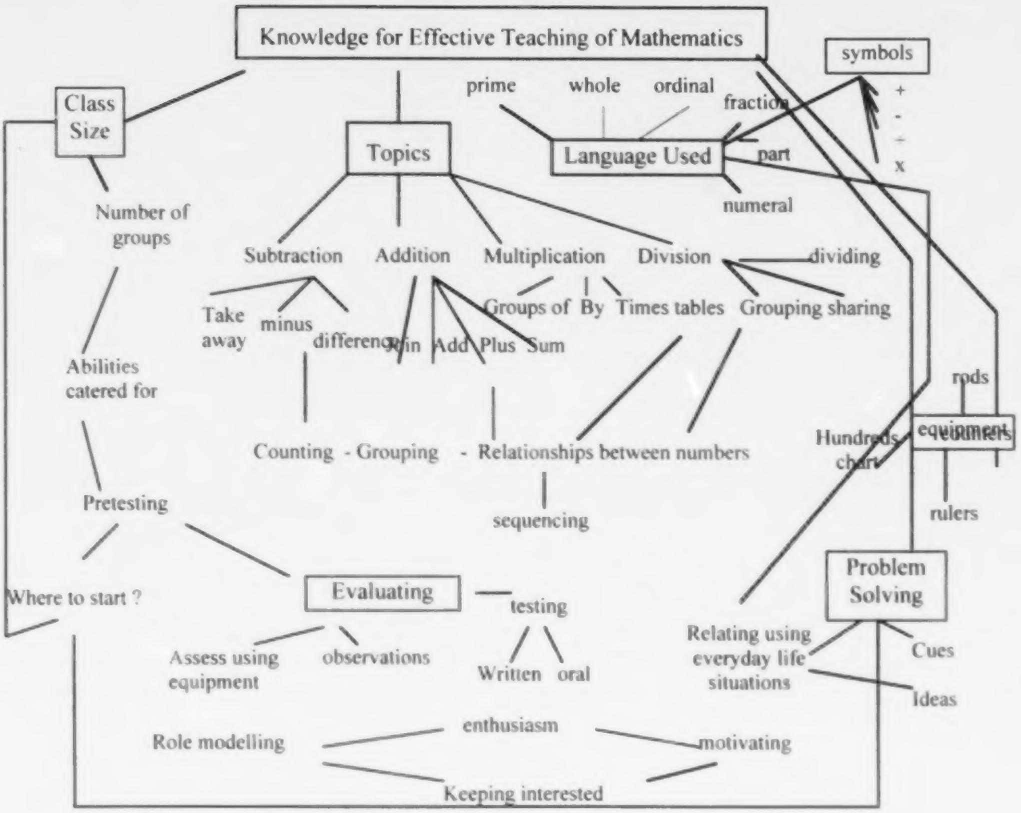
Knowledge for Effective Teaching of Mathematics
symbols
+
-
÷
x
Class Size
prime
whole
ordinal
fraction
Topics
Language Used
part
numeral
Number of groups
Subtraction
Addition
Multiplication
Division
dividing
Take away
minus
difference
Join
Add
Plus
Sum
Groups of
By
Times tables
Grouping
sharing
Abilities catered for
rods
Hundreds chart
equipment
counters
Counting - Grouping - Relationships between numbers
sequencing
rulers
Pretesting
Problem Solving
Where to start ?
Evaluating
testing
Relating using everyday life situations
Cues
Assess using equipment
observations
Written
oral
Ideas
enthusiasm
Role modelling
motivating
Keeping interested

Figure 1. Concept map constructed by Theresa during the first concept mapping session.

Another feature relating to initial concept maps that is highlighted in Table 1 is the minimal level of crosslinking. Crosslinking between the different types and sources of knowledge occurred in 7 teachers' initial concept maps (see for example, the crosslink between "problem solving" and "where to start?" in Figure 1). While more crosslinks were evident from features associated with pedagogical knowledge than any other type or source of knowledge, these were still proportionally low given the high frequency of key nodes associated with this type of knowledge. This small number of crosslinks indicates that there may be a lack of integration between the different types of teacher knowledge.

It will be noted from Table 1 that the mean number of key nodes and node links categorised as content knowledge actually decreased in the second concept mapping exercise. This is because the frequency with which this type of knowledge was represented on the concept maps remained relatively stable from one mapping session to the next for *most* teachers. However, there was one exception to this which accounts for the decrease in the overall frequency of features related to mathematical content knowledge being represented on the second round of concept maps. The initial concept map constructed by Theresa contained a large number of key nodes and node links in the category of content knowledge (see Figure 1 and Table 2), but during the second concept mapping session Theresa constructed an entirely new map that contained few features of the first (see Figure 2). Hence, while examples of content knowledge dominated the first map, the second map contained few examples of content knowledge. Instead, Theresa's second map indicates that she is now much more focused on her knowledge of how children learn mathematics—the stages of mathematical development and the strategies children use to help them solve mathematical problems at each stage.

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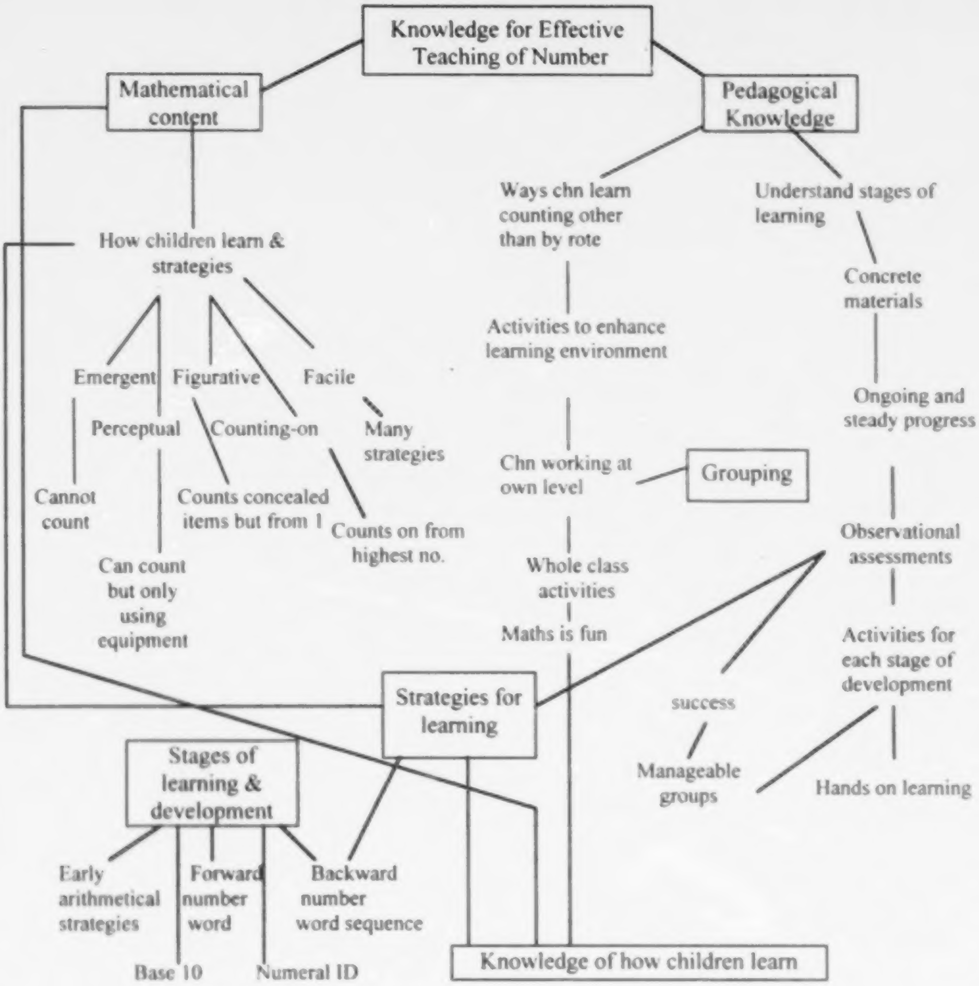
Knowledge for Effective Teaching of Number
Mathematical content
Pedagogical Knowledge
How children learn & strategies
Emergent
Figurative
Facile
Perceptual
Counting-on
Many strategies
Cannot count
Counts concealed items but from 1
Counts on from highest no.
Can count but only using equipment
Ways chn learn counting other than by rote
Activities to enhance learning environment
Chn working at own level
Grouping
Whole class activities
Maths is fun
Understand stages of learning
Concrete materials
Ongoing and steady progress
Observational assessments
Activities for each stage of development
success
Manageable groups
Hands on learning
Strategies for learning
Stages of learning & development
Early arithmetical strategies
Forward number word
Backward number word sequence
Base 10
Numeral ID
Knowledge of how children learn

Figure 2. Concept map constructed by Theresa during the second concept mapping session

The refocusing of attention towards children's cognition was noted in the second concept maps of many teachers. This is evident from a comparison of the means presented in Table 1. Theresa's initial map (Figure 1) was quite typical of other teachers' maps in that it does not contain any reference to children's cognition. However, her second map (Figure 2) contains 3 key nodes and 19 node links in respect to this type of knowledge. This indicates a major reconceptualisation of her view towards teaching mathematics.

Another trend in concept map changes is the slight increase in the level of integration between the different types of knowledge. This is evidenced by the increase in the frequency of crosslinks. While the number of crosslinks increased overall, concept maps constructed by three teachers contained no crosslinks even after the second mapping session. These teachers tended to be those who had experienced CMIT for a shorter period of time.

Summary and Conclusion

The evidence derived from an analysis of the concept maps indicates that Count Me In Too *did* have an impact on the professional knowledge of teachers. In particular, there was an

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increase in teachers' knowledge of how children learn mathematics and, to a lesser extent, their pedagogical knowledge. This was evident in the second concept mapping exercise which indicated an increase in the frequency of key nodes and node links associated with these two types of knowledge. The fact that the degree of knowledge integration only increased slightly from the first to the second mapping exercise overall, demonstrates how difficult it is to achieve such an outcome even by classroom-based professional development programs that are implemented over an extended period of time.

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Teachers, Students and Research: One Possibility for Teaching Early Numeracy

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Three interlocking papers have been written for the symposium 'Researching a Systematic Early Numeracy Initiative'. The initiative referred to is the mathematics project 'Count Me In Too' (CMIT), developed for the early years of New South Wales government schools. The three papers look at the effect this project has had on teachers and students, and how the project is situated within the body of research on the early years of schooling. This paper is written in response to an invitation to discuss these three papers with a view to forming some questions that may be useful to ask at the symposium.

Mapping Children's Stages of Arithmetical Solution Methods

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An analysis of the age at which students' demonstrate different stages of arithmetical solution strategies (based on Steffe's psychological models) was used to develop a baseline in evaluating the impact of a professional development program in early number learning. This baseline of performance, matched to ages within the cohort, was used as a global determinant of growth in increased efficiency of solution strategies due to the Count Me In Too early numeracy program operating in these schools.

Reviewing Literature Relevant to a Systemic Early Numeracy Initiative: Bases of CMIT

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A systemic initiative in early years mathematics—Count Me In Too (CMIT), is briefly described. This is followed by a review of links between the research base of CMIT and other research in early number. The review is organised into five areas: assessing students' knowledge and its progression; counting strategies; abstract composite unit; development of tens and ones knowledge; and knowledge of numerals. The review serves to locate the theoretical bases of CMIT more broadly in early number research.

Count Me In Too (CMIT) is a systemic initiative in early years mathematics which has operated in government schools in NSW since 1996 (Bobis & Gould, 1998, 1999; Stewart, Wright, & Gould, 1988). A basic goal of the initiative is to enhance teachers' understandings of young children's arithmetical strategies. After its initial trial in 13 schools in 1996, CMIT has received increased government funding each year. By the end of 1999, CMIT had been fully implemented in approximately 300 schools, and by 2003 will be available for implementation in all primary and central schools (approx. 1800). CMIT is also being implemented in other Australian school systems and in New Zealand on a large scale (500 teachers). Finally, CMIT theory and methods are now being applied systemically in NSW and elsewhere, at higher grade levels in primary and secondary schools (Year 7).

Development of CMIT involved application of a coherent program of research into young children's number learning and included the development of a learning framework in number which plays a key role in CMIT (NSW DET, 2000; Wright, Martland & Stafford, 2000). Presentation of an overview of this framework (Wright, 1998a) resulted in calls for examination of possible links between the framework and the underlying program of research on one hand, and other research. This paper has the purpose of examining such links. In this examination the learning framework in number and the underlying research will not be described in detail but will be alluded to as necessary. Descriptions of the research bases of the multiplication and division aspects of CMIT and their links to other research are available elsewhere (Mulligan, 1998; Mulligan & Mitchelmore, 1997) and will not be addressed here.

The examination of links between the research base of CMIT and other research is organised into five areas. These areas were determined by (a) identifying literature judged to be particularly relevant to CMIT, and (b) perusing the identified literature in order to identify common themes or issues. Thus these five areas provide a convenient way to organise the paper. Nevertheless, the areas are interrelated and many of the reports discussed here are relevant to more than one of the areas. The five areas are: assessing students' knowledge and its progression; counting strategies; abstract composite unit; development of tens and ones knowledge; and knowledge of numerals.

Assessing Students' Knowledge and its Progression

Young-Loveridge (1991) describes a longitudinal study of the development of early number knowledge. Sixty-eight children were assessed on five occasions — near the time of their 5th, 6th, 7th , 8th, and 9th birthdays. Of particular interest is the close similarity of some assessment tasks used by Young-Loveridge and the standard assessment tasks used in CMIT —tasks which are prominent in the research programs underlying development of CMIT.

Young-Loveridge selected six of the items she used in the assessment at five years of age, for which the children's scores correlate highly with their overall scores on each of the five occasions of assessment. These six items are, in Young-Loveridge's terms (pp. 110-112), forming sets, numeral identification, pattern recognition, rote counting, sequence forwards, and enumeration. The assessment of 5-year-olds also includes number sequence backwards (p. 12) and addition and subtraction with concrete and imaginary objects (p. 14-15), which also feature prominently in CMIT assessment. Young-Loveridge's study provides a very strong endorsement of the CMIT assessment tasks.

Three studies which focused on school entry knowledge and its progression during the first year of school are of interest because of similarities in their conclusions, and their relevance to CMIT. These studies are the UK study by Aubrey (1993), the New Zealand study by Young-Loveridge (1989), and the NSW study by Wright (1991, 1994). These studies detailed the wide range of levels of students' knowledge, and showed that current curricula were unsuited to many students, particularly the more knowledgeable students.

> The demonstration of such early competencies poses challenges to the conventional reception class curriculum which follows a sequence of sorting, matching and classifying, joining and separating sets, counting and ordering, recognising and writing numbers 0-10, where simple mathematic relationships may be demonstrated through the use of concrete material. ... [R]eception-age children clearly enter school having acquired already much of this mathematical content. (Aubrey, 1993, p. 39)

Denvir and Brown (1986a; 1986b) describe a hierarchical framework of 47 early numeracy skills. In this framework one skill is prerequisite to another if there is a logical reason why it depends on the other and all or nearly all children who succeeded on the harder task also succeeded on the easier one. The framework was used to chart the performance of seven pupils interviewed approximately six monthly over a period of two years. Denvir and Brown also describe two teaching studies and the use of the framework to map each participant's performance by showing skills acquired at pre-test, skills acquired at post-test, skills acquired at a delayed post-test, and skills taught (i.e., between pre- & post-test). In the case of one child the authors concluded that

> the acquired skills, whilst consistent with the hierarchical framework were not the ones that had been taught ... [and] whilst a child's performance ... will suggest which skills are likely to be acquired, it cannot be predicted precisely which skills a child will learn. (p. 152)

This finding, and Young-Loveridge's (1989) finding that "certain skills were learned over the first year of school by considerable numbers of children even though they had not been taught these skills by their teachers" (p. 56), highlight the separateness of what is taught and what children learn. As well, all of the above studies lend support to the instructional approach in CMIT. This approach highlights the need for teachers to take full account of students' current strategies and knowledge levels, and emphasises the role of problem solving and reflection in the development of more sophisticated strategies and in the extension of knowledge. As well, the approach can be contrasted with a narrow, lock-step approach emphasising learning of specific skills.

Counting Strategies

Carpenter, Fenema, Franke, Levi and Empson (1999) present an overview of the Cognitively Guided Instruction approach (CGI) to early number (see also Carpenter, Fenema & Franke, 1996; Fenema et al., 1996). CGI includes a model of a progression of strategies children use to solve addition and subtraction word problems—direct modelling, counting and derived facts. In this model "counting" refers to the relatively sophisticated strategies of counting on and back. Strategies involving other forms of counting; for example counting perceptually available items, may be used at the level of "direct modelling". Many of the strategies and observations about strategies in CGI feature in CMIT literature and its research base. The observation for example, that "[C]ounting strategies represent more than efficient procedures for calculating answers to addition and subtraction problems. They indicate a level of understanding of number concepts and an ability to reflect on numbers as abstract entities" (Carpenter et al., 1999, p. 28) is entirely consistent with CMIT's third level of early arithmetical strategies of "Counting On and Back" (also referred to as "Advanced count-by-one strategies"), and corresponds to what Steffe and colleagues called counters of abstract unit items (Steffe, von Glasersfeld, Richards, & Cobb, 1983, pp. 66-72), the sequential integration operation (Steffe & Cobb, 1988, p. 150), and the initial number sequence (Steffe, 1992, pp. 92-95):

> The deeper question … about how the number words come to stand for summations of units, is really the heart …. The semantic links of the number words change radically as the child develops more sophisticated unit types and associated conceptual structures. The semantic link between a number word and the concept of summation of units can only be made on the level of abstract units, and it is only then that a number word can stand for such a summation of units. (Steffe et al., 1983, p. 27)

In similar vein, Carpenter et al. observed that,

> although children frequently use fingers with Counting strategies, the use of fingers does not distinguish Counting strategies from Direct Modelling strategies. … Fingers may be used to directly model a problem or to keep track of the steps in a counting sequence. (1999, pp. 23-4)

This is entirely consistent with descriptions in CMIT which differentiate use of fingers at the Perceptual (Level 1), Figurative (Level 2) and Counting On and Back (Level 3) levels. Each is associated with profoundly different ways children think about numbers. In describing figurative counting (CMIT Level 2) Steffe (1992) wrote:

> This is quite distinct from a child simultaneously putting up four fingers and then three fingers as replacements (emphasis in original) for the hidden items, and then taking the fingers as a collection of perceptual unit items for counting. This latter coordination of finger patterns and the counting scheme is within the province of children with perceptual counting schemes. (p. 89)

As described above in the area of children's counting strategies, links between the CGI literature and the research base of CMIT can be discerned. Fuson's research on counting and related areas of early number is closely linked to the research base of CMIT.

> Children in the United States display a progression of successively more complex, abstract, efficient, and general conceptual structures for addition and subtraction. Each successive level demonstrates cognitive advances and requires new conceptual understandings. … The work [i.e., as presented in a book chapter] on counting and cardinal conceptual units, … cardinal conceptual operations, … and cardinal conceptual structures … was stimulated by Steffe and is a summary of his work and my own related work. (Fuson, 1992, p. 250)

The reports by Gray (1991) and Gray & Tall (1994) are particularly relevant to CMIT and its extension to grade levels above K-2. These focus on the strategies used by children to solve addition and subtraction tasks. The tasks were classified as Stage 1—addition and subtraction facts to 10, and Stage 2—addition and subtraction facts in the range 10 to 20. Seventy-two participants were selected from two schools considered to be typical English schools, as follows; six from each school in each of the six age ranges of 7+, 8+, … 12+. Each of the 12 sets of six students consisted of two children at each of three teacher-defined ability levels—below average, average and above average. Participants' addition strategies were classified as one of: known fact, derived fact, count-on, or count-all. Participants' subtraction strategies were classified as one of: known fact; derived fact; count-up or count-back; or take-away. Count-up and count-back are considered analogous to count-on. The take-away strategy is considered analogous to count-all and involves three forward counts from one (e.g., counting out a known minued, then counting out a known subtrahend from the minued, then counting the remainder to obtain the unknown difference).

Gray describes these four-level classifications of addition and subtraction strategies as preferential hierarchies. If unable to solve a task by immediate recall (of a known fact) the child reverts back to what might be regarded as a preferred level. Gray identifies "two distinct approaches to the regression" (pp. 569-570):

> The first makes use of other known knowledge, the deductive approach [used mostly by above average and average children]. The second is dominated by the use of counting, the procedural approach [used mostly by below average].
>
> What has become fairly clear ... is that the below average ability child is neither successful at learning the number bonds nor in making use of the ones that they do know ... .[For younger below average children] memory is abandoned for a procedure that involves the use of physical or quasi-physical objects. The bits they do know do not appear to be held together, with the result that this change in strategy may involve the child in long sequences of counting. ... However, by the end of the middle years of schooling such children feel secure, often confident, in their procedure.
>
> In contrast, condensing the long sequences appears to be almost intuitive to the above average child. This eventually becomes the cornerstone to their higher level of attainment; they can take short cuts and operate with increasing levels of abstraction.

Gray's research underlies the importance of children learning to count-on (Level 3 of CMIT's early arithmetical strategies) and also the importance of children progressing to levels where they routinely use "non-count-by-ones" strategies, can instantly recall facts, and also use these to derive other facts (i.e., Level 4).

Thompson (1995) describes a study of the solution methods of 59 Year 2 children (6- & 7-year-olds) on simple addition, subtraction and multiplication problems. Findings suggest that as children progress through school they continue to use counting as an important part of their problem solving repertoire, combining these counting skills in idiosyncratic ways, other learned skills and acquired knowledge. Thompson argues that the importance of counting as a basic building block of numerical understanding suggests that teachers of young children may need to place greater emphasis on the development and integration of counting skills during the first few years of school. Thompson's findings and recommendations are generally consistent with the emphasis in CMIT on developing increasingly sophisticated counting and other strategies in early number learning.

Abstract Composite Unit

Steffe's (Steffe & Cobb, 1988) psychological construct of "abstract composite unit" (p. 334), and related constructs such as "numerical composite" (p. 335) and "iterable unit" (p. 335) underlie CMIT's models of the development of children's knowledge of addition and subtraction, tens and ones, and multiplication and division. These constructs are prominent in research involving analysis of children's thinking in measurement (Clements, Battista, Sarama, Swaminathan, & McMillen, 1997) and geometric (Battista & Clements, 1996; Battista, Clements, Arnoff, Battista, & Borrow, 1998; Wheatley & Reynolds, 1996;) contexts, as well as in number contexts (e.g., Hunting & Davis, 1991; Jones et al., 1996; Lamon, 1994; 1996; Saenz-Ludlow, 1994; Watanabi, 1995).

According to Lamon (1996), "the ability to form and operate with increasingly complex unit structures appears to be an important mechanism by which more sophisticated reasoning develops" (p. 170). Lamon uses the notion of composite unit in studying children's partitioning strategies (1996), and the development of proportional reasoning (1994). A report by Jones et al., (1996) for example, presents and applies a framework of the development of children's numerical thinking spanning the period from pre-place value and initial counting to the understanding of three-digit numbers. The framework consists of five levels across four key constructs of counting, partitioning, grouping and number relationships, and draws on Steffe's notions of numerical composites and abstract composite unit (p. 311). Watanabe (1995) described relationships between children's fraction concepts and whole number concepts and presented case studies based on three children's solutions of two tasks which stimulate children to think in terms of fractions. The study identifies four types of unit coordination schemes (i.e., strategies) used by the children, viz one-as-one, many-as-one, one-as-many, and many-as-many.

Wheatley and Reynolds (1996; see also Reynolds & Wheatley, 1996) used the construct of abstract composite unit to explain children's geometric thinking as well their number thinking. They describe

> a relationship between a child's ability to construct abstract units from nonrectangular shapes and their use of ten as an abstract unit in adding and subtracting whole numbers. ... We conjecture that constructing abstract units is quite a general and significant mathematical operation which transcends number. (pp. 67-68)

Empirical evidence in support of this conjecture is drawn from observations of four children over a three year period. The study argues that "classroom activities which encourage the construction of units are likely to be useful to children in coming to act mathematically" (p. 82).

Finally, Battista, Clements and colleagues used the construct of abstract composite unit and related constructs in describing students' understanding of 3D arrays of cubes (Battista & Clements, 1996), development of length concepts (Clements et al., 1997) and spatial structuring of 2D arrays of squares (Battista et al., 1998).

Development of Tens and Ones Knowledge

The model of the development of children's knowledge of tens and ones used in CMIT is based on research by Steffe and Cobb (1988) and a study by Cobb and Wheatley (1988) which draws on the research by Steffe and Cobb (Cobb & Wheatley, 1988, p. 1). The model involves "three increasingly sophisticated concepts of ten ... numerical composite, abstract composite unit, and iterable unit" (Cobb & Wheatley, 1988, p. 4). Descriptions of children's conceptual structures for multidigit numbers—unitary multidigit, sequence-tens and ones, and integrated sequence-separate tens (Fuson et al., 1997) correspond to concepts of ten described by Cobb and Wheatley (1988) and related constructs (Steffe & Cobb, 1988) as follows: unitary multidigit corresponds to numerical composite and the initial number sequence; sequence-tens and ones corresponds to abstract composite unit and the explicitly-nested number sequence; and integrated sequence-separate tens corresponds to iterable unit. As well, there is accordance between Cobb and Wheatley's (1988) analysis of concepts of ten and Kamii's (1986) levels of children's counting by tens. Related to these correspondences is the concurrence among several researchers in descriptions of two distinct types of strategies children use in 2-digit addition and subtraction. Thus Beishuizen's (1993, p. 295) two categories of Dutch second-graders' strategies, that is, 1010 (the split method) and N10 (the jump method) correspond respectively with (a) collections-based and counting-based interpretations of 2-digit numerals (Cobb & Wheatley, 1988), (b) the partial sums and cumulative sums categories of additive strategies (Thompson, 1994, p. 333), and (c) the combining tens and ones and incrementing types of invented algorithms for multidigit addition and subtraction (Carpenter et al., 1999, pp. 70-73).

Knowledge of Numerals

From the perspective of CMIT, students' developing knowledge of numerals is considered to be an important aspect of early number development and to deserve a distinctive research focus. Nevertheless, research literature on knowledge of numerals in early number seems to be rare. Wright (1998b) for example, describes observations relating to young children's knowledge of numerals and its relation to number word knowledge. In CMIT, knowledge of numerals includes: identifying (reading or naming); recognising (selecting a named numeral); writing; sequencing (e.g., putting numerals from 26-35 in correct sequence); and ordering (e.g., putting 10, 28, 21, 30 in ascending order). A view held in CMIT is that it is appropriate to teach knowledge of numerals directly, and that learning about numerals can significantly precede learning place value. Thus young children should be taught to read 2- and 3-digit numerals for example, prior to formal teaching of place value, and this can constitute an important basis for learning place value. This accords with reports by Wigley (1997) and Hewitt and Brown (1998) of systematic approaches to teaching children to name numerals as an important basis of early numeracy.

Conclusion

This paper has reviewed research literature in early number learning judged to be particularly significant to the CMIT initiative. Thus the paper serves the purpose of locating the theoretical bases of CMIT more broadly in the relatively extensive body of research in early number learning that has been conducted in the last 15-20 years, both in Australia and New Zealand, and elsewhere.

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Symposium

The Discursive Production of Agency and Identity in Mathematics

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Emergence of Mathematical (In)Competence and Identity

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This paper is an inquiry into the development of mathematical self-concept or how students perceive themselves in relationship to mathematics. The inquiry is centred on two classroom episodes which involve changes in the nature and direction of conversation between students and the teacher. We suggest that if the episodes were typical of patterns of interaction in a class, they might significantly impact on the students' perception of their own competence in mathematics.

Introduction

In this paper we explore the notion of mathematical identity or self-concept (Jungwirth, 1996). The inquiry is set in the context of a Year 11 Geometry and Trigonometry class who were studying a topic on vectors. Taking a situated view of learning, we interrogate classroom dialogue for potential influences on students' perceptions of self, drawing on excerpts of conversation that we see illustrate (a) the attribution in social interaction of (in)competence and (b) the demands of two students being responded to over the needs of another.

Our situated perspective draws on social constructivist theory, especially on the *emergent perspective* of Cobb (1998), Cobb, Boufi, McClain and Whitenack (1997) and Cobb, Gravemeijer, McClain and Whitenack (1997). Accordingly, we see learning, including how students' perceive they stand in relationship to each other and to mathematics, to involve both social and cognitive processes. However, our focus in this paper is on the social dimension of learning where, consistent with social constructivism, we assume that the social practices in the class were founded on the teacher's and students' individual experiences (and understanding of those experiences) prior to coming together as a group, and continually evolved in their ongoing classroom activities.

Our intentions in writing the paper are (a) to provoke, you, the reader into considering the pedagogical and instructional implications of the episodes that we present and (b) to contribute to the debate on how to improve mathematics teaching/learning--by considering an area which seems to be given relatively little attention in the literature: namely, the development of mathematical identity. While writing the paper, we were aware that, while students' mathematical identities are constructed through their doing mathematics, our own identities as researchers are shaped by the decisions we make and actions we take at every stage of the research act. Therefore, we make explicit our choices and the limitations, as we came to see them, of our methodology. We see this as a way of highlighting our concern with representing fairly the teachers and students who agree to participate in our research.

Research Method

The site of the research was a Year 11 mathematics class of 18 students in a private college for girls in Western Australia. Pat (first author) took the role of participant-observer (Atkinson & Hammersley, 1994), to observe whole-class discussion and serve as an assistant teacher during small-group work, over one month. The main source of data for this paper was audio-recorded classroom conversation. Three small audio-recorders captured, we assume, most of what was said by seven students in small-group work, as well as the voices of the teacher (Mr C.) and the students during whole-class discussion. Other data were Pat's field notes, journal entries, a video-recording of each lesson from a corner of the classroom, photocopies of students' written work and students' responses to a questionnaire ascertaining perceptions of their own learning (Forster, 1999).

Our research methodology (like the perspective we took on students' learning) was based on social constructivist principles: the inquiry involved personal, reflective sense-making and critical dialogue (by email) between us--centred on our written interpretation and successive reinterpretations of events. Thus, our analysis was motivated by writing:

> writing as a method of inquiry ... moves through successive stages of self-reflection ... the field-workers texts flow from the field experience, through intermediate works, to later work, and finally to the research text ... Thus do fieldwork and writing blur into one another. (Denzin and Lincoln, 1994, p. 10)

The classroom episodes presented in the paper were purposively selected (Cohen & Manion, 1994) on the basis that they illustrate students interacting in problematic ways with each other and the teacher: ways that potentially could impact on the formation of self-concept. With the text of the episodes in front of us, we chose narrative as the genre for the inquiry and decided to write in the first person 'my' and 'I' (who is Pat--as participant-observer in the Year 11 class and principal writer of our co-constructed analysis). These choices allowed the interpretative nature of inquiry to be brought to the fore and brought the inquiry into the realm of fifth generation qualitative research (Lincoln & Denzin, 1994) with its issues of representation and legitimation. We address these issues now, albeit briefly because of the restricted length of the paper.

In presenting classroom life through only two short episodes, I am sharply aware that they represent to a very limited extent how 'it really was' in the class. Individual students would at one time be supportive of others, at other times be dismissive, so the excerpts of conversation--plucked from their context of 'dialogue over time'--are not to be read as portraying regular classroom practices.

In addition, I do not claim that my analysis accurately reflects actual effects on self-concept as experienced by students, immediately or in the long term. Even while an observer in the class, my understanding of what students were thinking and experiencing was, as for all estimations of 'the other', at best second order (Steffe, 1995)--an interpretation of their interpretation of events. To overcome this limitation, Guba and Lincoln (1989) recommend that analysis is member checked. However, I did not adopt this option for an important reason. I felt it would be unethical to present the students with analyses which they could take as criticism of the teacher. Besides, I do not claim that the inferences made are accurate but instead propose them as possibilities. Therefore the quality criterion which I pursued is that the analysis is *practically reasonable* (Fenstermacher, 1994)--in the opinion of others. To meet this criterion, interpretation is the consensual view of both authors; as well, analysis involved critical reinterpretation for progressive subjectivity, and I asked a teaching colleague (of English Literature) as well as the teacher of the Year 11 class (Mr C.) to read the paper. They did not disagree with the inferences drawn.

Theoretical Framework

I see students' learning largely in terms of social constructivist principles: that learning involves processes of active individual construction and, simultaneously, processes of enculturation (Cobb, Gravemeijer, et al., 1997). Social constructivism draws attention to cognitive development and to social processes: they are seen to literally not exist without each other, and to be reflexively, although indirectly related (ibid). Piaget's reflective abstraction, or re*cognition* of regularities, for example, of what it means to add or multiply, links cognitive and social activity (Cobb, Boufi et al., 1997). So, in a social constructivist account, merely being 'carried along' by mathematical discourse does not automatically constitute individual understanding.

From a social constructivist perspective, mathematical discourse, as well as mediating students' conceptualisation of mathematical relationships, is characterised by what I think of as a second layer of regularities and patterns. Cobb (1998) refers to these regularities as *social norms* and *sociomathematical norms*. Bauserfeld (1995) describes how the genesis of these regularities is "covert and subconscious" (p. 155): they start with "routinized definition" of a situation, although the patterns emerge interactively and are influenced by all participants. Social norms can be subject to intervention so that the teacher might encourage students in particular types of behaviour, for example, to always answer with extended explanations. Gradually these behaviours, as individuals re*cognise* them, can emerge to be accepted as normal among the class (e.g., Cobb, 1998; Cobb, Boufi, et al., 1997).

Thus, my reading of social constructivist theory led me to view mathematics learning as (a) conceptual development, or the student building on prior knowledge, reflectively, through social activity; and (b) as coming to know how to act in relationship to others, or the student building on previously known social practices, subconsciously and interactively in participation with others.

Self-concept

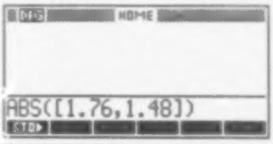
My inquiry was informed by a number of other studies that address students' self-concept, and these were based on a range of methodologies. Jungwirth (1996) uses discourse analysis to explore gender-related 'differences in confidence', in regard to learning mathematics. Klein (1999) draws on her observations of primary school classrooms to consider how constructivist teaching approaches might mediate against students' confidence and disposition towards mathematics. Roth (1998) uses phenomenological accounts by a female student and her male teacher to depict how the student felt about herself, and how she felt the teacher perceived her. Coulter (1999) uses stories of adults who were held back to repeat a grade as the basis for his inquiry into the long-term effects--on self-concept--of streaming. Murray & Whitehead (2000) provide an autobiographical account of Murray's struggle, as a person of mixed race, to establish his own identity.

The order I have chosen to list the above references is intentional and is according to the methodologies, which range from researchers commenting on self-concept from a viewpoint that is external and distanced from the students, to a student account of what identity is and meant to him. I see that each of the methodologies can alert us to different aspects of mathematical identity or self-concept. However, in offering the following analyses, I acknowledge again the limitations of my external, researcher's point of view.

The Attribution of Competence

March 2nd. It was the first lesson on vectors for the Year 11 class and my first day of observing the class. Mr C. started by introducing the terms vector, component, vector magnitude, and vector direction, and then specified new notations, including the graphics calculator function ABS for finding (vector) magnitude (see Figure 1).

ABS is absolute value, which means magnitude or size, and is under the -*x* key.



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ABS([1.76,1.48])
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Figure 1. Extract from the worksheet showing instructions for finding magnitude.

I watched the action and then, when students started on questions from their text, I moved among them, looking at their work and talking with them. After a while, I heard some excitement and then a laugh--it was Katie and Jenny explaining something to the teacher. They had programmed their calculators to automate the calculation of absolute value (see Figure 2). Mr C. asked Jenny if she would explain the aplet or program to the class at the end of the lesson. She said she would.

1. Mr C: ... Right, now we have stopped, Jenny has volunteered to tell us
2. something that she has discovered. So come up Jenny.
3. Katie: Yea. Go Jen.
4. Jenny: ... and I put in a little aplet ... so the aplet I got was, I put in ABS
5. brackets then 'I' and 'J', and I just said 'M' for magnitude ...
6. Mr C: Okay. Would everyone like to try it? Does everyone think it is a good
7. idea?
8. Ann: I think it's brilliant.

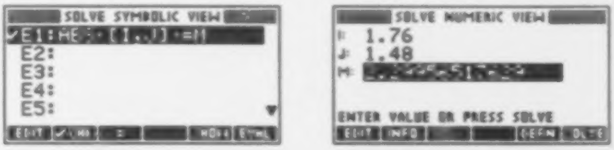

Figure 2. The symbolic and numeric screens for the ABS Aplet.

So, Jenny had "discovered" (line 2) something, it was a "good idea" (line 6-7), in fact it was "brilliant" (line 8). And I was impressed. That evening I transcribed the tape to see how the discovery had happened. This is what was recorded:

9. Jenny: Katie, do you reckon you could put this into our G and T things, if you went
10. 'ABS brackets i j'?
[pause]
11. Katie: You have to make an M.
12. Jenny: Yes. That would be okay. M could work. Then I could go '= M' [see Figure 2]

Later:

13. Mr C: How did you do those ones?
14. Jenny: We made an aplet.
15. Mr C: Did you?
16. Jenny: Oh we used M because we couldn't use the little line bars [the | | symbol
17. for magnitude]. We just did ABS, then I and J.
18. Mr C: Show me how it works.
19. Jenny: It works like this. You just enter I and J.
20. Mr C: Would you like to show everyone else?
21. Katie: [laughed]
22. Mr C: I think it is pretty clever
23. Katie: It's Jenny's idea.

Later:

24. Jenny: Yea. Back to my aplet.

I am wondering if you, the reader, have the same sense of surprise that I experienced on reading the text. The first thing I noticed was that Jenny changed from using the inclusive "we" (line 14, 16-17) to the personal "my" (line 24), thus changing from expressing shared ownership to claim sole ownership of the aplet. She continued using "I" (line 4) in explaining the aplet to the class, even though it was Katie who had suggested "You have to make an 'M'" (line 11). ['=M' was needed for the program to run.] However, it was Katie herself who assigned all credit to Jenny with "It's Jenny's idea" (line 23). At this signal the teacher also moved from using what I take was the inclusive "you" (line 13, 15) to promote the aplet to the class as Jenny's discovery (line 2).

This episode brought to life for me the observation of Erickson (1986) that "assessed ability is socially constructed ... rather than an attribute of the person" (p. 125). Jenny's ability was socially assessed as "pretty clever" (line 22), as up to making discoveries (line 2), "good" (line 6); and "brilliant" (line 8). Wouldn't such descriptors mediate students' perceptions of themselves (as mathematicians) in a favorable way? Conversely a repeated pattern of non-recognition might mediate against students' beliefs in their own capabilities.

In reading the episode, I recalled Davis's (1996) recommendation for teaching to involve a careful "hermeneutic listening" (p. 261): a listening to the text of students' conversations so that, for example, answers in terms of "we" (line 14, 16) are taken to imply joint effort. The episode also raises issues of what responsibilities students have to each other when working together, especially if shared work is brought into the public domain; and how might awareness of those responsibilities be fostered?

A Turn in Conversation

March 12th. It was the beginning of the lesson and a student, Amanda, asked Mr C:

1. Amanda: You know how you gave us an example where forces were going in the
2. opposite direction.
3. Mr C. Like that (drawing a diagram on the board--see Figure 3)?

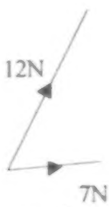

Figure 3. Diagram showing 12N and 7N forces.

4. Amanda: Yes. Sort of. Can you still solve it like that or do you have to move the vector (her
5. voice dropped away)?
6. Mr C: ... I think the question Amanda is asking, can we resolve the two vectors where
7. they are or do we have to shift one of the vectors and join it onto the end of the other
8. one. When you did the questions last night, what method did you use (to the class)?
9. [pause] Jenny?
10. Jenny: I put the 7 Newtons on the end of the 12 Newtons one.
11. Mr C: So you did the head to tail kind of thing?
12. Jenny: Yes.

There was more discussion, then Mr C. asked a student to come up and draw the head tail diagram on the board. He specified an angle of 40° between the forces then asked students to solve the problem:

13. Mr C: Everyone tell me what the resultant is. In component form.
14. Katie: In component form?
15. Jenny: Do we have to use component form?
16. Mr C: Oh sorry. You can or not.
17. Jenny: Can't we just use the cosine rule?
18. Mr C: Who prefers to use cosine rule?
19. Jenny & Katie: Yes. It's so much quicker.
20. Amanda: But we want to learn how to do vectors [the component method].
21. Mr C: You want to learn how to do vectors? You want to learn the other way?
22. .. Just get the resultant. Use whatever way you choose. You choose.
23. Katie (to Jenny): I don't know how to do the vector way.
24. Jenny: Well, you would use the 12 in components and that one [the 7 Newton vector].

Later, using student feedback which relied on the cosine rule, Mr C. wrote the solution on the board. When everyone was working again, Amanda asked him on a one-to-one basis about the problem. He discussed it with her.

My first interpretation of the episode is that it was a 'war of words' (Lakoff & Johnson, 1980), but it isn't clear to me if anyone perceived others as the enemy. The first shot was fired with Katie's challenge (line 14). Jenny quickly followed with "Do we" (line 15), then "Can't we" (line 17), which could be seen to have consolidated their position. The strength of the onslaught meant it was a struggle for the mediator Mr C. to keep his neutral stance (line 16, 18). Amanda attempted to restate her case but no-one supported her, despite her plea with the inclusive "we" (line 20). However, that anyone was a winner was questionable (line 23), especially as the final outcome was that a problem-solving method was used which was already very familiar to students.

My second interpretation is that the dialogue was 'dance' (Lakoff and Johnson, 1980). The problem was that when Amanda tried to join the company she only 'sort of' knew the set routines and didn't know how to ask the right questions (line 4). As often happens in this situation, someone will step in and ask a question for you, but mightn't get it quite right (line 6-8). Then, all you can do is stand at the side and watch the choreography and it sometimes ends up that you can't remember the elements of the question that you originally wanted to ask (line 20).

My third interpretation is that the dialogue was building self-concept and I ask, did Jenny and Katie learn that two people can achieve more than one? Did Amanda learn that it can be lonely and frustrating in a mathematics classroom? Would she have felt more empowered if she'd been invited to restate at the beginning her original question (line 4)? Did the practical action of the teacher of asking a student who could be relied upon to answer (line 9) further constitute her own sense of competency and at the same time mediate against that of others?

I want to remind you that "metaphor provides a way of *partially* communicating unshared experiences" (Lakoff & Johnson, 1980, p. 225). So, to make it clear, I did not consider that the classroom was a battlefield, but I did see that some students joined the dance more often than others. So, if we say that a sense of agency in a mathematics classroom arises from construction of a learner as one with the right to speak and be heard (Klein, 1999), then which learners are asked or offer to speak needs to be in teachers' awareness. Also relevant to the situation is the importance of helping students develop linguistic skills so that participation is open to them (Cobb, 1998; Taylor, 1996), and that learning in co-participation with others implies a concern for others' learning, which needs to be brought into students' awareness (Tobin, 1998).

Conclusion

In re-viewing the two episodes a theme that emerged is how students work in relationship with each other. In the first, I cast as problematic Jenny and not Katie receiving public acclaim for the design of the aplet, and suggested that students have responsibilities when they choose to work together. In the second, I questioned that Amanda's query was lost in the classroom debate, and highlighted that students also have responsibilities towards each other in the whole-class domain. The implication for teaching is that recognition of responsibilities needs nurturing, and this might be achieved for example, by making ways of acting the subject of discussion.

Both episodes display the crucial role a teacher plays in regards to who speaks and the direction that discussion takes. I identified the practices of the teacher filling in questions for students and choosing students who can be relied on to answer, and that these practices might need to be scrutinised, even though they are practical in so far as keeping the discourse moving. I take these practices as examples of social norms to which Cobb (1998) refers. They become part of the classroom culture. The implication for teaching is that it is desirable for teachers to be critically aware of their ways of responding to students and that these ways might subtlety impact quite significantly on students' perception of their own capabilities. Critical awareness might be achieved through systematic reflection on practice (Schön, 1987)--through which I uncovered some previously unrecognised regularities in my own teaching!

In the analysis I have been careful not to impute intention, by any of the students or the teacher. In other words I don't believe there was purposeful commendation or recognition of some students and not others, by the teacher and by students of each other. They were carried along by the discourse.

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Is there more to Numeracy than Meets the Eye?: Stories of Socialisation and Subjectification in School Mathematics

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In keeping with movements in society generally there has been, over the past several decades, an increasing interest in the social aspects of learning mathematics in school and using mathematics beyond school to meet the demands of life, work and leisure. In this paper I attempt to draw connections between the two; how what is learned in school, and how it is learned, might affect its competent and critical use after school. I argue that classroom practices of *socialisation*, as in social constructivist, Vygotskian and socio cultural psychological approaches to teaching and learning mathematics do not tell the whole story; stories of *subjectification* must also be told to help us understand how it happens that some school leavers are able to use mathematics to libratory and powerful ends and others in limited ways that deprive them of choice.

A constant in our world beyond 2000 is that the rate of change is exceptional and accelerating. Diverse aspects of these changes are taken up and selectively represented in the media via information and communication technologies. Our social world is data-rich and dynamic and demands new skills of school leavers who will need to be able to critically interpret and evaluate vast amounts of information and act decisively and justly. The workplace has become a knowledge economy where workers are problem identifiers, problem solvers and strategic brokers who work in small teams on short term projects; workers of the future will need the social skills that will enable them to work productively with others, be able to broker consensus, communicate ideas and give and solicit help as they generate joint products (Education Queensland, 1999).

Mathematically, students exiting the schooling system will need to know the mathematics and have the social adaptability, flexibility and confidence to use this knowledge in dynamic and powerful ways. One's standard of numeracy might be seen as a measure of one's ability to work *with/in* the mathematics discourse in a specific site; *with* the mathematics points to the necessary knowledge of content, the ability to reason, solve problems, communicate, connect mathematical ideas and applications and *in* mathematics points to one's positioning by oneself and others in the discursive field, as agentic and competent in performing the required mathematical tasks. In this paper I attempt to suggest some important features of the social context of school mathematics that might have a bearing on how well school leavers acquit themselves in the world of work and leisure beyond 2000.

Given the emphasis on mathematics as a social practice beyond school, I first of all examine the theoretical assumptions and practical imperatives of the (social) constructivist and Vygotskian inspired classroom practices in teaching mathematics in school. I argue that the notion of teaching and learning as *socialisation* framing classroom activities is useful in orienting students to powerful mathematical ideas and ways of engaging in reasoning and debate. However, in maintaining the psychological individual/society dualism, socialisation theories inevitably though unintentionally turn to notions of individual pathology to explain how it happens that many students do not learn the mathematics, nor manage to establish themselves as agentic and competent users of its powerful ideas. I argue that were we as educators and researchers to open our eyes to processes of *subjectification* also framing and influencing all that happens in the classroom social context, we may be able to take steps to interrupt at least those immediate practices that potentially disenfranchise many learners.

Framing my analysis of theory and practice throughout the paper is an inclination to accept that indeed there is much more than meets the eye to ensuring that all children leave school with the necessary intellectual and social skills needed for the postmodern world. Whereas in the past it was considered enough to know the facts and skills and pass the exams, it is now more widely recognised that these competencies often came at a cost; for example, there are many girls/women who do quite well in the exams yet do not choose careers where higher levels of mathematics are needed (Johnston, 1994), preservice teachers who are anxious and underconfident about teaching mathematics and too many children "who come to school enthusiastic and eager to learn mathematics and ... leave school with quite negative attitudes" (Australian Education Council, 1990, p. 31). Socialisation into the content and reasoning processes is important, but it may be that attendant, often invisible, processes of subjectification selectively position students within the discursive field; each of these factors may be co-constitutive and influential regarding agency with/in mathematics.

Teaching and Learning Mathematics: Socialisation

All contemporary learning theories consider that social interaction, in one form or another, is important to cognitive development. However, the nature, role and importance of social interaction is theorised differently in the various approaches to teaching mathematics in the classroom. In this section of the paper I examine the role of social interaction in Piagetian *social* constructivism where the social context is instrumental in providing challenges and perturbations as a catalyst to individual cognitive reorganisation, and Vygotskian approaches where social interaction in the form of "mediation by materials, tools, peers and teachers is seen to be constitutive of learning" (Lerman, 1998, p. 147). Each of these is premised on processes of socialisation to bring about cognitive and developmental growth.

The Social Context Facilitates Individual Knowledge Construction

Although Piaget's name is rarely mentioned in the mathematics classroom, his notions of development and the construction of knowledge have had a significant influence on understandings and assumptions that frame educational practice. Piaget's constructivism holds that learners have to construct their own knowledge; its appeal lies in images of learners actively engaged in knowledge construction and contrasts sharply with prior notions of learning as passive reception and retention of others' preformed knowledge. The knowledge constructed is an internal conceptual structure, or schema, built up piece by piece as the learner reorganises activity on the basis of experience. "*Knowledge*", says von Glasersfeld (1989, p. 124) who further developed Piaget's work, "refers to conceptual structures that epistemic agents, given the range of present experience within their tradition of thought and language, consider *viable*" (emphasis in original). Thus knowledge is not a *representation* of an independently existing *real world* but is made up of conceptual structures which prove to be adapted to, or fit within, the subject's range of experience.

Piaget did not speculate on how teachers should teach to ensure knowledge growth, though many educators have incorporated a well defined role for social interaction in the negotiation of meaning and *fit* with prior experience. In the literature (Davis, Maher, and Noddings, 1990) there has been an emphasis on establishing a mathematical community that would provide objects to be used in investigations, have lots of teacher-student interaction for purposes of diagnosis and guidance, encourage student talk, model mathematical thinking and promote questions and comments that help community members challenge and defend their own constructions. Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti, and Perlwitz (1991a, p. 6), who have been instrumental in adapting Piagetian notions of knowledge construction for teaching, stress the importance of construction and acculturation: "In the course of classroom social interactions, the teacher and students mutually construct taken-to-be-shared mathematical interpretations and understandings".

Regardless of the theoretical adequacy, or otherwise (see Lerman, 1996) of drafting social aspects of knowledge growth onto Piaget's autonomous cognising subject who constructs knowledge from experience, social constructivist approaches such as those of Cobb and his colleagues have developed and emphasised the importance of mathematical content, of having students speak their developing constructions and of student *engagement* in learning generally. However, my argument in this paper centres on the notion that engagement can be empowering or not for students and should not be understood as always a positive force in numeracy development; if the teacher understands knowledge as purely cognitive, and the learner as naturally rational and autonomous, s/he may structure the social context in ways that unknowingly limit personal sense-making and empowerment. For example, Yackel and Cobb (1996) tell the story of Donna, who changes an answer after repeated questioning by the teacher although she was correct in the first instance. The teacher berates Donna for not standing firm on her convictions regarding the answer and tells her "So you should have said, Mr K. Six. And I can prove it to you". He then adds "I've tried to teach you that!" According to the teacher's reading of the situation, Donna has let him down in that she did not behave rationally and autonomously in strongly asserting and reasserting her answer. In remaining blind to the power relationships operating between teacher and student in the social context of the classroom, the teacher is able to blame Donna for not learning what he has *tried* to teach. The effect of interactions such as this could be, even though Donna knows the mathematical content (she has agency *with* the maths), that she also learns to doubt herself and her ability to participate correctly in the discursive field (she lacks agency *in* participation).

Social Life is Primary in the Construction of Knowledge

Jaworski (1994) claims that the move to an *inherently social* view of knowledge construction paralleled a move away from constructivist notions of development centering on internal mental structures developed through experience, to Vygotskian notions of cognitive abilities and capacities formed and constituted in social phenomena. For Vygotsky, a theory of cognition is a social theory where the personal is imbricated with the social and the public such that mental function cannot be considered independent of that interaction. Wertsch and Tulviste (1992, p. 548) state:

> Vygotsky's account of culture suggests that humans are never as autonomous and as free of outside interference as it might at first appear. Instead, human mental functioning, even when carried out by an individual acting in isolation, is inherently social, or sociocultural, in that it incorporates socially evolved and socially organised cultural tools.

Language for Vygotsky was fundamentally communicative and social, rather than merely a tool for reflecting cognitive processes (Burman, 1994). Language carries the social and cultural inheritance of the communities in which an individual participates and plays a central role in the development of consciousness.

The idea of consciousness formed and constituted in social phenomena through mediation by cultural tools and communication brings a new dimension to classroom practice in that it makes clear that mathematics education is not a neutral process. Students and teachers bring subjectivities and socio-cultural perspectives and values to bear on educational tasks and so it becomes important to both engage and *embrace* all learners in the classroom community. However, it is one thing to recognise and celebrate difference and quite another to ensure that this recognition leads to teaching practices that support numerate behaviour in the classroom. If we accept that learning is a process of internalisation, then internalisation has produced these differences and we must accept that what we started with, as in constructivist understandings of the individual, was biological (Henriques, 1984, p. 21). The notion of internalisation of knowledge through social practices is deterministic. As Gee (1995) says, humans use language and other sign systems as social tools in interaction and internalise patterns of tool-within-contexts-of-use as pieces of intra-mental furniture that bears the hallmarks of interactive use, but there is no space for individuals to critique or question the formulations they have swallowed.

The Learner in Context

Theories of socialisation are based on the notion of a rational humanist individual who acts according to constructed knowledge. The individual learner is an *epistemic agent, naturally* striving to resolve cognitive conflict, and a state of equilibrium (von Glasersfeld, 1989). In the classroom context it is taken for granted that learners behave rationally as Cobb (1986, p. 303) states: "it is assumed that children's behaviour is rational" and they are "expected to show intellectual and social autonomy in the classroom". Cobb et al. (1991b, p. 174) emphasise that: "The teacher trusted the children to resolve their problems and they trusted her to respect their efforts". Practice is based on humanist understandings of the individual and the environment or context of learning is assumed to be challenging yet supportive of all learners.

In many such classrooms reference is made to communities of collaborative practice where learning through doing, argument, experimentation and risk-taking are highly valued; they are valued because involvement in these processes is seen to be constitutive of students with inquiring minds who seek and find resolutions to meaningful problems. For example, the Australian Mathematics Education Program (1982) was based on the understanding that students who learn mathematics in challenging and problem solving contexts will be able to apply constructed knowledge and skills in practical ways, have a deeper understanding of the mathematics involved and show significantly increased ability to recall or reconstruct the mathematics. Again, such proclamations take for granted that active engagement is necessarily an *enabling* experience for students; they ignore how problem solving and investigations as commonly practised in schools may merely reinforce traditional binaries of correct/incorrect answers and successful/unsuccessful students.

A poststructuralist view of the social construction of knowledge holds that all knowledge is discursively constituted, that the learning context can be empowering or not, and that individuals are not essentially rational nor autonomous. All learners are constituted through discourses wherein relationships of power/knowledge position them as agentic or not; agency is thus problematic, it is fragmentary and transient, because it is discursively produced. Weedon (1987) makes clear that the use of the word discourse in poststructuralist theory infers more than ways of thinking and producing meaning as often used in psychological discourses: "Discourses", states Weedon (1987, p. 108) "constitute the 'nature' of the body, unconscious and conscious mind and emotional life of the subjects they seek to govern". This notion of constituted subjectivity goes against that of the rational choosing individual of socialisation theories. That is, persons will choose to do things, but always within the bounds of their discursive production; for example, it is unlikely that the young Aboriginal girls playing across the road in the park will choose to run for Prime Minister of Australia or become an airline pilot as such possibilities, for them, have probably not been discursively produced. As Henriques et al. (1984, p. 117) state: "The subject itself is the effect of a production, caught in mutually constitutive web of social practices, discourses and subjectivity; its reality is the tissue of social relations". This theorisation of the learner has important implications for numeracy beyond 2000.

Theories of subjectification provide useful analytical tools for making clear the coercive power of all discourses, including school mathematics. The tools of subjectivity and positioning with/in discourses enable teachers and researchers to examine teaching-as-usual for its constitutive effects. Theories of subjectification do not hold that teachers are shaping the *internal* beings of their students but rather that students, and teacher, are *subject to* relationships of power/knowledge within the discursive matrix. Poststructuralism does not allow the external/internal nor the passive/active divide. People are always learning, through positioning in overlapping discourses, to know themselves at times in powerful and libratory ways, and at other times in ways that deprive them of choice and the possibility of operating in powerful ways (Davies, 1994). This has important ramifications for teaching school mathematics and numeracy in school and beyond, though it does not carve a linear and seamless link between the two.

Agency, or power, in a poststructuralist sense, is always spoken of as "relationships of power" (Foucault, in Bernauer and Rasmussen, 1987, p. 11). Power is not possessed, it is not a commodity or gift. It is sustained in relations not persons. As Applebaum (1995) suggests, it is perhaps helpful to think of a physics of power instead of an economics of power; power is in all relationships, similar to gravitation among physical bodies. The imperative becomes not to establish who *has* power, but rather *how does power operate*? In all contexts, including the classroom, power operates to structure the possible field of action of others. Foucault (1982, p. 789) states: "It is a total structure of actions brought to bear upon possible actions; it incites, it induces, it seduces, it makes easier or more difficult; in the extreme it forbids absolutely; it is nevertheless always a way of acting upon an acting subject or subjects". Power relations in the classroom are constitutive of learners and degrees of agency with/in school mathematics.

In the past we have understood agency to lie in teachable attitudes and skills, and ultimately to ne an individual attribute or quality. We have become used to speaking in psychological terms of motivated/unmotivated, autonomous/dependent learners of mathematics. If, on the other hand, we see agency as discursively determined, we are obliged to take more seriously the extent to which classroom practices actually foster numerate behaviour in students. Numerate behaviour reflects a certain agency with mathematics and comprises intellectual and social aspects of knowing mathematics. An important issue here is that agency with/in mathematics is closely tied to having authorship of constructions and meanings and signifies "the capacity to speak/write and be heard, to have voice, to articulate meanings from within the collective discourses and beyond them" (Davies, 1991, p. 52). Teachers then need to examine very carefully whether classroom practices, for example completing a blackboard of *sums* or another page in a textbook, position the students as appropriately numerate for a world beyond 2000.

Interestingly, the extent to which students can establish themselves as numerate in the classroom has no certain relationship with particular teaching methods, resources or grouping arrangements a teacher chooses to use. Rather, the question is always how a discourse operates to positively position as many students as possible; all teaching methods and resources have empowering or oppressive potential. In the end, it is important that students exiting school for a postmodern world have come to know doing mathematics as a social practice, in school and beyond, and themselves as competent and agentic users of its powerful ideas. Educators and researchers over the past score of years have emphasised the importance of embracing and engaging students in learning mathematics, though in taking agency for granted they have neglected to examine the many ways in which classroom activities and processes limit the extent to which students are able to establish themselves as competent. For example, many teachers seek to make learning mathematics more enjoyable for students by taking them outside the classroom to, let's say, measure objects and complete graphs, though it is not at all clear that traditional power relationships between teacher and taught are interrupted. If not, students are in no real sense able to establish themselves as competently agentic and thus continue to come to know mathematics as comprising disconnected bits of information and skills, and themselves as passive receptacles of this arguably limited and limiting notion of mathematical knowledge.

Agentic numerate individuals, lifelong learners with inquiring minds, live out their apprenticeships, partly at least, in school. I would argue that inquiring, investigative, problem-solving minds are discursively constituted – they are not physiological or psychological attributes of individuals. To have agency, even if only for a moment, is to be recognised by oneself and others as one who can and should have voice within the discursive community, and also to be positioned as one who can interrupt and question taken-for-granted ways of operating. In school, learners need to experience such ways of operating with/in mathematics and to articulate how well classroom activities facilitate their developing mathematical understandings and ways of making sense of experience. The teacher might engage with students in making the operation of the discourse problematic; in this way school mathematics is recognised as a social practice and this recognition may be constitutive of inquiring and questioning mathematicians of the future.

Conclusion

Usher and Edwards (1994, p. 25) explain the foundations on which theories of socialisation are based:

> The very rationale of the educational process and the role of the educator is founded on the humanist idea of a certain kind of subject who has the inherent potential to become self-motivated and self-directing, a rational subject capable of exercising individual agency. The task of education has been understood as one of 'bringing out' of helping to release this potential.

I have argued that this particular understanding of the human subject is dangerous in that it takes the ability to be agentic as given; it diverts attention from all manner of educational and other social pressures that render this problematic.

I have used the poststructuralist notion of individuals constituted through discursive practices to suggest that many invisible, and at times oppressive, features of teaching interactions can variously position learners in school classrooms. While it is important that learners of mathematics construct powerful mathematical ideas and are socialised into ways of reasoning and conjecturing in mathematics, these observable features of learning conceal their constitutive powers. I am referring here to countless elements of interactions that might affect whether or not students come to know themselves, viscerally, as mathematically competent and agentic; for example, if the teacher always defers to the Question-Response-Evaluate questioning sequence, or always works from a textbook, the students are positioned as dependent and having to reproduce others' knowledge. In such cases, there is little room for student authorship of ways of making sense of mathematics and coming to appreciate its inherent integrity, nor of realising themselves as competent and agentic in its articulation. Although teachers are in no position to defer to student input regarding *what* mathematics is learned, there may be much to be gained from students' invaluable insights regarding the often unseen and unrecognised power relations affecting *how* it is learned.

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Mental Effort and Errors in Bracket Expansion Tasks

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Previous research has shown that when students expand brackets, errors tend to cluster around key positions due to increased working memory load. This study found that by neutralising the effects of position, the occurrence of error clusters was reduced. Furthermore, a self-rating mental effort instrument was employed which found a positive correlation between errors and mental effort. This instrument also detected subtle variations in mental effort between groups of varying mathematical ability.

The cause of mathematical errors can be quite complex, as errors can be caused by limitations in working memory (Kintsch & Greeno, 1985) as well as by lack of knowledge. Considerable research has been conducted into the nature and cause of mathematical errors. Hitch (1978) found that the solution to a mental calculation, such as 347 + 189, was highly dependent upon the problem solver's ability to hold and process information. If any of the initial information or interim subtotals were forgotten, errors would occur. Consequently, Hitch argued that mental arithmetic errors were caused by decay in the storage of problem information. More recent research by Ashcraft, Donley, Halas and Vakali (1992), and Logie, Gilhooly and Wynn (1994), has confirmed the link between calculation errors and loss of information. Furthermore, Campbell and Charness (1990) found that when students were mentally required to square large numbers (such as 74) by using a calculation algorithm, errors had a higher chance of occurring at "heavy traffic stages in the squaring task" (p. 887). Campbell and Charness argued that these stages corresponded with the high demands of keeping track of subgoals and their results. In addition, Ashcraft et al. (1992) reported that the retrieval of basic addition facts could also be affected by working memory demands. An increase in problem difficulty corresponded to less accuracy in recalling number facts. This last finding is of some significance as it suggested that working memory load affects the retrieval of information as well as its storage.

Much of the research into the relationship between working memory and errors has been conducted in the domain of mental arithmetic. However, evidence of this relationship has also been found in other areas of mathematics such as arithmetic word problems (Fayol, Abdi & Gombert, 1987), geometry (Ayres & Sweller, 1990; Ayres, 1993) and algebra (Ayres, In press). Fayol et al. employed word problems and found that the way that the problem texts were organised led to significant performance differences. Changing the order of the wording of the problems, for example, led to more errors and a reduced performance. Fayol et al. argued that certain text structures forced the participants to problem solve in a manner (bottom up) which overloaded working memory and forced errors. Similarly, Ayres and Sweller (1990) identified a specific error profile in two-move geometry experiments which required the compulsory calculation of a subgoal first before the ultimate goal could be reached. It was discovered that students made more errors in the calculation of the subgoal stage than on the goal stage. The design of these geometry experiments was such that all problems were counterbalanced. In other words, the application of a theorem such as the "angle sum of a triangle is 180 degrees" occurred an equal number of times as a subgoal and as a goal. Consequently, if all cognitive demands were equal, it was expected that errors connected to the triangle property would be equally distributed over both stages. This was not the case, as more errors were reported at the subgoal stage. Consequently, it was argued that errors were not necessarily made because the geometry theorems were poorly learnt, but because the subgoal stage demanded more working memory resources and forced errors.

Error clusters was also identified by Ayres (In press;) on bracket expansion tasks of the type: -3 (-4 - 5x) - 2 (-3x - 4). These tasks require the following four operations to be completed: -3 * -4 (Operation 1), -3 * -5x (Operation 2), -2 * -3x (Operation 3), and -2 * -4 (Operation 4). It was discovered that more errors were made during the expansion of the second bracket compared with the first bracket, and more errors were made during the second operation (Operations 2 and 4) compared with the first operation (Operations 1 and 3) within each bracket. Nearly all errors made involved the manipulation of signs. The clustering of errors indicated that some students could multiply -3 and -4 together correctly when they appeared during the first or third operation, but made errors (-12) when they appeared during the second or fourth operation. To explain this result, Ayres argued that more decision-making processes are made on the second operation within each bracket and during the expansion of the second bracket due to the dual-role of signs which link together brackets and operations. Consequently, as a result of increased decision making, working memory load was not equally distributed over the four operations. At points where the load was heaviest, information was either lost from working memory or incorrectly recalled from long-term memory and errors resulted.

To investigate the working memory load explanation further, Ayres (In press) completed an additional experiment using a dual-task methodology (see Ashcraft, 1995). Prior to completing a specific operation within a bracket problem, students were given a random sequence of letters to remember, which they were required to recall after completion of the computation. It was found that recall varied across the four operations. In particular, recall connected to the fourth operation was the poorest, suggesting that this operation caused the heaviest working memory load. Whereas, this result was consistent with a working memory load model, two anomalies were identified. Firstly, variations in recall were only found on operations that included an x-term. For operations that were purely numerical, no variation in recall was found. This was surprising, as it was found in earlier experiments that error clustering would occur regardless of the inclusion a pro-numeral. Secondly, the students made few calculation errors. This was also unexpected as students were required to simultaneously remember a random sequence of letters and complete a calculation. To explain these anomalies, Ayres suggested that because students were only required to calculate one operation per problem (instead of four), the overall working memory load could have been reduced.

The findings described in the previous paragraph are interesting, because if asking students to complete only one operation per bracket problem reduces working memory load and error rates, then this strategy may have the potential to be an effective instructional technique. Cognitive load theorists such as Sweller argue that instructional techniques are most effective when cognitive load is reduced (see Sweller, 1999). Consequently, this study was designed to investigate further the effects of requiring students to calculate only one operation on bracket expansion tasks that normally require four. More specifically it attempts to examine under this condition:

- whether error clusters appear;
- how students rate their own mental loads per calculation;
- whether error rates and self-rating mental loads vary according to mathematical ability.

Method

Participants

Sixty two students from Grade 8 of a Sydney high school participated in this study. On the basis of Grade 7 mathematics results the students had been streamed into three groups on their entry into Grade 8. The group of highest general mathematical ability is referred to as Group 1 (n =23), the next group as Group 2 (n=22) and the final group as Group 3 (n=17). All students had prior experience of expanding brackets of the type used in this study.

Materials and Procedure

A set of eight problems, each containing two brackets, was designed (see Table 1) and recorded on a work sheet. For each problem, one specific operation was targeted as the designated calculation for that problem. These operations were flagged for the students' benefit by drawing an arrow downward from the relevant term within the bracket. Below the arrowhead was drawn a dotted line indicating where the student was required to complete the given computation. For example, the fourth operation (-3 * -7) was the targeted calculation in Problem-1 (see Table 1) and consequently an arrow was drawn downward from the "-7" in the bracket. This indicated to the student that they were required to complete this calculation only. Overall, eight calculations were required. To detect error clusters without bias, the problem set consisted of a number of counterbalanced pairs. These pairs featured the calculations "-4*3" (operations 1 and 2 in problems 2 and 8), "-3*-7" (operations 3 and 4 in problems 6 and 1, "-3*-8x" (operations 1 and 2 in problems 5 and 3), and "-2*-9x" (operations 3 and 4 in problems 4 and 7). This design ensured that a calculation including a pro-numeral (x) and without a pro-numeral occurred in all four operational positions.

Table 1
The Problem Set Used in the Study

Problems	Targeted Operation	Code
1. 5 (3x - 4) - 3 (6x - 7)	4th	Op. 4
2. -4 (3 + 6x) + 3 (2x - 4)	1st	Op. 1
3 -3 (2 - 8x) + 3 (8x - 7)	2nd	Op. 2x
4. -4 (3x - 1) - 2 (9x - 5)	3rd	Op. 3x
5. -3 (-8x + 2) + 3 (2x - 4)	1st	Op. 1x
6. 5 (3x - 4) - 3(-7 + 6x)	3rd	Op. 3
7. -3 (2 - 8x) - 2 (-5 + 9x)	4th	Op. 4x
8. -4 (6x + 3) - 2 (4x + 5)	2nd	Op. 2

To measure mental load, a self-rating technique developed by Paas and van Merriënboer (1994) was employed. Using a 7-point Likert Scale (ranging from 1- very easy to 7- very difficult) students were asked to self-rate the mental effort involved in each calculation. Students were asked to do this immediately after the completion of each calculation. A designated space was provided on the work sheets for each problem. Before commencing the problem set, a practice example was given by the researcher. During this phase instructions were given and students were allowed to ask questions. Calculators were not allowed and sufficient time was given for students to complete the problem set. It is worth noting that this technique has been used successfully over the last six years in measuring the effectiveness of instructions (see Sweller, 1999). However, as far as the author is aware, it has never been applied in the context suggested in this study.

Results and Discussion

Analysis of Errors

The number of errors made by each student was recorded according to their positional operation and whether they included a x-term or not (see Table 2). A total of 60 sign errors were made. Very few arithmetical errors were made (10%) and were not included in the analysis of the data as they were not the focus of this study. The mean number of sign errors made were 0.3 ($SD = 1.4$), 1.1 ($SD = 1.3$) and 1.7 ($SD = 1.7$) for Groups 1, 2 and 3 respectively. A one-way ANOVA revealed significant between group differences: $F(2, 59) = 6.77$, $p < 0.01$. A post-hoc Tukey-HSD test revealed that the highest ability group (Group 1) made significantly fewer errors than the lowest ability group (Group 3), as might be expected. Group 1 also made fewer errors than Group 2, and Group 2 made fewer errors than Group 3, although these results were not significantly different.

To examine differences between the operations themselves, a number of paired t-tests were completed. No significant differences were found between Operation 1 and Operation 2 ($t = 1.00$), Operation 3 and 4 ($t = 1.00$), and Operation 3x and Operation 4x ($t = 0.33$). However, a significant difference was found between Operations 1x and 2x, ($t(61) = 4.8$, $p < 0.01$), indicating that students made more errors multiplying -3 and - 8x together when it appeared as the second operation rather than the first. Furthermore, the total number of errors made in the first bracket was not significantly different to the total number of errors made in the second bracket: $t(61) = 1.49$, $p = 0.14$). As no differences were found between the two brackets and Operations 3 and 4 (with or without x's) the results are not consistent with the Ayres (In press) study which found large differences on these comparisons when all operations were completed for each question rather than just one. Nevertheless, the significant difference found between Operations 1x and 2x, suggests that Operation 2x may have provoked an increase in working memory load at this point. It is also worth noting that operations with x caused significantly more errors than operations without x under a paired t-test: $t(61) = 4.18$, $p < 0.01$.

Table 2

Frequency of Errors made on each Operation by Group

	Op. 1 -4*3	Op.1x# -3*-8x	Op. 2 -4*3	Op. 2x -3*-8x	Op. 3 -3*-7	Op. 3x -2*9x	Op. 4 -3*-7	Op. 4x -2*9x
Group 1	0	0	1	3	0	1	0	1
Group 2	1	0	2	7	4	5	0	6
Group 3	0	1	0	9	4	5	4	6
Totals	1	1	3	19	8	11	4	13

Note. x indicates that a pro-numeral was present in the operation.

Analysis of Self-rated Mental Effort

Self-rating mental effort scores were recorded for each individual operation (see Table 3). A Cronbach alpha run on this data gave an alpha coefficient of 0.90 indicating that the instrument had high internal consistency. Generally students did not rate these tasks as difficult. A maximum score of 2.2 on any operation indicated that the students rated the tasks as easy to fairly easy. However, a MANOVA (Hotelling's Trace) using the eight operations as dependent variables found a significant difference between the groups: T = 1.98, p = 0.02. Uni-variate tests (see Table 3) revealed that operations 1x, 2, 3 and 4x had p-values of less than 0.01 and therefore contributed to the overall group differences. In these cases, the high ability group (Group 1) exhibited significantly lower scores than the other two groups. This was consistent over the other four operations as well, except for Operation 4 (-3*-7), where Group 2 rated their mental effort lower than the other groups.

Table 3

Self-rating Mental Effort for each Operation

	Op. 1 -4*3	Op.1x -3*-8x	Op. 2 -4*3	Op. 2x -3*-8x	Op. 3 -3*-7	Op. 3x -2*9x	Op. 4 -3*-7	Op. 4x -2*9x
Group 1	1.4	1.4	1.3	1.8	1.3	1.4	1.8	1.4
	(0.82	(0.50)	(0.47)	(0.74)	(0.88)	(0.59)	(1.23)	(0.58)
Group 2	1.8	2.1	2.1	2.2	2.1	2.1	1.6	2.2
	(0.81)	(0.75)	(1.02)	(0.81)	(0.75)	(0.90)	(0.91)	(1.10)
Group 3	1.9	2.3	1.9	2.1	2.2	1.7	1.8	2.2
	(0.86)	(1.16)	(1.03)	(1.03)	(1.48)	(0.85)	(1.07)	(1.03)
Combined	1.7	1.9	1.8	2.0	1.8	1.7	1.7	1.9
	(0.81)	(0.89)	(0.92)	(0.86)	(1.10)	(0.82)	(1.07)	(0.95)
F values	1.75	7.13**	5.24**	1.56	4.92**	3.67*	0.46	4.94*

Note. Standard deviations recorded in parentheses; *p < 0.05; **p< 0.01.

To investigate the differences in self-rating mental load within the problem-set, a number of matched t-tests were run on the counterbalanced pairs (see Table 4).

Table 4

Paired t-test Values on Paired Groupings for Mental Effort Measures

	Group 1 t values	Group 2 t values	Group 3 t values	Combined t values
Op 1 v Op 2	0.70	1.67	0.39	1.01
Op 1x v Op 2x	2.86**	0.90	1.33	1.31
Op 3 v Op 4	2.02^	2.13*	1.07	0.60
Op 3x v Op 4x	0.16	0.66	2.58*	1.25
Bracket 1 v Bracket 2	0.41	1.06	1.05	0.88
Numerical v Algebraic	0.56	3.58**	0.55	2.54**

Note. **p < 0.01, *p < 0.05, ^p = 0.06.

Although a number of significant differences were identified on these comparisons, there was little consistency between the groups. For example, in the comparison between operations 3 and 4 (no x's), the top group rated Operation 4 (1.8) more difficult than Operation 3 (1.3); whereas Group 2 rated Operation 3 (2.1) more highly than Operation 4 (1.6). The combined group data produced only one significant difference; viz, operations with algebraic terms were rated more difficult than purely numerical operations. Nevertheless, this result was consistent with the error data, which indicated that more errors were made on the operations that included an algebraic term. The error data also revealed that more errors were made during Operation 2x rather than 1x. However, only Group 1, who made few errors, ranked Operation 2x more difficult that Operation 1x. Despite the variations between the groups, there was a correlation of 0.65 ($p = 0.08$) between the error rate and the mean mental effort over the eight operations. Although this result fails to reach significance at the 95% level, it is fairly close and may indicate a real relationship between mental effort and the occurrence of errors.

General Discussion

Examination of the error profiles revealed that errors were not generally clustered around key points such as Operations 2 and 4 which tends to happen when students calculate all four operations (see Ayres, In press). This study therefore provides some evidence to support the hypothesis that asking students to complete one calculation only per question, reduces the overall working memory load. However, the data did not totally support this argument because one error cluster was observed when students made more errors on Operation 2x than 1x. It is worth noting that " -3*-8x" includes two negatives and a x term. Studies have shown that many students have great difficulty understanding the basic concepts of algebra (see Küchemann, 1981; Herscovics & Linchevski 1995; MacGregor & Stacey, 1996) and negative numbers (see Gallardo, 1994; Herscovics & Linchevski, 1995). It is therefore feasible that the manipulation of an algebraic expression with two negative numbers is fairly complex and may have exerted a heavier load on working memory than the other combinations. This increased load may have re-acted more directly with the bracket task to cause the error cluster. As the results are not conclusive, it is too early to assess the potential of a single calculation technique as an instructional method. Future studies might explore this idea further by directly comparing the single calculation method with the traditional full bracket expansion method.

The use of the self-rating mental effort instrument proved reliable. The instrument demonstrated a high internal consistency and was sensitive enough to identify differences between groups. For example, the group with the highest mathematical ability rated the overall task less demanding than the other two groups, as might be expected. A positive correlation (0.65) was found between errors and mental effort, suggesting that, as mental effort increases, errors are more likely to occur. Generally, calculations that included pro-numerals were rated as requiring more mental effort and tended to promote more errors. As self-rating scales of mental effort were found to be useful in this study, they may have a greater role to play in the learning and teaching of mathematics.

Finally, the use of errors in the teaching of mathematics has long been valued by mathematics educators (see Borasi, 1994; Ashlock, 1986). Some researchers (Brown & Burton, 1978; Resnick, Nesher, Leonard, Magone, Omason, & Peled, 1989) argue that error analysis is an important diagnostic tool in identifying common misconceptions and wrongly applied strategies. This study on errors, like others, has uncovered some useful information about student performance and perceptions. It is therefore argued that research into errors should be continued, as it provides a rich source of knowledge about the processes and influences involved in mathematical problem solving.

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Developing Task Specific Criteria: A Preliminary Report

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A case study methodology was utilised to document the processes involved in the development and marking of extended response tasks within a systemwide numeracy assessment. One of the tasks included in the final test is used to exemplify how numeracy and measurement requirements can be satisfied within this process. This paper is a preliminary report of a research project being undertaken as part of the development of the pilot of the NSW Department of Education and Training Secondary Numeracy Assessment Program.

Secondary Numeracy Assessment Program (SNAP)

The *Secondary Numeracy Assessment Program* (SNAP) is a systemwide assessment of students' numeracy skills at the beginning of Year 7. It consists of three sections: short responses, connected responses, and extended responses. The focus of this research project is the development of scoring rubrics for the two extended response tasks that would be professionally marked.

Students are given twenty minutes to work on each of the extended response tasks. Each student is provided with a plastic instrument sheet, which features a ruler, a protractor, a 2 mm grid and a 1 cm grid. Triangles, as shown in Figure 1, is one of the tasks in the 2000 pilot.

Extended Response Task 2: Triangles

Part A

Using your plastic instrument sheet, draw three different types of triangles.

Name the types of triangles you have drawn.

Explain how your triangles are different from each other.

Use appropriate mathematical terms in your work when naming and explaining.

Part B

If one triangle has a large perimeter than another triangle, will it always have a larger area?

Answer Yes or No and give reasons for your answer.

To do this task, you need to:

1. Draw some triangles
2. Show their areas and perimeters
3. Use the correct units of measurement for the areas and perimeters.
4. Use the plastic instrument sheet to help you.

Figure 1. Final version of Triangles task.

Teachers, parents and students are provided with detailed reports of each student's achievement on the numeracy aspects assessed in the SNAP paper. One of the reports provided for teachers indicates how students achieved on each item and task in the test. Teachers are expected to use this information to support them in planning relevant learning experiences for the students in their classes and school.

Students' Work

Good tasks can provide information about the extent of the student's knowledge being assessed and give information about a number of mathematical ideas, and the extent to which the student has integrated them and is able to use them in new situations (Webb & Briars, 1990). Open-ended questions generate a variety of mathematically valid responses, which differ only in the quality of understanding displayed (Clarke, Clarke, & Lovitt, 1990).

With richer responses from students given open-ended questions, teachers need to develop skills in interpreting evidence and using the results (Bryant & Driscoll, 1998). Assessing student's written work on a problem could be done using analytic scoring, focused holistic scoring or general impression scoring. Analytic scoring involves the use of a scale to assign points for certain aspects thus it becomes possible to identify specific areas of strength and weakness. Focussed holistic scoring enables a numerical score to be assigned to the total solution based on specific criteria. It is most appropriate when requiring a general rating of the processes used and where reliability of the scoring procedure is important. General impression scoring is based on implicit criteria to rate a total solution numerically (Charles, Lester, & O'Daffer, 1987).

Open-ended tasks have the potential to enrich assessments as they reveal different information from closed tasks. Teachers are able to apply scoring rubrics to open-ended tasks. The scoring allows the tasks and student performance to be evaluated (Sullivan, 1999). Well-designed tasks with appropriate scoring rubrics can elicit information about students' deeper understanding of mathematical concepts and provide reliable and valid data for measurement purposes (Callingham, 1999).

Stephens and Sullivan (1997) conducted research on the viability of using open tasks for system-wide assessment. Teachers were able to consistently apply a scoring rubric to the tasks, which allows the tasks and student performance to be evaluated and independent judgements to be validly made.

In analysing students' responses to performance assessment tasks, Peressini and Bassett (1996) found that inadequacies in test construction are more explicit in open-ended tasks than traditional mathematical tasks, enabling the teacher to enhance the quality of the open-ended tasks through refinement based on evidence from student responses. Unexpected responses to open-ended tasks can be examined to ascertain whether the unexpected response is due to the construction of the task.

The Research Project

How can extended response tasks be scored to provide appropriate, reliable and valid information to teachers on the numeracy skills of each student?

A three-phase research project is being developed. Phase One involves the development of the tasks and accompanying scoring rubrics. A single embedded case study provided an appropriate methodology for research into how extended response tasks could be scored. The phenomenon under study, the development of the means of scoring extended response tasks, is not readily distinguishable from its context, which is the processes of developing the Secondary Numeracy Assessment Program (Yin, 1994). The case is one complete cycle of development for the pilot in 2000 involving selecting two tasks from over twenty tasks. The researcher is an active participant and leader within the team developing the assessment. Triangulation of data has been achieved through collecting data on the tasks and development process from different periods of time (trialling, retrialling, and review) and from different persons involved in the process. Peer examination, involving asking colleagues to comment upon findings as they emerge occurs at each stage of the development process.

The Rasch model has been used to inform decisions about the appropriateness of tasks and criteria during the first phase.

> The Rasch model is different from other models including latent models. It is primarily designed to aid test constructors in the process of constructing measurement variables. The primary difference is

> that it is a model of intent. That is, it has evolved from a theory or theoretical position. As such, it has come before any data are collected. The data are then collected and compared to the model, which is a mathematical rendition of the intention. If the data do not accord with the model, then this is evidence that the data do not reflect the intention, and further qualitative work is required. (Tognolini, 1996, p. 29)

Phase Two is the marking of the tasks using these scoring rubrics. Markers in both schools and a central location will use the marking procedures to mark the students' work. An evaluation involving survey and focus group discussions will be conducted with the school based marking process. The central marking operation involves monitoring through analysis of markers' responses. QUEST and RUMM are the two software programs used for Rasch analysis of dichotomous and polytomous data within this project.

Phase Three is the reporting of the students' results to schools and parents. The reports for schools will include a table indicating the scores for each student on each criteria. Both qualitative and quantitative research methodologies will be used to investigate the use and implications of the report data for the extended response tasks.

The Process for Developing Tasks and Criteria

Developing, panelling and trialling initial tasks. Contract and team writers developed more than twenty five tasks. A panel of numeracy, curriculum and equity experts critiqued the tasks in terms of quality and appropriateness for a Year 7 numeracy assessment. Tasks were subsequently modified or rejected. Sixteen of the accepted tasks were trialled. About two hundred students, representing various demographic and ability groups, attempted each task.

The Triangles task, as shown in Figure 2, is located within the mathematics key learning area. This task is a good question (Clarke, Sullivan, & Spandel, 1992) that requires more than recall or repetition of a fact or procedure to complete the task, is open-ended and has the possibility for the student to learn about mathematics in doing the task. The skills required to attempt the task are identified in the *Mathematics K-6* (1989) and *Outcomes and Indicators for Mathematics K-6* (1998) as being appropriate for students in Stage 3 (Years 5 and 6). It is expected that all students would be able to start the task as some of the required skills are appropriate for Stage 1(Years 1 and 2).

Extended Response Task 2: Triangles – Part A	*Extended Response Task 2: Triangles – Part B*
Draw at least 3 different types of triangles. Label your triangles. Explain how your triangles are different from each other. *Remember* • *Use appropriate mathematical terms in your labels and explanations.*	If a triangle has a large perimeter, will it have a larger area? *Remember:* • *Show how you worked out your answer, including calculations.* • *You could use the triangles in part (a) to help you answer this question.* • *You may need to draw some more triangles to test whether your answer is correct.*

Figure 2. Trial version of Triangles task.

Observing students completing tasks. Personnel with expertise in numeracy and/or assessing open-ended tasks observed students attempting the tasks. These observations were conveyed to the development team. Some tasks were rejected based on these observations. In trialling, all students attempted the Triangles task and only a few students asked for clarification about the task.

Developing draft criteria. Eight tasks were selected for developing draft criteria. The numeracy skills needed to complete each task were identified. Student work samples were examined for evidence of those skills. While students would have had to use certain skills to complete the task, there was not always evidence in the student's work that the skill had been used, the extent to which the skill had been used, or how the skill had been used. There were other skills that were evident only in some students' work, even though other students had provided work that indicated a higher level of numeracy achievement. Draft criteria could be developed for five tasks.

An extensive list of skills (Figure 3) could be generated for the Triangles task and evidence of these skills was found in the student work samples.

Drawing triangles
Labelling triangles
Measuring angles
Measuring area of triangles
Measuring perimeter of triangles
Use a ruler to measure length
Use a grid to measure area
Checking answer to problem
Using calculations to solve problem
Give reasons for solution
Use square centimetres
Using drawings to solve problem
Know the units for length are cm
Know the units for area are cm2
Solving problems
Use terminology
Use centimetres
Identifying angles
Definitions of triangles
Properties of triangles

Figure 3. Skills for Triangle task.

Marking criteria were developed which markers were able to use consistently. Criteria A1 and B7 are shown in Table 1.

Table 1
Triangles: Trial Marking Criteria for Criteria A1 and B7

Part	Criteria	Scoring	Descriptor
A1	Construction of triangles	0	No triangles drawn
		1	1 triangle drawn matches label
		2	2 different triangles match labels
		3	3 different triangles match labels
B7	Uses drawings	0	None drawn
		1	Drawings with errors, irrelevant or lacking information
		2	Drawings accurate, relevant, appropriate information

Marking trialled tasks using draft marking criteria. Four tasks were marked. Only one group of four markers marked each task. Each group had a team leader who provided training on the criteria and practice marking to check consistency of application of criteria. The markers worked with one team leader referring any questionable work samples to the team leader. At the conclusion of marking, the markers provided comments and recommendations on the wording and appropriateness of the tasks, the appropriateness and quality of the criteria, and the marking procedures utilised. One task was rejected as a result of observations during marking.

Analysing data of student achievement on tasks. The results for three tasks were analysed using QUEST and RUMM analysis. The data for each criterion for the three tasks was examined for reverse thresholds, inappropriate item fit, spread of thresholds and mean ability of groups for each score of each criteria. The analysis of the marking data revealed that the majority of the criteria satisfied the measurement requirements. For Triangles, Table 2, Figure 4 and Figure 5 show that criteria B7 satisfied these requirements while criteria A1 has reverse thresholds and an unacceptable fit. Suggestions were made for the other criteria, which indicated that a valid set of criteria could be developed for this task. Numeracy and measurement aspects were considered in selecting the two tasks for the pilot.

Table 2
Percentages of Students Achieving each Score for Criteria A1 and B7

Criteria	Score 0	Score 1	Score 2	Score 3
Criteria A1	32%	18%	18%	32%
Criteria B7	37%	38%	25%	0%

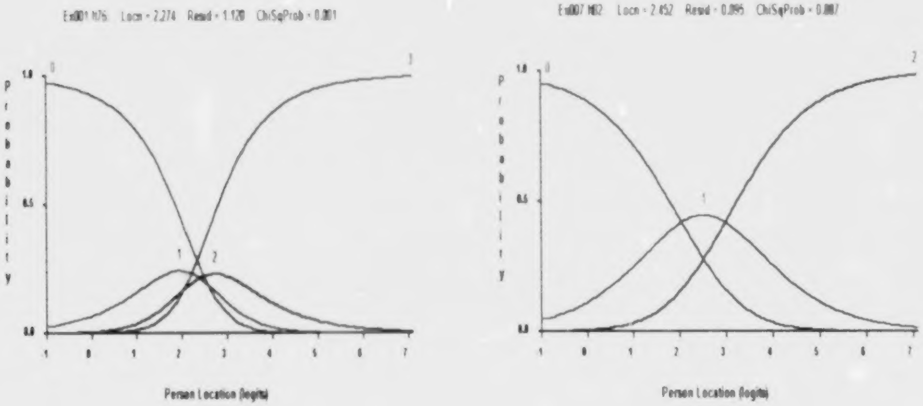

Figure 4. Category probability curves for trial criteria A1 and B7.

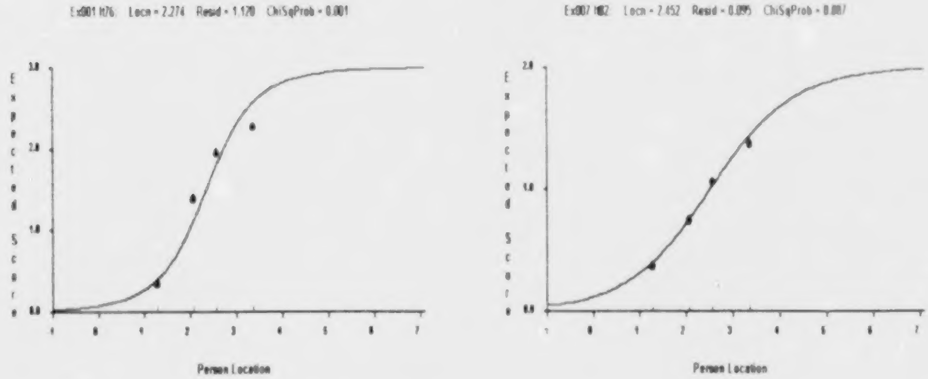

Figure 5. Item characteristic curve for trial criteria A1 and B7.

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Refining tasks and criteria. Changes were made to the layout and wording of the tasks based on the information from the trialling and analysis of the trialling data. The retrialled (and final) version of the Triangle task is shown in Figure 1. Revised criteria were developed for marking the retrial. Table 3 shows the redefined A1 criterion.

Table 3

Triangles: Retrial Marking Criteria for Criteria A1 and A4

Part	Criteria	Scoring	Descriptor
A 1	Drawing different triangles	0	No triangles or only 1 triangle drawn or draws triangles which are similar
		1	Draws 2 triangles which are not similar or draws two different triangular shapes
		2	Draws 3 dissimilar triangles
A4	Uses knowledge of shape definitions	0	No evidence of shape definitions
		1	One shape definition correct
		2	Two shape definitions correct
		3	Three shape definitions correct

Retrialling tasks. Both tasks were retrialled with more than one hundred and sixty students in four schools. The four schools were diverse in their student demographics and student ability based on past performance of Year 7 students at the schools.

Marking of retrialled tasks. Again, each task was marked with only one group of markers using similar procedures to the first trialling process. Extensive notes were taken during the marking process for use in developing the detailed marking procedures.

Analysing retrial data and student work samples. As for the first trialling process the marking data was analysed using QUEST and RUMM. The criteria were modified and recoding or rescoring occurred with the criteria. Acceptable criteria have an appropriate spread of thresholds, no reverse thresholds, the mean ability increasing with score within the criterion, and appropriate fit for the criterion.

Analysis of the retrial data indicated problems with some of the criteria for Part A of the Triangles task. Figure 6 shows the reverse thresholds for criteria A4. These criteria were redefined and all the work samples rescored. Discussion of the work samples led to further modifications and rescoring. The revised criterion for A4 is shown in Table 4. The data was analysed to reveal acceptable statistics, as shown in Figure 6, for the criteria. The validity of the criteria for a numeracy assessment was paramount in making the modifications to the criteria. The skills being assessed by the criteria needed to be appropriate numeracy skills and the criteria needed to be valid instruments to measure the numeracy skills indicated.

Developing draft marking procedures. The agreed criteria were elaborated with detailed explanations, examples and references to actual student work samples. A draft was developed and critiqued through the use of contentious work samples and questions of clarification.

Table 4
Triangles: final version of marking criteria for criterion A4

Part	Criteria	Scoring	Descriptor
A4	Explanation of different types of triangles	0	Incorrect descriptions/definitions
		1	Writes 1 or 2 descriptions/definitions of geometrical properties of their triangles which are correct
		2	Writes or indicates 3 descriptions/definitions of geometrical properties of their triangles which are correct and consistent with both the labels and drawings

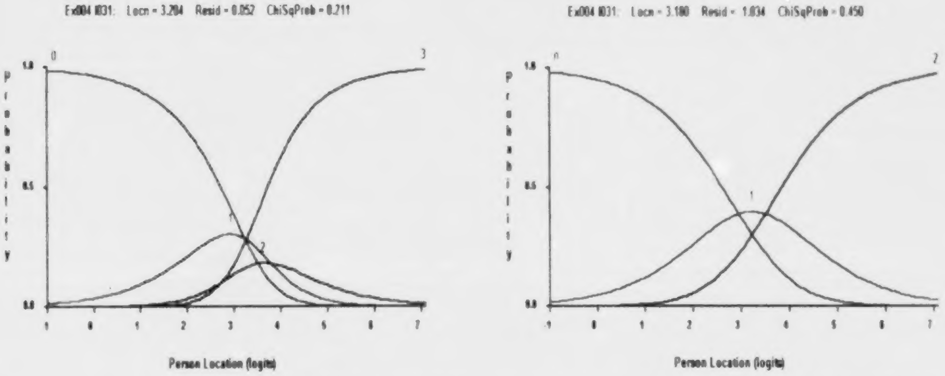

Figure 6. Characteristic probability curves for two different versions of criteria A4.

Refining marking procedures. Experts in numeracy, literacy, curriculum, assessment and measurement reviewed the marking procedures. The document is currently being finalised with modifications based on three review cycles.

Marking of extended response tasks. The marking procedures will be used to mark the extended response tasks in the pilot of SNAP. Teams of markers, both in selected schools and at a central location will be trained. The marking process will include vertical and horizontal audits for reliability and consistency of marking.

Recommendations for Future

The data from this case study indicated that the process up to the first trialling phase was useful in selecting and refining the tasks. As indicated in other research, the students' responses provided information that could be used to enhance the quality of the tasks. The process of retrialling enabled the enhanced tasks to be used for refining the criteria.

Further research is needed on the aspects of task design which impact on student achievement. Research on the differences in the teaching and assessing contexts would provide valuable information on why student achievement in assessments may not be consistent with teachers' expectations. The data from this project revealed a difference between the perceived difficulty of these tasks and the actual difficulty of the tasks. There were indications that teachers may actively assist students when doing numeracy assessment tasks in the classroom. In assessment situations students are expected to work without this assistance. This has possible implications for classroom-based assessments of what students are able to do.

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Construction of a Numeration Model: A Theoretical Analysis

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This paper analyses the major concepts and processes within decimal-number numeration in terms of the cognitions inherent in their structure to determine how they are related to each other, how they are different and, if possible, to determine which are more difficult conceptually. It synthesises this analysis into a cognitive model that provides a framework for decimal-number knowledge.

For my doctoral dissertation, I developed a test to assess students' understanding of place value and their ability to identify, regroup, count, order, and estimate decimal numbers (limited to tenths and hundredths). This test was administered after formal instruction was complete to determine what structural knowledge of decimal numbers the students had abstracted. In undertaking the structural analysis of the numeration processes, I drew heavily on the properties of the decimal number system and the following research.

In their analysis of the semantic features of whole numbers and decimal fractions, Resnick et al. (1989) argued that the two domains shared the semantic features of base, order, place value, and multiplicative structure (a property that is derived from the a result of the notion of base 10 embedded in the decimal number system). However, Resnick et al.'s list of semantic features did not provide the cognitive processes (e.g., determining number names, determining the size of a number, determining whether two numbers are the same or different in value, changing the name of a number without changing its value) that are required to process decimal numbers.

Children's ability to interpret the part/whole subconstruct of fractions is highly dependent on their notion of a unit (Steffe, 1986) and their ability to partition (Kieren, 1980; Lamon, 1996; Pothier & Sawada, 1983) and reconstruct units (Behr et al., 1992; Nik Pa, 1989; Steffe, 1986). Steffe (1986) identified four different ways of thinking about a unit, namely, *counting (*or *singleton) units*, *composite units*, *unit-of-units* and *measure unit,* with each type representing an increasing level of abstraction.

Behr et al. (1992) analysed the complexities involved in *unitising* (identifying singleton units) and *reunitising* (identifying composite units and unit-of-units) whole numbers and decimal numbers (which require invoking the measure unit to relate the part to the whole). (In this paper, the term "unitising" is meant to include the opposite process, namely, partitioning.) They claimed that the ability to reunitise (i.e., to change one's perception of the unit) requires a flexibility of thinking that may be beyond young children. This has importance for hundredths (and thousandths, etc) which need to be thought of as a number of hundredths sometimes and as a number of tenths at other times. Similarly, tenths need to be thought of as a number of tenths or as a number of hundredths.

To these notions of unit, I added a focus on the concomitant processes that are used to derive either "superunits" (i.e., larger numbers) or "subunits" (i.e., smaller numbers) (Baturo, 1998). I argued that, in decimal-number numeration, generating superunits requires an understanding of grouping units by powers of 10 whilst generating subunits requires an understanding of partitioning units by powers of 10. Halford (1993) claimed that transforming units and keeping track of the transformations requires a great deal of flexible thinking and increases cognitive load.

These research findings provided me with a cognitive scaffold for undertaking an analysis of the cognitions required for semantic knowledge (understanding) of the central concepts and processes of decimal-number numeration, namely, place value, number identification, regrouping, counting, ordering, and estimating. No cognisance was taken of syntactic knowledge (e.g., reunitising tenths as hundredths by inserting an "imaginary zero" after the tenths digit).

Structural Analysis

Cognitions required for place value. Understanding place value means associating place names with particular *positions* on a continuum, and knowing the *order* of the place names. For example, to write the standard number (i.e., 4.25) when given a set of jumbled places (e.g., 2 tenths, 5 hundredths, 4 ones) requires semantic knowledge of the place names, their position, and order – not a reliance on the syntactic "left to right" order in which the digits of a number are usually written. Understanding place value also means identifying those numbers whose values were unchanged by the insertion of zero in the leftmost or rightmost places (e.g., 03.78 and 3.780). This item requires a clear understanding of the *order* of the places so that internal insertions which alter the original place values can be recognised (as opposed to the zero insertion at the rightmost place of a decimal number which is the syntactic feature associated with renaming one decimal place as another). In their study of decimal-number computation, Hiebert and Wearne (1985) drew attention to the difficulties students have in knowing when zeros can be inserted without changing the value of the given number.

Like all other digits that comprise a given number, zero has a cardinal value as well as a positional value in relation to the number. Thus, in 74.05, the 0 means 0 tenths and indicates that, in the continued grouping of 7405 hundredths, there were no tenths left after grouping to make ones but there are 5 hundredths remaining. Therefore, from a place value aspect, 0 should be considered in the same way as other digits. However, as the identity element for addition, zero could be taken to mean "having no effect" because the addition (or subtraction) of 0 will not change the number to be operated on. Hence, 74.05 could be thought of as 70 + 4 + 0 + 0.05 or as 70 + 4 + 0.05 (employing the additive property of the decimal number system) showing that 0 tens has no effect in this situation and can be omitted from the calculation. However, students often translate "having no effect" as "zero means nothing (and can therefore be ignored)" (Baturo, 1998); overgeneralising of this rule can lead students to interpret 74.05 and 74.5 as having the same value. Thus, the use of zero in a number can invoke inappropriate access.

The "value" part of place value is dependent on the *base* and either grouping or partitioning the unit in terms of the base to make "superunits" or "subunits" respectively. Inherent in the processes of grouping and partitioning is size and a direction in which the change in size is represented. For example, grouping in terms of the base (10 in our system) makes a number 10 times larger which, in the decimal number system, is indicated by a movement of digits one place to the left. Conversely, partitioning in terms of the base makes a number 10 times smaller and this is shown symbolically by a shift of the digit/s one place to the right. Thus, a leftwards direction is associated with becoming larger in value whilst a rightwards shift is associated with becoming smaller. Therefore, place value is *continuous* (i.e., across whole-number and decimal-fraction places), *bi-directional* (to the left is 10 times larger in value – ×10; to the right is 10 times smaller in value – ÷ 10), and *exponential* (i.e., nonadjacent places are related by powers of 10 – 10^2, 10^3, etc.)

In Baturo (1998), I defined the continuous, bi-directional, and exponential properties of the decimal number system as multiplicative structure. That is, if a given number is multiplied by powers of 10 (base), it becomes larger in value, not from a change of digits but from a change in position. Conversely, if a number is divided by powers of 10, it becomes smaller in value because of the change in position.

Cognitions Required for Number Identification

Numbers can be represented in pictorial, word, or digit form. When presented in digit form (e.g., 4.7, 6.39), the name of the endmost decimal-fraction place must be known in order to identify the numbers as tenths and hundredths respectively (i.e., *position* and *order*). When decimal numbers are presented in word form, the fraction name can be seen so it is merely a process of relating the fraction name to the appropriate fraction place (*position*). However, when presented in pictorial form (e.g., a prototypic 10 × 10 grid for hundredths), an ability to *unitise* (in conjunction with the measure unit) is required as well as a knowledge of position and order (to record the identified fraction correctly).

Cognitions Required for Regrouping

Numbers presented in pictorial, word or digit forms can be regrouped. Underlying the regrouping process is the notion of conservation of number which allows mathematicians flexibility in problem solving (e.g., knowing that 1 can be renamed as 10 tenths or 4 quarters, etc. without changing the value of the given number). Thus regrouping is one of the most powerful processes in mathematics. There are two main categories of regrouping tasks –those that require the given "unit" to be "superunitised" (i.e., transformed to a larger unit) and those that require the given unit to be "subunitised" (i.e., transformed to a smaller unit).

Solution paths (See Figure 1) show that, for each word/digit representation, the following cognitions are employed: (a) *additive structure* (e.g., 86 t = 80 t + 6 t); (b) *equivalence* (e.g., 80 t = 8 ones – superunitising; 1 one = 10 t – subunitising); (c) *position*; (d) *order*; and (e) *base*. However, when the number to be regrouped is in pictorial form, different cognitions are required, namely, *unitising* and *reunitising* (which requires the measure-unit). Figure 2 shows the complexity of thinking (drawing on Behr et al's, 1992, numeration) required for the simplest (Baturo, 1998) of these types, namely, forming subunits (e.g., renaming 2 tenths as 20 hundredths).

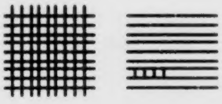
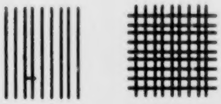
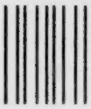
Superunitising		Subunitising	
86t 3h = __	7 t 14 h = __	2.09 = __ t __ h	0.52 = 4 t __h
86 t 3 h	7 t + 14 h	2.09	0.52
= 80t + 6t + 3h	= 7 t + 10 h + 4 h	= 2 ones + 9 h	= 5 t + 2 h
= 8 × 10t + 6t + 3h	= 7 t + 1 t + 4 h	= 2 × 1 one + 9 h	= 4 t + 1 t + 2 h
= 8 × 1 one + 6 t + 3 h	= 8 t + 4 h	= 2 × 10 t + 9 h	= 4 t + 10 h + 2 h
= 8 ones + 6 t + 3 h	= 0.84	= 20 t 9h	= 4 t 12 h
= 8.63			
	74 h = 70 h + 4 h = 7 t 4 h		3 t 8 h = 30 h + 8 h = 38 h

Figure 1. Classification of the regrouping items and their possible solution paths.

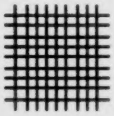


First: Unitise as tenths

10 × 1-unit is unitised as 1 × 10-units (which becomes the measure unit to which the shaded parts are related) so 2 is thought of as:

$2 \times {}^{1}/_{10}(1 \times 10\text{-units})$ or ${}^{2}/_{10}(1 \times 10\text{-units}) = 0.2$.

Second: Reunitise as hundredths



1 × 10-units is reunitised as 1 × 10 × 10-units by partitioning (mentally or physically) each tenth into 10 equal parts.

1 × 10 × 10-units can then be reunitised as 1 × 100-units (which becomes the new measure unit) so 2 becomes 20 and can be thought of as:

${}^{20}/_{100}(1 \times 100\text{-unit})$ or $20 \times {}^{1}/_{100}(1 \times 100\text{-unit}) = 0.20$

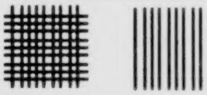
Figure 2. Cognitions involved in reunitising a pictorial representation of tenths as hundredths.

Cognitions Required for Counting

Counting forwards and backwards can be considered as the repeated addition or subtraction of an iterated unit. There are basically two types of tasks, namely, when the iterated unit is given (e.g., count in tenths from 6.7), or when the iterated unit has to be identified (e.g., continue the sequence, 6.5, 6.6, 6.7). I designated these types as Counting A and Counting B respectively. For Counting A types, a knowledge of *position* and *order* is required to identify the digit to which 1 tenth (or 1 hundredth, etc) has to be added and a knowledge of *order within places* (0<1<2<3...<9) is required to determine the direction (forwards/larger or backwards/smaller). In some cases (e.g., 6.9), the addition of 1 tenth will result in the need to *reunitise* so that the next number becomes 7 and not 6.10. For Counting B types, the iterated unit and the operation (+, −) needs to be determined. Determining the iterated unit requires a knowledge of *position* and *order across places* (i.e., tens>ones>tenths>hundredths) to decide which place/s remain constant and which place changes first.

Cognitions Required for Ordering

Ordering can be thought of as determining which of two or more given numbers is the smaller/est or larger/est in value so, basically, there is only one form of activity. The most common activities given to students are: (a) to select, from two given numbers, the number that has the larger or smaller value; and (b) to arrange three or four given numbers in ascending or descending order. Both types of activity involve comparing one number with another and require a knowledge that only like places (*position*) can be compared, combined with a knowledge of *order across places* (e.g., tens>ones>tenths>hundredths) and *order within places* (0<1<2<3...<9). However, pictorial comparisons of decimal fractions require perceptual as well as cognitive processes (see Figure 3).



The internal partitions of each shape need to be removed mentally to allow a focus on the relative size of the shading.

Figure 3. Comparing pictorial representations of unlike decimal fractions.

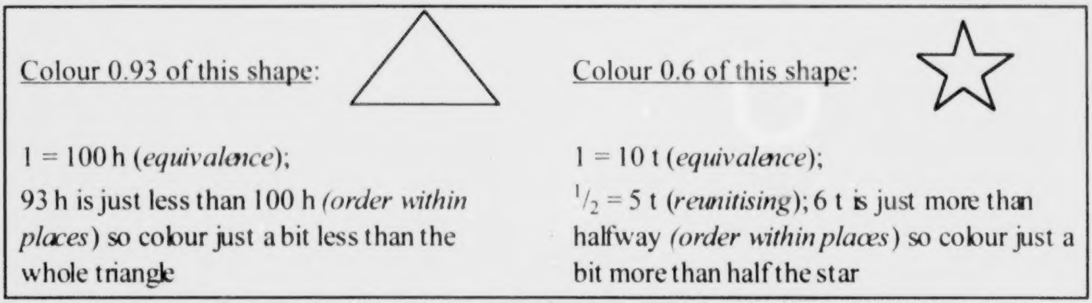
Cognitions Required for Estimating

This category subsumes approximating of which there are two types of activity (designated as Approximating A and Approximating B respectively), namely, to approximate a given decimal number to the nearest whole number (or tenth, etc.) or to identify a given number that has been approximated to the nearest whole number (or tenth, etc.). The A type is closed in that it can produce just one correct answer whilst the B type is open as there will be a range of numbers that will satisfy the condition. For the A types (e.g., round 6.2 to the nearest whole number), the following thinking may be employed.

> 6.2 has 6 ones and 2 tenths (*additive structure* – 6 ones + 2 t); 6.2 comes between 6 ones and 7 ones (*order within places* – identifying the parameters); 1 = 10 tenths (*equivalence*); $^1/_2$ = 5 tenths (*unitising* – related to *base*); 2 tenths is less than 5 tenths (*order within places*) so it is less than halfway between 6 ones and 7 ones; therefore 6.2 is closer to 6 ones than to 7 ones.

For the B types (e.g., what number could be approximated to 6), a knowledge of *order within places* is required to know that there are numbers that are both smaller and larger than 6. *Unitising* the whole as halves to determine the range of numbers on "either side" of the 6 is required in conjunction with *reunitisng* (halves as tenths) In effect, this type of approximation is the opposite process to that required for the ordering task in which the parameters of a range are given and selection of a number that falls within that range.

Whilst approximating activities produce exact answers, estimating activities usually produce answers that are inexact. When applied to pictorial representations, a knowledge of *order within places*, *equivalence* and *base* are required (see Figure 4).



Colour 0.93 of this shape:

1 = 100 h (*equivalence*);

93 h is just less than 100 h *(order within places)* so colour just a bit less than the whole triangle

Colour 0.6 of this shape:

1 = 10 t (*equivalence*);

$^1/_2$ = 5 t (*reunitising*); 6 t is just more than halfway *(order within places)* so colour just a bit more than half the star

Figure 4. Possible solution paths for nonprototypic estimation tasks.

Summary

Table 1 summarises the cognitions embedded in decimal-number numeration processes. It shows the structural complexity of regrouping and estimating, and the comparative structural simplicity of ordering.

Table 1

Cognitions Embedded in the Decimal-Number Numeration Processes

	Numeration Processes					
Cognition	PV	NI	RG	CO	OR	ES
Position	✓	✓	✓	✓	✓	✓
Base	✓	✓	✓	✓	✓	✓
Order	✓	✓	✓	✓	✓	✓
Equivalence	✓	–	✓	–	–	✓
Unitisation	–	✓	✓	✓	–	✓
Reunitisation	–	✓	✓	–	–	✓
Additive structure	–	–	✓	✓	–	✓
Multiplicative structure	✓	–	✓	–	–	–

Note. PV = place value; NI = number identification; RG = regrouping; CO = counting; OR = ordering; ES = estimating; – = not required.

Table 2 provides the test results from the six classes (two schools) involved in the main study of my PhD. The results generally support the hypotheses that regrouping and estimating are structurally complex whilst ordering is comparatively simple.

Table 2

Class Test Means (%) With Respect to the Decimal-Number Numeration Processes

	Numeration Processes					
Class	PV	NI	RG	CO	OR	ES
6A	39.7	68.5	39.8	63.4	73.6	57.9
6B	39.5	64.8	30.4	67.4	73.0	57.9
6C	48.1	69.6	45.8	74.9	71.6	74.6
6D	30.2	50.1	39.1	57.8	71.3	46.3
6E	43.2	62.4	30.5	57.0	58.8	44.8
6F	48.9	67.6	34.0	66.2	73.4	52.7

Constructing the Numeration Model

The test results showed that students' understanding of the cognitions embedded in the numeration processes varied considerably. For example, Table 2 revealed that place value knowledge was generally very poor. However, Table 3 shows that this was due mainly to the items related to multiplicative structure – students' understanding of position and order was generally reasonable. The disparity in means indicates that multiplicative structure is structurally more complex, and at a higher level, than position and order.

Table 3
Cognitions Embedded in the Place-Value Category of the Test

	Class					
Cognition	6A	6B	6C	6D	6E	6F
Position	79.3	76.9	81.5	61.6	72.4	76.9
Order	65.5	64.6	67.1	37.7	64.6	61.2
Multiplicative structure	21.8	15.8	25.6	20.7	27.3	36.5

As a result of similar analyses of the other numeration processes, I categorised the cognitions into the following three levels.

- Level 1 is associated with *position*, *base* and *order* and, as shown by Table 1, these cognitions are required for all processes. Therefore, without these cognitions, students would have no chance of processing decimal numbers. On the other hand, having Level 1 knowledge alone is not sufficient for a full understanding of all decimal-number numeration processes. For these reasons, I defined this level of cognitions as *baseline knowledge*.
- Level 2 is associated with unitising and equivalence. One or both of these cognitions appears to be required for Level 3. For this reason, I defined this level of cognitions as *linking knowledge*.
- Level 3 is associated with reunitising, additive structure and multiplicative structure. These cognitions appear to provide the superstructure for integrating the other levels and, for this reason, were defined as *structural knowledge*.

As a result of the foregoing analyses, I constructed a numeration model (Figure 5) to show the levels of cognitions and how they appear to be connected within and between the levels.

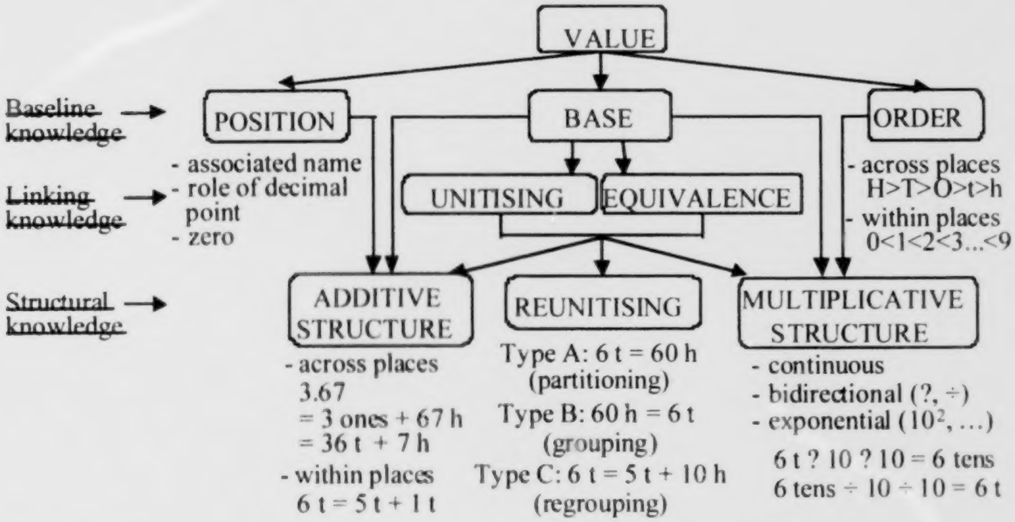

VALUE
Baseline knowledge
POSITION
BASE
ORDER
- associated name
- role of decimal point
- zero
Linking knowledge
UNITISING
EQUIVALENCE
- across places H>T>O>t>h
- within places 0<1<2<3...<9
Structural knowledge
ADDITIVE STRUCTURE
REUNITISING
MULTIPLICATIVE STRUCTURE
- across places
3.67
= 3 ones + 67 h
= 36 t + 7 h
- within places
6 t = 5 t + 1 t
Type A: 6 t = 60 h
(partitioning)
Type B: 60 h = 6 t
(grouping)
Type C: 6 t = 5 t + 10 h
(regrouping)
- continuous
- bidirectional (?, ÷)
- exponential (10², ...)
6 t ? 10 ? 10 = 6 tens
6 tens ÷ 10 ÷ 10 = 6 t

Figure 5. Cognitions and their connections embedded in the decimal number system.

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The model portrays position, base (10) and order as the baseline knowledge from which all decimal-number numeration knowledge is derived. These are elementary cognitions that have a unary structure, that is, they consist of static memory-objects (Derry, 1996), for example, the names of the places and their positions. Inherent in the notion of position is knowledge of the associated place names, the role of the decimal point, and the effect of zero in a variety of places (external and internal). Zero plays an important role in the syntactic knowledge associated with abstract reunitising (i.e., inserting a zero after the tenths digit makes hundredths and, conversely, cutting off the 0 hundredths makes tenths), thinking which may or may not be associated with the semantic knowledge required for reunitising tenths as hundredths and hundredths as tenths.

Equivalence and unitising (Level 2 linking knowledge) are shown as emerging from the notion of base. These cognitions are dynamic in that one object is transformed to another through some operation. Therefore, they are binary in nature and, according to Halford's (1993) complexity metric, represent relational mappings.

Reunitising, additive structure and multiplicative structure (Level 3 structural knowledge) emerge from an amalgamation of Level 1 and Level 2 knowledge. They incorporate ternary relations, that is, they are transitive in nature (e.g., 6 tenths × 10 = 6 ones; 6 ones × 10 = 6 tens; ∴6 tenths × 10 × 10 = 6 tens). Halford (1993) claimed that ternary relations are the basis of system mappings and consequently are much more complex cognitively than binary relations. To add to the complexity, some cognitions (order, additive structure and multiplicative structure) encompass different understandings – a consideration across places and within places (additive structure), different types of reunitising processes (Baturo, & Cooper, in press), and the continuous, bidirectional and exponential properties of multiplicative structure.

For the students in my study, additive structure tended to dominate multiplicative structure (e.g., associating +, – or ×, – instead of ×, ÷ relationships for growth to the left and right respectively). This may have been the result of overextended representation of numbers with base-10 blocks. With respect to reunitising, the students' results indicated that Type C regrouping was more difficult than the other two when decimal numbers in digit form were being considered. However, when decimal numbers were represented in pictorial form, Type B reunitising (i.e., grouping to form "superunits") was found to be more difficult. I believe that this phenomenon could be attributed to teaching that focuses on grouping whole numbers *but not decimal numbers* and partitioning fractions but not whole numbers. Teaching needs to provide the reverse experiences as well, namely, partitioning whole numbers to make other whole numbers (e.g., thousand to hundreds) and grouping fractions to make larger fractions or whole numbers.

The numeration model described in this paper is proposed as a signpost for future research in the domain of decimal-number numeration. As my test results showed, the students had major problems in processing decimal numbers and their teachers were unsure as to how decimal-number numeration processes could be taught effectively, indicating that there continues to be a need for vigorous research in this area.

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Modes of Representation in Students' Explanations

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Mathematics curricula commonly promote the development of problem solving, reasoning and communication skills. Students need to be able to communicate their mathematical ideas; written communication is therefore an important part of the assessment process in mathematics. When writing explanations students need to consider and choose an appropriate mode of representation for their response. This paper examines students' modes of representation used in solutions for two problem solving tasks and considers the issue of judging mathematical communication.

Communication is an integral feature of current curriculum reforms in mathematics (Australian Educational Council, 1991; Department for Education, 1995; Ministry of Education, 1992; National Council of Teachers of Mathematics, 1998). Communication is a key aspect of learning. Students need to communicate if they are going to make sense of mathematics; communication has both cognitive and social significance in the classroom (Hiebert, 1992). It is through communicating mathematical ideas that students become engaged in 'doing' mathematics. "Doing mathematics means agreeing on assumptions, making assertions about relationships, and checking if the assertions are reasonable" (Hiebert, 1992, p. 444).

> Communication in and about mathematics serves many functions. It helps to (1) enhance understanding, (2) establish some shared understandings, (3) empower students as learners, (4) promote a comfortable learning environment and (5) assist the teacher in gaining insight into the students' thinking so as to guide the direction of instruction. (Mumme & Shepherd, 1990, p. 18)

It is asserted that the use of assessments in mathematics should be consistent with the goals of educational reform and reflect current curriculum statements (Begg, 1998). These goals reflect a change in emphasis from traditional content to the processes. Assessment tasks should therefore measure achievements in the mathematical process skills such as problem solving and communication. They should be carefully constructed so that students are encouraged and motivated to solve problems and communicate their mathematical thinking (Clarke, Waywood, & Stephens, 1993; Miller, 1991; Szetela & Nicol, 1992). An appropriate task should also provide quality insights into students' mathematical understanding National assessments in New Zealand in recent years have shown a trend to provide students with more authentic assessment tasks and to have students write explanations and justifications as part of the solution process (Bicknell, 1998). Questions are more likely to be constructed so that they allow students to construct solutions and provide visible records of their thinking processes, instead of simply recording an answer.

Problems that require students to give explanations and justifications are usually cognitively more complex. "Cognitively complex tasks should be challenging and unusual so that students have to construct strategies and procedures for solving the problems; rote methods should not suffice" (Magone, Cai, Silver, & Wang, 1994, p. 320). The tasks should also allow multiple forms of representation, multiple solution methods or even multiple answers. However, Linn, Baker and Dunbar (1991) warn that it cannot be assumed that open-ended tasks require more complex cognitive processes than response-choice items. An analysis of a task's complexity can be based on the task features that make it complex and a consideration of actual student responses (Magone, Cai, Silver & Wang, 1994).

Recent developments in cognitive science highlight the value of examining more than the correctness of answers to problems. It is important to also examine characteristics of the solution including problem solving strategies, errors and representations (Resnick, 1988). Representational tools are a valuable aid to the thinking and communication processes. The National Council of Teachers of Mathematics (1998) identifies representations as one of its standards and states that: "Mathematics instructional programs should emphasize mathematical representations to foster understanding of mathematics"(p. 204). Students may use multiple representations but it is the act of representing the problem that helps students to articulate, and clarify ideas; the ideas can then be evaluated and extended.

Allowing students to choose modes of representation makes it more likely that students will give an indication of their problem-solving processes; it helps them to show their solution strategies (Magone, Cai, Silver, & Wang, 1994). An examination of modes of representation is a valuable part of the process of describing student performance qualitatively (Cai, Magone, Wang, & Lane, 1996). Explanations and justifications can be reliably and efficiently judged, providing diagnostic information on the level and quality of individual students' understandings (Niemi, 1996). The information from the analysis of students' responses can then be used in classroom instruction so that together the teacher and students develop a qualitative framework for interpreting and evaluating explanations

The Study

The findings that follow are taken from a larger study which focused on the writing of explanations and justifications. The research was conducted at a large provincial co-educational secondary school and involved 36 students from two classes. The students completed a problem-solving task sheet of five problems. These items were modelled on externally-mandated national examination questions from the New Zealand School Certificate examination in mathematics. The first part of the examination uses response-choice items and short answers. The second section has more cognitively complex tasks.

The problems presented to students were, in contrast to short-answer and multiple choice items, 'open-ended'. Open-ended tasks according to Cai, Lane and Jakabcsin (1996) are more informative and make mathematical communication part of the assessment. They require students to not only produce answers but to show solution processes and give justifications for answers. In the tasks provided, students were required to not only select and produce an answer but to show their working; to explain and justify their answers. Follow-up semi-structured interviews were conducted to find out more information about the student's responses. The students were able to refer to their solutions during the interview.

Two problems from the study are presented and a qualitative analysis of student responses provided. Students' responses were analysed for the quality of the mathematical argument and the mathematical correctness; this has been reported elsewhere (Bicknell, 1998). This paper focuses specifically on the modes of representation used for the explanations.

Results and Discussion

Problem 1

The first problem (Figure 1) was a measurement problem designed to assess students' understanding of volume and their ability to choose and justify selecting a particular thickness (ie the concept of average). They were also required to provide an explanation and finally give a justification for the choice of degree of accuracy.

Problem 1:

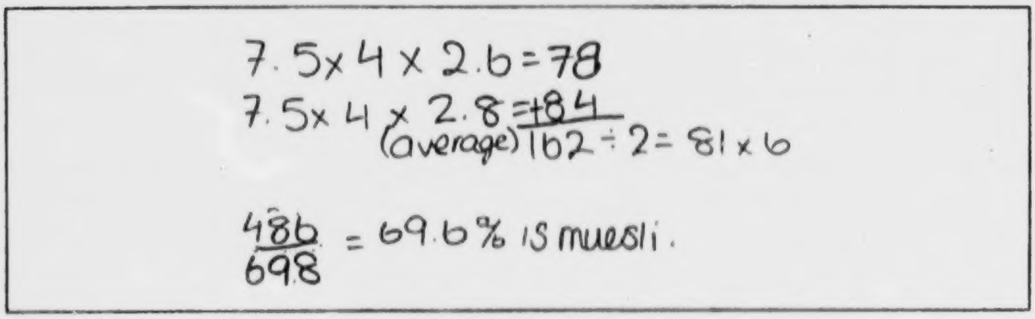
Derryn investigated the packaging of snack bars. She measured the packet with a ruler and found that it was 16.5 cm long, 9.2cm high, and 4.6 cm wide. She calculated the volume to be 698 cm^3 (3sf).
The packet had 6 muesli bars in it.
Each muesli bar was 7.5 cm long, 4 cm wide but the thickness of the bars varied between 2.6 cm and 2.8 cm.
Find the volume of the 6 muesli bars as a percentage of the volume of the packet.

- *Explain what you are calculating at each step and show your working.*
- *Round your answer appropriately, stating the degree of accuracy*
- *Justify why you have chosen this degree of accuracy.*

Figure 1. The measurement problem.

Responses were analysed and categorised according to the mode of representation used. The students' modes of written representations were categorised as: symbols only; symbols and words; or a combination of symbols, words and diagrams. Only three of the students provided a response using symbols only, devoid of any verbal explanation of what they were calculating. The remainder all included verbal explanations with 56% of the students writing an explanation using symbols and words. A combination of symbols, words and diagrams was the mode of representation chosen by 36% of the students.

Kim's solution was an explanation using symbols only (Figure 2). She lacked confidence in her ability to give a satisfactory explanation and was unsure as to whether her chosen mode of representation was sufficient. In the follow-up interview Kim reflects on her answer: "I'm not quite sure whether I've explained it. It's a bit messy and I'm missing the labels so people won't know what I'm doing. I don't explain at the start what I'm doing."



$7.5 \times 4 \times 2.6 = 78$

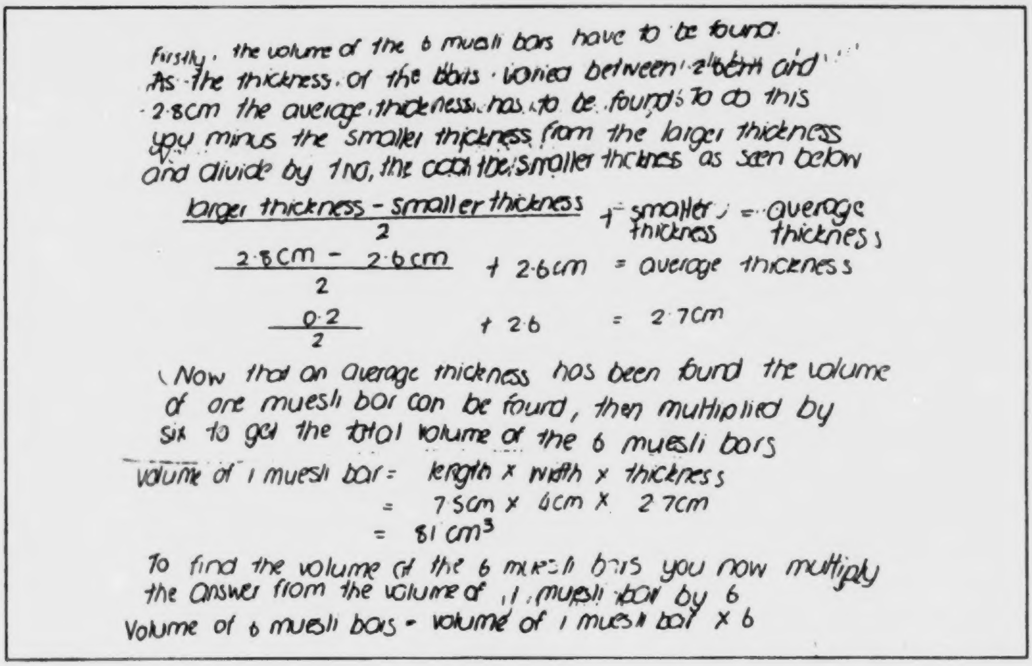
$7.5 \times 4 \times 2.8 = \underline{+84}$

(average) $162 \div 2 = 81 \times 6$

$\frac{486}{698} = 69.6\%$ is muesli.

Figure 2. An explanation using symbols.

Jessica's answer (part of which is shown in Figure 3) is an example of a comprehensively written explanation using symbols and words. When interviewed Jessica revealed that she was uncertain about whether this mode of representation was what was expected and was concerned that she had written too much. She recognised that she invariably wrote excessively and liked to write in prose so that an examiner was clear about what she had to say.



Firstly, the volume of the 6 muesli bars have to be found.
As the thickness of the bars varied between 2.6cm and
2.8cm the average thickness has to be found. To do this
you minus the smaller thickness from the larger thickness
and divide by two, the add the smaller thickness as seen below

$$\frac{\text{larger thickness} - \text{smaller thickness}}{2} + \text{smaller thickness} = \text{average thickness}$$

$$\frac{2.8cm - 2.6cm}{2} + 2.6cm = \text{average thickness}$$

$$\frac{0.2}{2} + 2.6 = 2.7cm$$

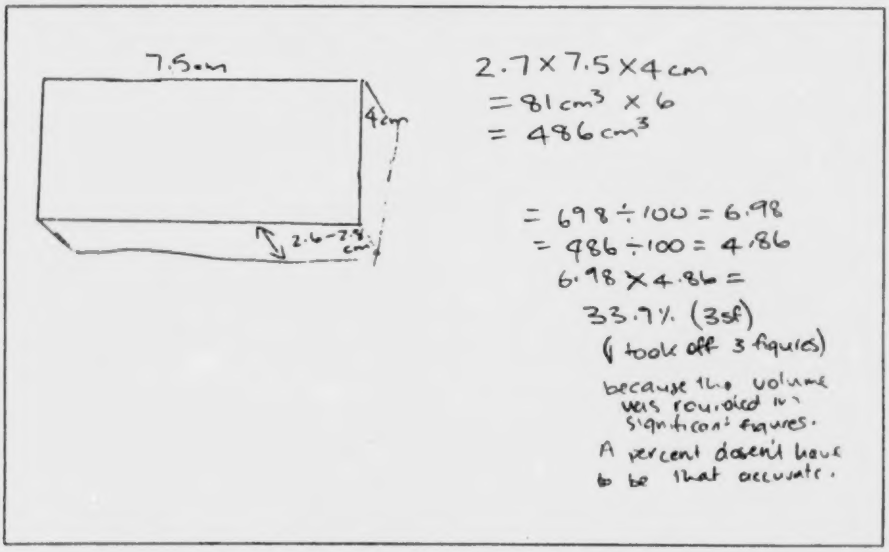
Now that an average thickness has been found the volume
of one muesli bar can be found, then multiplied by
six to get the total volume of the 6 muesli bars

volume of 1 muesli bar = length × width × thickness
= 7.5cm × 6cm × 2.7cm
= 81 cm^3

To find the volume of the 6 muesli bars you now multiply
the answer from the volume of 1 muesli bar by 6
Volume of 6 muesli bars = volume of 1 muesli bar × 6

Figure 3. An explanation using symbols and words.

Many students included a diagram in their responses. Jan's is one example (Figure 4). Jan justified her use of a diagram and commented: "I like using diagrams; it helps you know which bit you're talking about." Another student explained: "A scale-type drawing gives you a better picture of what you're doing. It shows you how to find things and if you are thinking appropriately."



$2.7 \times 7.5 \times 4$ cm
$= 81 cm^3 \times 6$
$= 486 cm^3$

$= 698 \div 100 = 6.98$
$= 486 \div 100 = 4.86$
$6.98 \times 4.86 =$
33.9% (3sf)
(I took off 3 figures)
because the volume
was rounded in
significant figures.
A percent doesn't have
to be that accurate.

Figure 4. An explanation that includes a diagram.

There were common concerns from the students about not knowing whether they had provided a satisfactory explanation and whether their chosen mode of representation was what was required. They really had little confidence that a teacher would accept their mode of representation as appropriate. Those who combined modes were "covering their options" and ensuring that their explanation was comprehensive enough.

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Problem 2

In order to answer this problem (Figure 5) the students had to firstly extract the two items of data needed and in their response convey the idea of quantity. Secondly, they had to give a process description explaining the calculation. The majority of students (89%) were clearly able to state the two items of data. The difficulty of defining and interpreting the scope of the explanation was illustrated by the number of students (7) who elaborated on how the data was to be collected. These ideas related to how to conduct a survey, data gathering techniques, and finding averages.

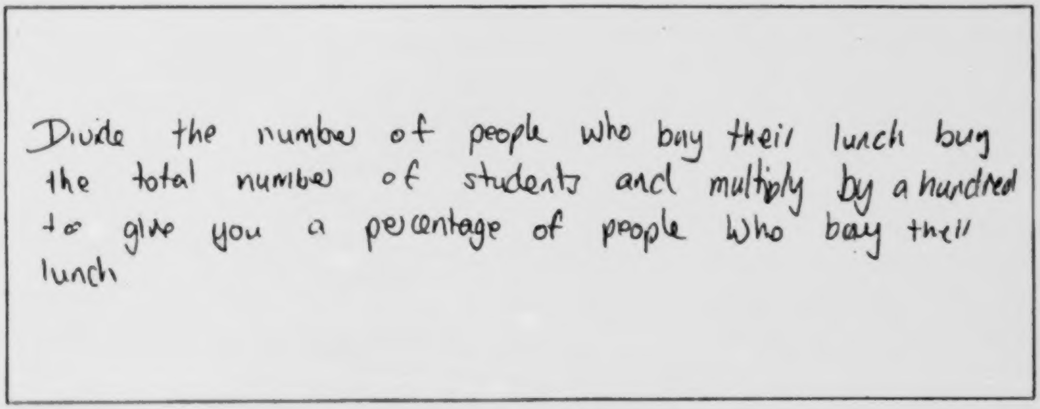
> Problem 2
>
> James and Richard are gathering data for a survey on students' lunchtime eating habits. They wish to find the percentage of students who buy their lunch from the school canteen.
>
> State the two items of data they need to gather and **explain** what calculation they need to make in order to be able to calculate the percentage of students who buy their lunch from the school canteen.

Figure 5. The statistics problem.

The responses were categorised according to the mode of representation, namely; the use of words, symbols, algebraic expressions and whether a worked example was used. He resulting distribution is shown in Table 1.

Table 1
Modes of Representation for Problem 2

Mode of Representation	Frequency
Explanation using words only	11
Explanation using words and symbols	15
Explanation using an algebraic statement	2
Explanation using words and an algebraic statement	2
Use of a worked example	1
No explanation	5



Divide the number of people who buy their lunch buy the total number of students and multiply by a hundred to give you a percentage of people Who buy their lunch

Figure 6. A verbal explanation.

Many of the students (30%) provided a verbal explanation as shown in Richard's example (Figure 6). Richard explains why he wrote his explanation exclusively in words: "I thought

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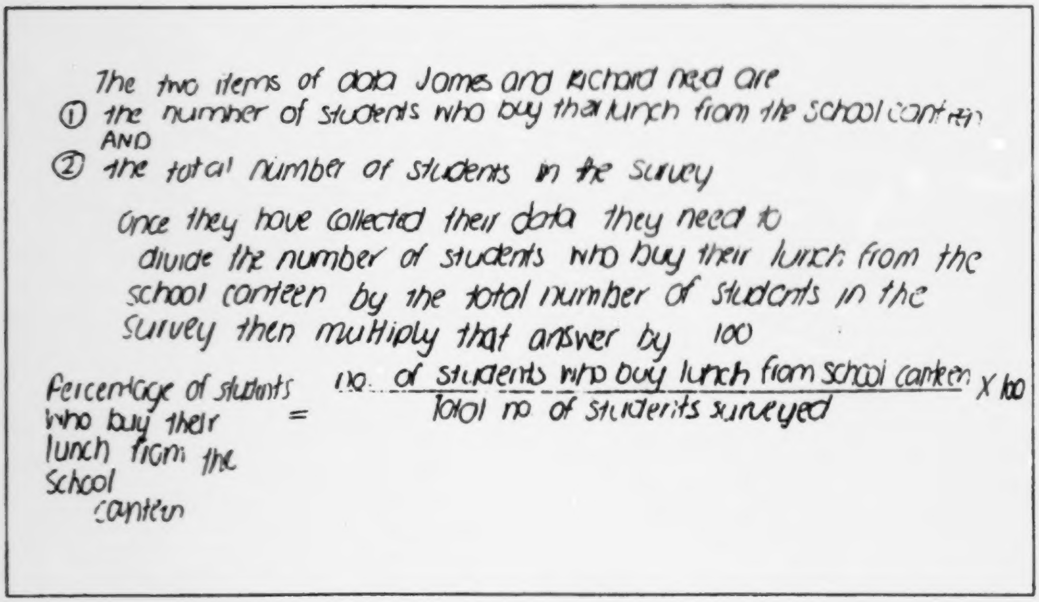
that I'd get more marks writing it down like that, but again I probably didn't really know. I saw it as a 50% chance of getting it right."

The preferred mode of representation was to combine both words and symbols (Figure 7). Paula explains: "I wrote it all down and then replaced what I'd said with an equation. I think that in order to make sure that you've got what the marker requires you need to write it all down."



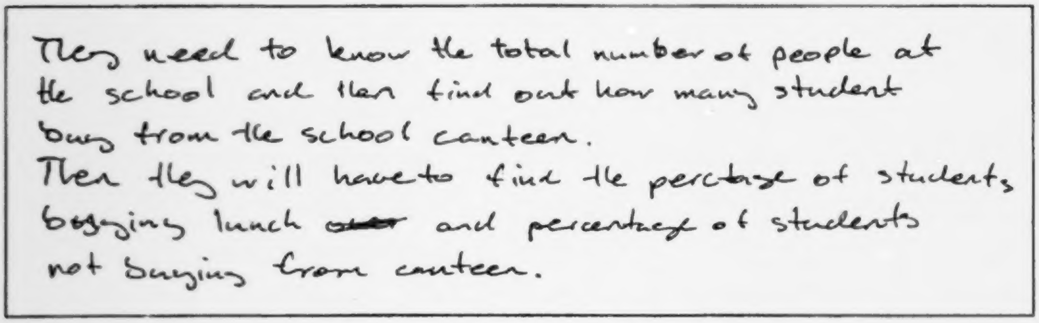
The two items of data James and Richard need are
① the number of students who buy thar lunch from the school canteen
AND
② the total number of students in the survey

Once they have collected their data they need to divide the number of students who buy their lunch from the school canteen by the total number of students in the survey then multiply that answer by 100

$$\text{Percentage of students who buy their lunch from the school canteen} = \frac{\text{no. of students who buy lunch from school canteen}}{\text{Total no of students surveyed}} \times 100$$

Figure 7. An explanation combining words and symbols.

Two students used only algebraic statements but explained the meaning of each variable, whilst five students did not attempt to explain the required calculation. A student who provided a comprehensive verbal and algebraic answer expressed doubt about whether this was appropriate because it all seemed a bit simple. In contrast, Mark believed quite confidently that his solution (Figure 8) was appropriate. He explained: "I answered the question because it didn't say solve it, it just says explain your calculations and I've done that and I've written it in sentence form."



They need to know the total number of people at the school and then find out how many student buy from the school canteen.
Then they will have to find the perctage of students buying lunch [illegible] and percentage of students not buying from canteen.

Figure 8. An incomplete verbal explanation.

Conclusions and Implications

Students represented solutions in a variety of ways; they used symbols only, words only or used a combination of symbols, words, and diagrams. The students chose varying forms of representation for each of the problems for a variety of reasons. Some students felt that one form over another would receive more marks and was therefore superior in assessment terms; others chose a form that they felt was sufficient to explain their problem-solving processes. A few students duplicated their explanations using different modes to ensure that they had provided what was expected; to ensure that they were covering all 'expected' options and therefore would not sacrifice any marks in the assessment process.

Students had personal preferences for particular modes of representation in their explanations. One of the concerns is that this preference was not based on a sound mathematical reason. There was a clear common concern about not knowing what was the expected mode of representation and the question surfaced: "Is one mode of representation superior to another?" They expressed doubts about whether in the assessment process their chosen mode of representation would gain full recognition. This element of insecurity and doubt about what is the best or preferred way for representing solutions has direct implications for teachers.

One of the teacher's goals in the mathematics classroom is to promote the development of communication skill. The teacher's role is to help students learn to use representations flexibly and appropriately in their explanations and justifications. Students need to develop a repertoire of representations so that they can choose an appropriate mode to support a particular solution. This goal can be achieved through presenting students with different levels and modes of representation and asking students to evaluate responses in terms of the quality of the mathematical communication.

The classroom situation should also provide students with the opportunity to share and discuss their alternative modes of representation. This process contributes to the development of sociomathematical norms (Yackel & Cobb, 1996). Teachers can help establish a normative understanding of what is an acceptable explanation and justification. Criteria can then be established for judging mathematical communications and for establishing high quality explanations using a variety of modes of representation.

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What is Taught versus What is Learnt: The Case of Linear Measurement

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This paper presents the findings of a study that examined the results of 120 students selected according to their performance levels in mathematics in Year 1 to Year 5. The results show that while most high-ability students had a conceptual understanding of length, the majority of the lower-ability students did not acquire important concepts relating to the linear nature of units and took longer to acquire basic measuring skills. This knowledge is essential for its application in perimeter, area and volume, and in topics that rely on the understanding of scales, such as directed number gauges, and graphs.

Introduction

While performance on mathematics tests have shown that some cultural groups in the United States have made significant gains in mathematics, Lindquist (1989, p. 4) commented that it is imperative to "...take a look at our low-ability students." A survey of the research literature reveals that scant attention has been paid to students who perform poorly in mathematics. For example, such students may take longer to complete tasks, may not understand mathematical concepts and may make errors in basic calculations. Lindquist suggests these students need a curriculum designed so that they can make sense of mathematics, instead of an emphasis on learning procedures that they may not understand or see as relevant.

Low-achieving students may face years of frustration studying material that they may see as irrelevant (Tayler, 1995). These students will require specific assistance from teachers, yet there seems to be little information to identify their particular difficulties. Additional research is required to pinpoint the specific problems of low-achieving students, particularly in measurement, because little appears to be known about the development of measurement concepts by these students.

Students, especially low-ability students, may be taught techniques for measuring lengths, rather than the principles that govern the measurement process. Such techniques may not provide the basis for developing ideas of units and scales or a framework for later measurement concepts. If students do not have an understanding of linear units they are unlikely to succeed in more complex areas of mathematics; for example, area and volume in which length units are extended to two and three dimensions respectively. Crucial to an understanding of measurement is the nature of the parts (or units) that are counted (Hiebert, 1984). The cognitive significance of the distinction between counting discrete objects and the 'counting' of a continuous property defined as length was noted by Piaget, Inhelder and Szeminska (1960) and, more recently, in a pedagogical sense by Wilson and Rowland (1992).

Researchers have suggested that measurement instruction should begin with informal units because the use of informal units promotes understanding of measurement processes by helping students to see the relationship between the 'continuous' nature of length and the 'parts' that may be counted (Wilson & Osborne, 1988). However, unless the process of measuring with informal units is linked to the construction of a scale, students may not see the relationship between this process and its formalisation as an abstract representation of standard unit, the centimetre, in the form of a ruler. Although rulers are "...an indirect method of laying down units of length end-to-end" and the numerals on the ruler indicate the number of unit lengths used (Thompson & Van de Walle, 1985, p. 8), students, especially low-achieving students, may not perceive this structure. Thus, even such a common tool as a ruler, may be understood only in a limited way.

This distinction between counting discrete objects and subdividing a continuous property into units is crucial to understanding the importance of 'zero' in its role as an indicator of no length. The importance of 'zero' in the construction of measurement instruments, such as rulers, tape measures, and student-constructed tools should be made clear to students (Thompson & Van de Walle, 1985). Ward's work (1979) with ten-year-old students on the construction of graphs has indicated the importance of the fundamental properties of scales. He also emphasised that students need to understand the point of origin and the iteration of measurement units. Familiarity with the process of 'renaming zero' is an important skill in flexible use of scales, as solutions can then be obtained by counting units or by subtraction. Such skills are important when a length to be measured cannot be aligned with zero, or when changes in measurements must be considered.

The aim of this paper is to investigate the performance of students in Years 1 to 5 on a variety of length measurement tasks using a common measuring tool, a ruler. The students were chosen by teachers to represent the range of mathematical ability in the different classes. The results of a subset of ruler and scale questions are presented; the items are part of a larger study of the development of children's understanding of linear measurement.

Methodology

The design of the study was cross-sectional; 120 students from Years 1-5 (aged 6 to 10 years) were selected from four state primary schools in a medium to low socio-economic area of Sydney. Each class teacher selected six students: one girl and one boy considered 'above average', 'average', and 'below average' in terms of mathematical achievement.

Table 1

The set of Tasks Involving Ruler Use

Task	Description
1	Draw a line 7 cm long above a ruler printed on the page (0 is aligned with the end of the ruler).
2	Draw a 9 cm line using a wooden ruler (0 is not aligned with the end of the ruler).
3	Measure a 6 cm line. (0 is not aligned with the end of the clear plastic ruler).
4	Measure a line that is longer than the ruler—Years 1 and 2 measured a 16cm line, Years 3, 4 and 5 measured a 26cm line—using the same ruler as for Task 3.
	The ruler used in Tasks 5 to 7 was a clear plastic strip on which a centimetre scale had been marked. Different modifications were made to this "ruler" in each task.
5	Measure a 9cm line with a "ruler" that had been cut at the 3.5cm mark. The scale was marked from 4 to 20cm.
6	Measure an 11cm line using a "ruler" showing centimetre marks without a corresponding numeric scale.
7	Draw a 6cm line above a printed "ruler". The marked scale, beginning with 6 and ending with 13 only comprises part of the line segment. A 3.5cm unmarked line extends to the left of the 6 and a similar line extends to the right of the 13.
8	Count five sea horses shown on a card and state what the '5' represents. Then explain what '5' on the ruler represents and identify a single unit.
9	Use 2 paper clips to measure a 28cm line; note there is a fraction of a unit remaining.
10	Make a ruler using paper clips as the unit of measure (given a long rectangular strip of cardboard and a supply of paper clips).

The subset of questions (see Table 1) was designed to investigate knowledge of rulers. The students were given four common classroom tasks (Tasks 1-4) involving length measurement (the technique of using a ruler), as well as four tasks (Tasks 5-8) that required the students to apply their knowledge of rulers in unfamiliar contexts (the understanding of using a ruler). Two additional tasks (Tasks 9-10) were given; these assessed students' use of informal units—in this case, paper clips—as a measuring tool.

The first researcher interviewed individual students towards the end of the school year (September-November) when they had completed a large part of the measurement program for the year. The interview tasks were designed to elicit information about the students' understanding of length measurement. Practical tasks using wooden or plastic rulers were used because the effectiveness of paper-and-pencil items for the assessment of mathematics knowledge has been questioned (Clements & Ellerton, 1995).

For all tasks except Task 2, clear plastic rulers marked in centimetres were used; these were made from overhead transparency film. For Task 2 a wooden ruler was used; on this ruler the zero mark was not aligned with the end of the ruler. The tasks have been grouped to emphasise their conceptual similarities. They were presented in the same order to all students but not in order given in Table 1 (the paper clip items were presented earlier) and the ten tasks were separated by other tasks not listed.

Results

Figure 1 shows the results for the first four tasks, those involving the technique of using a ruler. In general, all groups were successful on Tasks 1-3 by Year 5. The high-and middle-ability groups were successful from Year 3 on but few low-ability students were successful before Year 5. Students in the low-ability group made ruler-placement errors more often than the other groups, and these errors persisted into Years 4 and 5. Older low-ability students were more likely to express concern or confusion over where to start measuring, at zero, at one or at the end of the ruler. Even in Year 5 some of these students could not measure a line that was longer than the ruler. Faced with Task 2, Erin (Year 5), when asked why she started

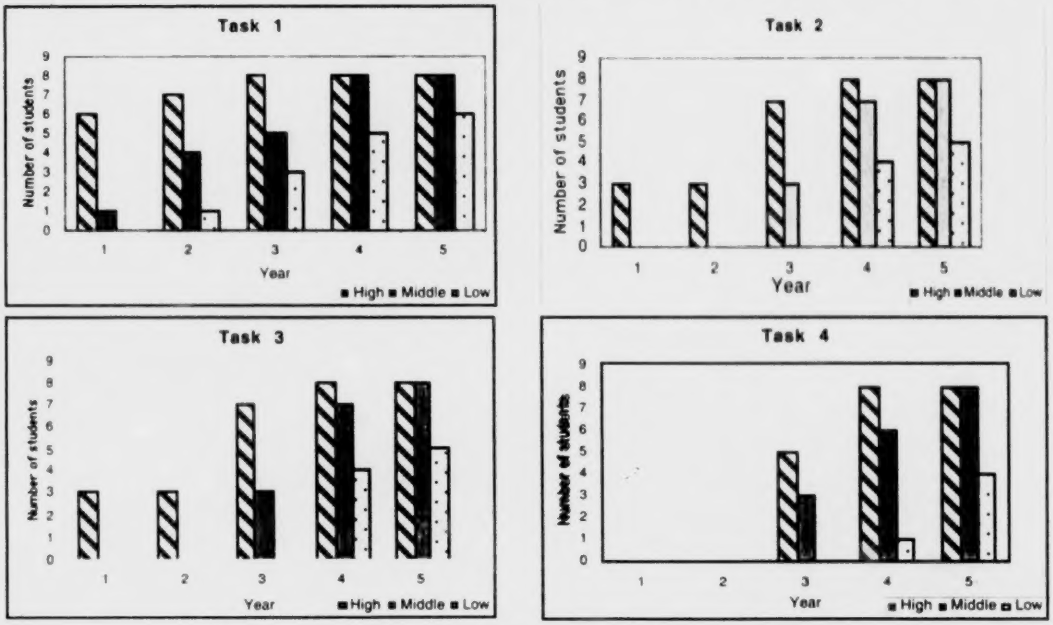

Figure 1. The results for Tasks 1-4—the technique of using a ruler.

to measure from zero replied, "Oh that's easy. You start at zero because the line hasn't made a centimetre yet, the line hasn't started." Soula (Year 4) demonstrated by drawing a line, marking zero, and moving along the line to where she said 'one' was. High-ability students were able to refer to movement along a line; e.g., "... you go along it to get to 1 ..."; or to use a physical movement; for example, finger or pencil. These explanations indicate that these students have acquired an understanding of the underlying concept of the linear nature of length measurement.

The children were asked whether they talked about the length of objects that they measured in their lessons. Peter from the top group in Year 5 replied: " No, we just used to count stuff, you know, ...like shoes and our hands, and them paddle pop sticks. I used to get things wrong 'cause I used to start at 1 like we counted. But I worked it out." Peter demonstrated how he had worked it out by using a ruler: "See? It's like you start at one, you get to two, but that's not two 'cause it's just one."

Figure 2 shows the results for Tasks 5-8—the understanding of using a ruler. Although Task 5 (the 'broken ruler') and Task 6 (the 'empty ruler') have appeared in research studies and basic-skills tests, there are very few similar examples in student textbooks. These tasks require students to have an understanding of how rulers are constructed and to apply their knowledge of what constitutes a unit.

All high-ability students in Years 4 and 5 successfully completed Tasks 5 and 6 but only about half the middle-ability group and almost none of the low-ability group were successful. Thus, for the previous set of tasks (1 to 4) the performance of middle-ability students was similar to the top group in Years 4 and 5. However, their performance on these tasks seems to indicate that they have not acquired an understanding of a unit scale. The distribution of results for Task 7 was similar to those for Tasks 5 and 6 but with a slightly higher difficulty level. However, Task 8, "the sea-horses" showed a very different pattern of results. This task indicates that it is not until Year 5 that high- and middle–ability students can explain the measurement unit. Only one low-ability student in Year 5 could point to the centimetre units on the ruler.

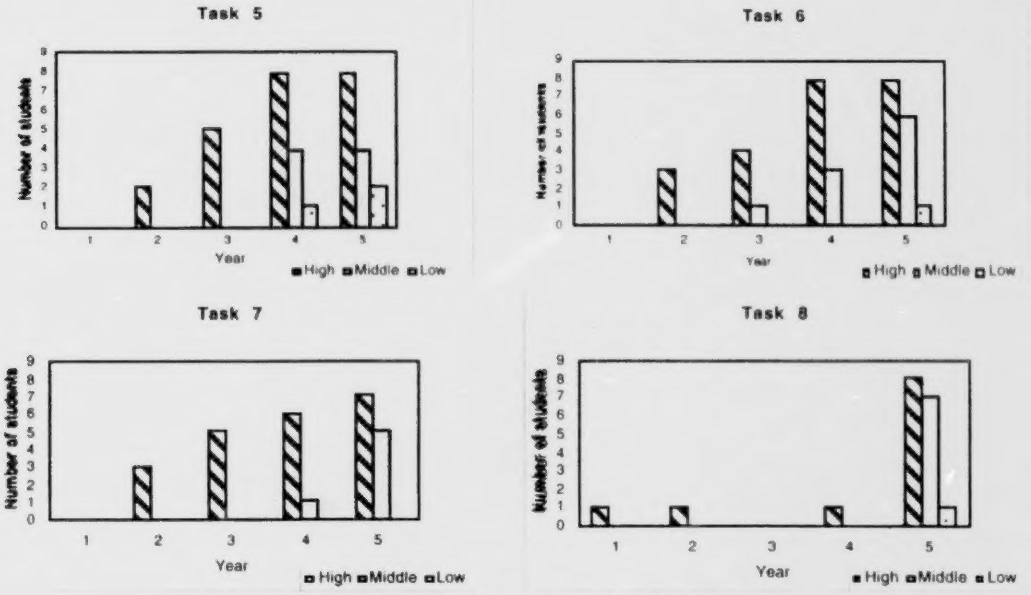

Figure 2. The results for Tasks 5-8—the understanding of using a ruler.

A common feature of students' explanations was a lack of insight into the nature of the units used to measure length. A number of students referred to procedures they had learnt and were confused about where to begin measuring in unfamiliar situations. For example, Jenny (Year 5) said that she started from zero "...because that's how you get the answer." William (Year 4), said, "You start at nought because Mrs. K. said to start there."

Asking children what the numbers refer to on a ruler (Task 8) was quite revealing; many students said that rulers "tell where lines end". Overall, 19 of the 72 students in Years 3 to 5 (26%) indicated that the "5" referred to 5 centimetres and showed a line 1 centimetre long on the ruler. The remaining students said that the "5" referred to the "5" on the ruler, or the unit mark the "5" was next to. Many of the latter students (54%), when prompted to show the "5" centimetres counted the unit markers, some unable to resolve the fact that they had counted six unit markers. Twelve students (17%) counted the spaces, recording what they had counted by colouring the spaces between the unit marks. This evidence seems to have important implications for both the teaching and testing of student understanding.

Measuring length with informal units could be considered a common task for younger students. However, if there are insufficient units, in this case paper clips, to align end-to-end along a line, most students cannot successfully measure the length of the line. The reason appears to be that students do not understand the structure and precision of unit iteration. For Task 9, measuring a line given two paper clips, high-ability students were increasingly successful from Year 2 to Year 5 because they measured accurately. Almost half (14) of the 33 successful students started to measure by using the two clips as the measuring unit. All but two of them switched to an 'alternating' strategy where they would mark and move one paper clip at a time. Comments were "It's easier this way"; "You can count them more easy this way"; and "...you don't get spaces this way." Middle-ability Year 5 students did this task well but only one low-ability student was successful.

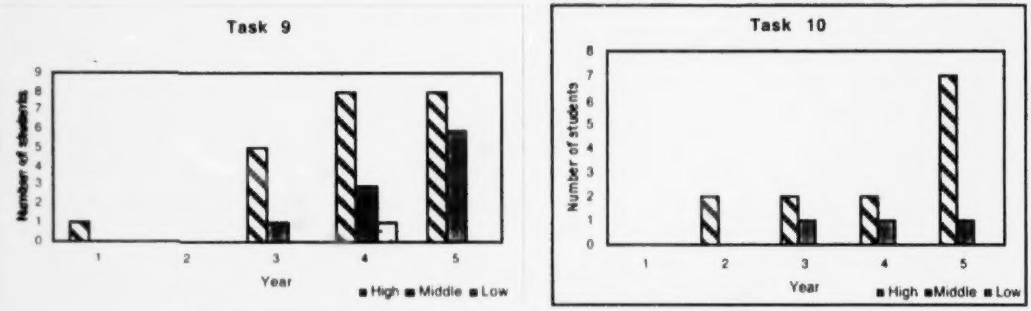

Figure 3. The results for Tasks 9 and 10—the use of informal units.

The responses of older low-ability students were more like those of the younger students. They were more likely to ask for more clips, showed less interest in accuracy, and were unaware of or unable to name the fractional part. In spite of the emphasis placed on informal units in the early years of school, a large proportion of students seem not to have learnt the concepts the use of informal units was designed to teach. More students were proficient with a ruler than with informal units, although two factors made the informal task difficult: provision of a limited number of units and inclusion of a fractional unit.

The distribution of results for the final task, Task 10, construction of a paper clip ruler, was quite similar to that for Task 8 (the "sea-horses"). Only about half of the students (49%) in Years 3 to 5 were able to make a ruler where the unit used was the length of a paper clip.

None of the lower ability students could construct such a ruler. The latter seemed unaware that the length of the paper clip was to be used; instead, they created units of arbitrary length using the paper clips as markers. The successful students were all aware of the need to mark off the length of the paper clips and paid attention to the detail of the ruler's scale; for example, "That's because you have to make them as long as the paper clip". Students who created arbitrary units, although aware of the features of a ruler, were unable to maintain the length of the informal unit.

Discussion and Implications

When the three sets of results (Figures 1, 2, and 3) are compared, there appears to be far greater change for all three groups on the ruler-technique and informal-units tasks. For the four tasks involving understanding a ruler there is a substantial change from Year 1 to Year 5 for the high-ability students, a more moderate change for the middle-ability students and almost no change for the lowest group. More improvement would be expected because only about half the Year 5 students gave correct responses on these tasks. The results are consistent with an hypothesis that teachers focus on the techniques for using a ruler rather than on understanding how a ruler is constructed.

The results clearly show that most students were able to measure and rule a line by Year 5, but they were less successful on tasks that required an understanding of linear units, such as Tasks 5-7 where the numeric scale did not start at one or was not given. In these more complex tasks many students reverted to counting from one or counting unit markers. Few students could explain the units on a ruler or construct a measuring tool with paper clips (Tasks 8 and 10). They seemed unaware of how to construct a ruler using the 'length' of the paper clips as the unit, even though many of them were able to find an object's length of by counting how many paper clips fitted along it.

It became clear that the common components of linear measurement—in this case *length* as represented by a line and *linear sub-units* of that line had not been made explicit when informal units had been introduced. The outstanding feature of the responses of the better students was that they seem to have extracted this information for themselves.

Conclusion

These results from this study show that in spite of the ease with which most students measured with rulers and counted informal units, many students did not understand the relationships between linear units and a formal scale. This is particularly evident with the lower students who take longer to acquire basic technical skills and who are unlikely to have developed an understanding of concepts they need for more complex measurement. The main errors made in the tasks involving ruler techniques concerned alignment. Such errors might arise from the emphasis on counting from one or, as found by Gravemeijer, McClain and Stephan (1998), confusion arising from methods of counting informal units such as paces.

Students used two main measurement strategies in this study: (a) counting informal units, unit marks, or unit spaces; or (b) aligning the ruler and reading the scale. Correct use of either strategy did not indicate that students understood linear measurement in the more complex tasks. An explanation for students' reliance on procedures might be the way that measurement is often taught. Worksheets and textbook exercises frequently involve counting informal units or the techniques of ruler use. Such an emphasis on techniques does not develop fundamental measurement concepts of unit size and the structure of the unit iteration. To develop such knowledge would seem to require practical experiences of measuring and marking units, followed by discussion about key aspects of such student-constructed scales. If students do not understand how scales are constructed, they will not have the basic knowledge to relate measurement of length and number lines, nor have the foundation to develop area, volume and other higher order mathematical applications.

While the majority of older students could use informal units to 'measure' a length, the process of counting them appears to have resulted in different interpretations from those expected by teachers. The emphasis on counting may obscure the *linear nature of the unit of measure* if it is not made explicit when informal units are introduced. A possible explanation would be that students attend to the action of placing or counting discrete units rather than their lengths. According to Wilson and Osborne (1988) neither zero nor the iteration of line segments can be made explicit when informal units themselves are counted, thus reducing the possibility that students are able to make the important link with the underlying linear unit concept (Hiebert, 1989).

In 1984 Hiebert pointed out that "...many elementary school students are quite proficient in a variety of standard measuring skills but lack an understanding of some of the basic concepts involved." (p. 19). In 1990 Webb and Briars wrote that teachers should know firstly that students are able to reliably and efficiently apply procedures, but more importantly they should know "what a student knows about the concepts that underlie a procedure" (p. 11). This research suggests that the situation is similar in Australia; few students interviewed could reliably demonstrate a deeper understanding of the concepts that underpin linear measurement. These findings have significant implications for teaching about the relationship between the construction and representation of informal and formal linear units.

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Key Factors in the Implementation of an Assessment Innovation: A Case Study

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The introduction of portfolios into the mathematics assessment programme of a New Zealand secondary school was closely monitored. The responses of teachers, students and parents are recounted in this paper. Inconsistencies in teachers' views and behaviour highlight the professionalism necessary for education reform.

It is difficult to successfully implement genuine reforms in education (Lieberman, 1998). The reform discourse can be stopped in its tracks by teacher unions, delayed, subverted, and even reversed (Broadfoot, 1998). Fullan (1993) relates three stories of failed implementation from the 1970s: open-plan schools, individualized instruction, and large-scale national curriculum efforts. Inevitably, the received curriculum differs from the official curriculum. The constraints of time, funding and talent conspire to produce mere approximations of the intended reform. Political interests, such as conservative schools keen to preserve traditional examinations, also blunt the impact of educational reform.

The portfolios innovation which was the subject of this study was initiated by a mathematics curriculum reform in New Zealand (Ministry of Education, 1992). The current curriculum requires that:

> Assessment should, as far as possible, be integral to the normal teaching and learning programme. Continuing assessment as part of the teaching and learning programme increases the range and quality of assessment which can be carried out for good diagnosis, and avoids the artificial intrusion on learning and teaching time which is associated with separate assessment sessions. Assessment should involve multiple techniques including written, oral, and demonstration formats. Group and team activities should also be assessed. …teachers should report what students have been working on, what they have achieved, and how well they have achieved it. A grade, level, or mark alone is insufficient (p. 15).

Faced with such curriculum requirements, teachers at the author's school were quick to support the introduction of portfolios as part of the mathematics assessment scheme. A portfolio was taken to be a collection of a student's work, often featuring problem-solving projects, selected by the student or prescribed by the teacher. Unanimity prevailed as the teachers could see little feasible alternative. And so the case study began, with a participant observer present for the first year and a mixture of quantitative and qualitative techniques to monitor developments.

Aims of the Study

The study sought to identify key factors facilitating change within the immediate school community. Teachers are central to educational reforms, but demands are also made on students and their parents. An attempt to gauge the response of each of these groups was made.

Methodology

A Mathematics Survey (Radalj, 1982) was employed to measure enjoyment of mathematics, perceived importance of mathematics, stress, and desire to improve mathematical skills. It was administered to all 500 students involved in the study at the very beginning of the school year, and again near the end of the year once all the portfolio work had been completed. A standard instrument to measure classroom environment, "What is Happening In this Class?" (Fraser, Fisher & McRobbie, 1996), was administered during the year to the students and also their teachers. Three other questionnaires for teachers, administered at intervals over a two-year period, comprised of open-ended questions. A postal survey of parents was also conducted.

Taped interviews were conducted with small groups of students representing every participating class. The student interviews sought to validate information from the questionnaires and to provide a forum for unanticipated student ideas. Summaries of the results for their own classes and of the teacher surveys were distributed to the teachers. Taped interviews with the teachers were conducted at the end of the first year of the innovation.

Results

All four of the attitudinal variables quantified by the Mathematics Survey declined as interest in school waned throughout the year. A subsequent administration found that attitudinal variables largely recovered at the beginning of the following year. Individual students were tracked using anonymous self-generated code numbers on every questionnaire completed. This enabled links to be made between attitudinal variables and the classroom practices apparent from the What is Happening In this Class? survey.

High scores for the Investigation and the Task Orientation constructs were the best predictors of decline in the attitudinal variables. This is a surprising finding as the two constructs seem antithetical. Task Orientation consists of items such as "Class assignments are clear so I know what to do", whereas Investigation consists of items such as "I carry out investigations to answer questions coming from discussions". It may be that extreme approaches at either end of the constructivist classroom spectrum reduce the students' self-reported enthusiasm for mathematics. Alternatively, it was the practice within the mathematics department to allocate experienced staff to both high-ability and low-ability classes, and this may have introduced a bias in this 15 class sample. Skillful and enthusiastic teachers may initially induce high expectations about such things as enjoyment of mathematics, so measures of absolute decline may be inappropriate.

Interviews with students revealed that portfolios had little declared impact. The portfolio tasks called on investigation and presentation skills already honed in primary school and used in other subject areas. There was inconsistent treatment of the portfolios by the teachers in respect of the information given, class time devoted to doing the tasks, marking, and storage of the accumulating work. But the students were unperturbed by the innovation. Written tests were sometimes preferred because they take less time and are not subject to thoughtful revision, but sometimes reviled as "boring". New Zealand students equate assessment with tests (Fitzsimons, 1997), and are oblivious to the idea that portfolio work influences their real marks and ultimately their life chances.

The students were anxious about unfairness: that students with access to computers at home might get better marks for neatness; that uncooperative parents may not enable high-scoring interviews; and that rural students do not have access to the junk mail that was the subject of one task. Although 90% of students in the school remedial reading programme are boys, the students failed to mention that there would be one gender particularly handicapped by the literacy demands of portfolios.

The students voiced a very functional view of mathematics, and relegated analysis and synthesis tasks to other subject areas. The need to do mathematics was primarily seen in terms of getting a job, as illustrated by this exchange between two fourteen-year-old boys:

> I just think there's not much point in doing algebra and that. ... The jobs you want to do don't need maths skills or anything. Like, algebra should actually be optional for kids that want to do it.
>
> Yeah, if they've already made up the decision what sort of job they want to do they should go and do like that sort of maths. What's required for that job.

Students were not sure of the standards for a 'good' grade, and preferred the familiar percentage marks to any other system. Some had not been adequately briefed about the 20% contribution the portfolio mark made to their final mathematics grade. They liked to know what was coming up next, and used the advance warning to pick things up for their portfolios. They enjoyed working together on portfolio projects in class, whereas set homework was "boring" and often not attempted at home.

One teacher when asked about the unanticipated spinoffs from using portfolios replied "The enthusiasm that has taken students beyond my anticipated level of achievement". Another teacher acknowledged that portfolios had changed his teaching practice and was unruffled with what he perceived as the failure of one portfolio task with his class, adding "It may be that [in future] the suggestion which is made for the formal writing I don't take up and I want to take up one of my own...". This is more than recognition of the success or failure of the innovation, but a self-acknowledgement of the teacher's professional ability. Such self-assessment is a goal in the development of any capable teacher.

Portfolios were used as a vehicle to incorporate new experiences into the mathematics classroom programme. Some of this was overt, such as a mandatory computer programming task and a statistics assignment that had to be done on the computer, but much of it was by extension of the allowable responses of the students. Exhortation and a long lead time resulted in many students using a wordprocessor to complete projects at home. Other technologies used included the Internet, still photography, glue guns, and presentation folders. Many of the portfolios incorporated novice attempts at using display technologies such as scanning, clipart, Powerpoint, and colour printing. Some mathematics teachers conceded that portfolios had forced them to take classes to the school library to undertake research, to do role plays in class, to 'brainstorm' in class, and to teach formal writing skills. Some teachers encouraged student self-assessment. Collectively, this is an extensive list of brave new adventures by teachers and students, all of which is apparent to anyone who reads the portfolios produced. The latent effect of this is to encourage experimentation by both teachers and students in subsequent rounds.

It had long been the policy of the mathematics department to encourage all junior students to enter a poster or project in the annual regional school competition. The level of participation and of project quality varied greatly, and the classes of some teachers produced very little. With the introduction of portfolios as part of the assessment scheme, an entry for the annual competition became one of the tasks required of every student. In the first year of implementation the Head of Department used three classrooms to display the projects, and five hundred students toured the exhibition. This invasion of teaching spaces certainly drove home the point that every student and, more significantly, every teacher had to attempt the task.

It transpired from interviews with the students that one teacher had not distributed the portfolio tasks sheet to each student in the class. Although the interviews were not intended as a monitoring exercise, student information-sharing produced a demand for equal treatment from the teachers. The problem was soon remedied.

During the second year of using portfolios, nine of the twelve teachers involved reported that the innovation prompted them to engage in focused discussion with colleagues. This took such forms as joint scrutiny of the work of students from two classes, brainstorming items for future use, planning programmes for weaker students, and sharing teaching strategies. Much of this was voluntary and not centrally organised.

Coordinators, designated Assistant Head of Department and paid a salary supplement of approximately 2%, were charged with drawing up the portfolio tasks, keeping everyone on track, and collating the portfolio marks for the students of one of the two year levels involved. The coordinators developed substantial teaching resources, for example a complete teaching programme for a week on spreadsheets. The coordinators noted inconsistent implementation and marking of portfolios and that some teachers felt 'pushed' into using portfolios, but they considered that the programme was worth continuing because it developed the students' pride in their work. They seem to have demonstrated the three components of Glatthorn's (1987) cooperative professional development: colleague consultation, coaching and reflection about practice.

Curiously, none of the other teachers acknowledged the help of the coordinators; their comments in interviews and questionnaires steadfastly focused on their own performance and that of their students. Kent (1985) found that "facilitators sometimes encountered jealousies of teachers who were not moving into new roles, because of the additional training and teamwork provided for these new roles" (as summarised by Nisbet, Dole and Warren, 1997, p.369). No such jealousies were volunteered in this study, perhaps because the author acted as a facilitator in the first year of the implementation, or perhaps because there was no training provided for the facilitators.

Holmes (1998, pp. 256-257) notes a trend in education "over the last twenty or so years" towards "denying the legitimacy of parents helping their children". Fink and Stoll (1998, pp. 299-300) discuss the community "contextural network" of schools and conclude that "innovations often fail because of the educators' inability or unwillingness to involve parents in meaningful ways in their development and implementation". The portfolios in this study sought to involve parents directly by such methods as mandating students to interview their parents about the mathematics parents actually use at work, and indirectly by providing shareware for home computer use and by requiring observations, such as sketching spider webs, to be recorded at home.

A postal survey of parents was conducted at the end of the first year of portfolios. On a score of 1 (Nothing) to 5 (Lots), the mean parent response to "How much involvement do you have with your son/daughter's mathematics homework?" was 2.19, with a modal score of 1. This was very close to a prediction survey conducted with the teachers. Comments by the parents included "maths homework doesn't seem to encourage parental involvement" and that parents "do not always understand the maths homework". The only parental contribution contemplated by four of the thirty parents who replied was hiring a private tutor for the student. However, some parents did report enjoying helping by being interviewed or assisting with mathematical research on the Internet. Parents appreciated knowing what was coming up in tests and projects.

Although many teachers reported no discussion with parents about portfolios, this was one teacher's experience:

> With my kids the parent involvement was very strong. ... I took this [the portfolio task sheet] to parent interviews with me and ran through it with them and if they [the student] had done a piece of work I took it along and showed it to them and said 'this is what we're doing and have these things to do'. Some parents said to me 'Oh, yes, I've got this up on the fridge'."

Discussion

The portfolios innovation did not produce better marks for the participating students in the traditionally-styled final examination. The same examination paper was used for two years in a row with little discernable change in results for the portfolio cohort. But there was little interest by the teachers in this result. After all, the curriculum was being delivered, most of the students had generated impressive files of "personal best" completed work and, as was found in the Vermont study (Koretz et al., 1993), there had been a greater emphasis on problem-solving. Some of the teachers felt that formative assessment, as represented by portfolios, had led to less superficial learning, making mathematics "more real".

The decision to use portfolios became securely established by promotion of some teachers, exposure of the students to what would be expected of them in future years, and the irreversible commitment of publishing the assessment scheme to parents at the beginning of the year.

Many teachers have reservations about portfolios; for example, regarding them as unsuitable for low-ability students. Literacy problems and inability to undertake independent research rendered investigation-style tasks "useless" for such students. One teacher of a low-ability class said that:

> For me, it [portfolios] was less to do with assessment and more to do with motivation, with actually handing in something that at the end of it the kids could look at the folder and say 'well, I actually achieved a few nice things throughout the year'.

Many different approaches were taken: ignoring or modifying the tasks; giving practice tasks first; strongly directing how to proceed in the investigation; and writing a different marking scheme more appropriate to the level of the class.

A number of the teachers taught the same class levels in other subjects such as Technical Drawing and Physical Education. This may have given them a better appreciation of the skills to expect of the students when it came to written prose, collating information from a variety of sources, and independent investigation. No formal comparison was made between written work in Mathematics portfolios and that in Social Studies or English.

Mathematics teachers are familiar with the expected results of an investigation. But they may have little on which to base their expectations of students' investigation and communication skills. At times this may result in confusion, with teachers impatient about the unexpected digressions of their students, and students who feel their creative work is undervalued. There was little evidence in the study that teachers scrutinised the written work their students produced in other subject areas.

Conclusions

The innovation in this case study is still in place. This implementation of portfolios in mathematics assessment allowed a great deal of scope for teachers to vary the programme. They could substitute the tasks, vary the marking schemes, collaborate with other mathematics teachers, and call on the expertise and encouragement of nominated expert colleagues. In short, the teachers were empowered. This is unlike the state-wide implementation in Vermont (Koretz, Stecher, Klein, McCaffrey, & Diebert, 1993) which was closely moderated but which found better correlation between writing proficiency and the portfolio scores than between mathematical proficiency and the portfolio scores (p. 86).

Teachers did not only face a curriculum demand for changes in teaching practice, but they had to deal with year-level coordinators who required marks for the stipulated tasks to be entered into a computer in time to produce the student reports. The teachers were conscious of the research programme being undertaken and were also reminded of their responsibilities at frequent departmental meetings. Intrusion into classrooms reached a higher level, and there was more professional discussion focused on the students' learning. The immediacy of this monitoring was vital in overcoming resistance to change.

Those parents who took an interest found their involvement legitimated by appropriate tasks. Many found their non-mathematical expertise was called upon, for example, recalling the names of teachers who featured in the students' mathematical autobiographies. Parents appreciated publication of the assessment scheme and were pleased to see home computers put to good use.

The students were outwardly indifferent to portfolios in mathematics. Some threw their completed portfolios into the rubbish bin when they received them at the end of the year. Others had invested greater effort into their portfolio and saw it as an interesting artefact to preserve. Many enjoyed the opportunity to be creative and to employ novel means of presentation for their mathematical work. Some even pursued mathematical investigations far beyond the expectations of their teachers. Many escaped the need to make judgements about their own work, their teachers not insisting on a declared rationale and careful selection of portfolio items.

The streaming of mathematics classes by ability accounted for a considerable amount of the variation in the students' responses to portfolios. But the influence of the teachers was also significant, a conclusion which is supported by research over many years (Hall, George & Rutherford, 1979). Although required to implement the change, each teacher adapted the programme to the needs of their students. Between teachers, inconsistencies in implementation became so great that the Head of Department decided the portfolio marks had to be re-scaled to the same mean and standard deviation as the class examination marks. But this was not seen as the failure of the programme.

It is notable that many factors did not impact on the implementation of portfolios. There was no externally-sourced professional development for the teachers, the only resourcing being a more focused application of the usual management salary increments within the mathematics department. The school administration played little role in the implementation. The indifferent response of most students and many parents made no impression. Other departments within the school, including the remedial reading programme, were not consulted. Some mathematics staff became persuaded of the benefits of portfolios, while others retained serious reservations but worked to modify the programme to be useful for their students.

It is hard to know whether a "portfolio culture" (Duschl & Gitomer, 1991) flourished, however briefly, in some classrooms. This would require that the students truly understood why portfolios were used, and had internalised a self-critical approach to their own learning. The test of whether a portfolio culture existed is the test of all formative assessment: did the teachers and students change their teaching and learning activities as a result of the assessment information they had obtained? (Black & Wiliam, 1998). In this study the teachers often spoke of changing their practice, but there was little evidence that the students undertook self-directed activity.

The factors which appear to have been most significant were the professionalism of the teachers and the openness with which the programme was managed. Implicit in many of the responses from the teachers involved was the idea that the teacher felt no compunction to deliver precisely the same programme as everyone else. The expertise of the teacher and the needs of the class were always factored into planned activities. Initiatives taken by teachers were shared formally and informally, and given a positive reception. Just as the portfolios were a celebration of the students' judgement and creativity, the implementation permitted the use of the same skills by the teachers.

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Are Teachers' Probability Concepts More Sophisticated than those of their Students?

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The probability concepts of a group of preservice primary teachers were investigated and classified in relation to known misconceptions as well as with a framework that identifies levels of understanding. These results were compared with the misconceptions exhibited by a group of 11 and 12 year old students. The major question investigated was whether these prospective teachers showed more sophisticated probability concepts than 11 and 12 year old students.

In New Zealand as in other parts of the world, probability is now included in the school curriculum from an early age. Prior to New Zealand's current curriculum statement (*Mathematics in the New Zealand Curriculum*, 1992), probability concepts were first introduced at the year 7 and 8 level, through the use of random-generating devices and moved quickly into developing a formula-based interpretation of probability. In the 1992 curriculum statement, probability is introduced from Year 1, and concentrates initially on developing an understanding of probability through language, specifically the understanding and use of subjective terms such as certain, possible, and impossible.

According to some writers (for example, Hawkins & Kapadia, 1984; Kapadia & Borovcnik, 1991; Konold, 1991), there are four different interpretations for probability. The *classical* interpretation (or *a priori* probability) is based on equally-likely outcomes; the probability of an event is defined as the number of desirable outcomes divided by the total number of possible outcomes. This allows probabilities to be calculated before any trial is made. The *frequentist* interpretation requires repeated identical trials in order to establish long-run relative frequencies which are used as estimates of the probability of the event in question. With this interpretation, it is meaningless to talk of the probability of an event in a single trial that will not or could not be repeated. *Subjective* or *intuitive* probability measures the belief in the truth of a proposition. Subjective probabilities can be based on information which is independent of any empirical data in a subject's mind or on empirical data from repeated experiments. This can lead to a subjectivist adjusting the strength of their belief, given new information such as results of actual trials (Konold, 1991). The fourth category of probability is *formal* or *theoretical* probability which has no justification for the numerical values obtained for probabilities in any case of application; rather it is an abstract system of rules and procedures.

Because many of the ideas of probability are counter-intuitive, teaching of probability presents its own problems (Ahlgren & Garfield, 1991). The intuitive ideas that students develop are known to be firmly held and particularly resistant to change (Hope & Kelly, 1983; Konold 1995). The use of contexts and drawing on students' experiences and understanding is recommended for enhancing the students' learning of mathematics. However, with regards to probability, some experiences are known to contribute to the development of inadequate types of probability reasoning, as opposed to what are the commonly-accepted correct (or normative) concepts and reasoning.

Conflicts can and do arise between intuitive interpretations and formal ones which have been acquired by instruction. Fischbein (1987) claims that learners may often have these contradictory interpretations co-existing. Even more problematic is that the intuitive concepts are generally held more strongly by the learner and these annihilate the formal conception. The nature of randomness and probability is difficult for students to grasp, in contrast to much of the mathematics which they learn using a deterministic approach (Kapadia & Borovcnik, 1991).

Some common misconceptions in probability have been classified by various researchers. The *representativeness* heuristic (Kahnemann & Tversky, 1972) indicates that people estimate the likelihood of an event on how well an outcome represents some aspect of its parent population and on how well it reflects the salient features of the process by which it was generated. For instance, in a sequence of outcomes from tossing a coin, a sequence with close to 50% heads and 50% tails would be seen as more likely to occur than a 'less balanced' sequence. The gambler's fallacy is an example of the representativeness heuristic being used: after a sequence of heads, a tail becomes more likely than another head. This is sometimes referred to as the *negative recency* effect. Another variation of the gambler's fallacy is the *positive recency* effect: there is a greater chance of the current trend continuing.

The *availability* heuristic (Kahnemann & Tversky, 1972), is when people estimate the likelihood of an event based on the ease with which they recall particular instances of the event occurring in the past. This generally results in significant bias because of the narrow experience of an individual. More particularly, a single event can take on inflated significance if it happens to us (Shaughnessy, 1993).

Konold (1995) has explored the way that people interpret probability values or view probability questions. He has defined a commonly-used type of reasoning as the *outcome approach* – people translate probability values into yes/no decisions or focus on the result of a single trial. This approach is at odds with a frequentist interpretation of probability: a person using the outcome approach to evaluate the prediction of a particular outcome would not be concerned about the relative frequencies over a large number of cases (Konold, 1989). Sometimes, in questions about the most likely and the least likely of a set of outcomes (such as a sequence of coin outcomes), students use two different types of outcome reasoning. These two types of reasoning result in inconsistent responses of which the students are not aware. Konold referred to this as the *M-L switch* – the students' responses indicate that they think more about the non-predictability of coin flipping rather than the probability of various sequences being obtained.

It is common for students to assume that random events are equiprobable by nature; Lecoutre (1992, cited in Sharma, 1997) refers to this as the *equiprobability* bias. It is particularly apparent when the students are using symmetrical random generators (such as dice or coins).

The language of probability is known to be problematic to learners (Fischbein, Nello, & Marino, 1991) even though they have dealt with countless situations involving uncertainty and have learned to use in common discourse words such as probable, random, independent, lucky, chance, fair and unlikely (Konold, 1991). Probability judgments are sometimes not made using any of the typical heuristics but rather using some personal belief system. One such example is that young children think that a six is the hardest number to throw on a dice (Amir & Williams, 1994). Another example described by Amir and Williams is the belief that the outcome from a random generator can be controlled, such as through the way that the generator is held, shaken, thrown or spun. Even in instances when the desired outcome does not eventuate, the belief is still maintained. The influence on outcomes from luck, superstitions, or God is another example of a belief system.

Anchoring (Tversky & Kahnemann, 1974) refers to the tendency to not move significantly from an initial estimate, in spite of evidence to the contrary. This is relevant to probability experiments where students are asked to estimate the number of outcomes that may occur; following the experiment, once the results are available, there is a tendency for subsequent predictions to stay closer to the first prediction (which may have been obtained subjectively) rather than use the available empirical evidence for an improved estimate.

Tarr and Jones' (1997) framework for assessing students' thinking in two probability constructs (independence and conditional probability) enables the thinking to be described across four different levels (subjective, transitional, informal quantitative, and numerical reasoning). An earlier framework developed by Jones, Langrall, Thornton and Mogill (1997) enabled assessment of probability thinking in four constructs (sample space, probability of an event, comparison of probabilities, and conditional probability), also across the same four levels as the framework of Tarr and Jones.

The Study

The aim of the research was to classify the probability thinking exhibited by a group of pre-service teacher education students and to compare these results with those of a group of Year 7 and 8 students (approximately 11 and 12 years old). The question of whether the teacher preservice education students have a more sophisticated understanding of probability has important implications for the mathematics education courses that students undertake during their preservice programme.

A group of 9 preservice teacher education students were interviewed individually. Three of the students were in their first year, while the other 6 were third (and final) year students. They were selected from a sample stratified by elected subject study area (either mathematics or some other subject) and gender. The interview covered aspects such as the interpretation of probabilities, outcomes from simple events (tossing a single coin, throwing a dice, drawing a counter from a bag) and outcomes of compound events (such as throwing 2 dice or 2 coins). Each interview took approximately 45 minutes, and was audiotaped then transcribed by the researcher for analysis.

The 11 and 12 year old students were part of a research project investigating the use of games for learning (Burgess, 1999). The 16 students were selected by the two classroom teachers as representing a range of mathematical ability within each class. They were given a written questionnaire to complete on various probability concepts. They were involved in playing two types of games in groups of four over 2 days, during which they were asked questions about aspects of their understanding. Following each game they were involved in group interviews to further explore their understanding in relation to probability aspects of the games. The interviews (during the games and following the games) were audio- and videotaped and transcribed by the researcher for later analysis.

Results and Discussion

The understanding of the 11 and 12 year old students was classified initially using the various categories of misconceptions described earlier. Table 1 below gives some examples of the types of understanding and misconceptions revealed by students through the written questionnaire and the interviews. The 'dominant reasoning' (if it existed) was determined by the researcher checking for a form of reasoning which was more prevalent than any other. The absence of an obviously dominant form of reasoning is shown in the table by '??".

Table 1
Examples of School Students' Reasoning and Misconceptions

Student no.	Reasoning, and misconceptions demonstrated	Dominant reasoning
S3	Gamblers' fallacy (negative recency); causal (can control dice outcome); Equiprobability bias with outcome approach.	equiprob. bias
S5	Causal; outcome approach; representativeness; equiprobability bias; language - outcomes have equal probability, none are more or less likely but some have better chance; normative reasoning.	outcome approach
S6	Normative; outcome approach; representativeness; equiprobability bias.	normative reasoning
S8	Outcome approach; gamblers' fallacy (positive recency); representativeness; causal (can control the dice); normative; equiprobability bias.	??
S9	Language difficulties; outcome approach; representativeness; causal (can control the dice); availability ("From my experience..."); equiprobability bias; language - (equal chance outcomes plus some more and less likely).	??
S15	Normative reasoning; equiprobability bias; belief system (luck?).	equiprob. bias
S16	Subjective terms not understood; outcome approach.	outcome approach

It is clear that with most students, a variety of probability misconceptions exist, even for those who demonstrate a dominant type of reasoning, including normative (such as student S6). It was not uncommon for students to demonstrate inconsistent types of reasoning for similar problems. For example, one question gave four different sequences of coin outcomes, and asked firstly for the most likely one (or were they all equally likely), and secondly for which of the sequences was the least likely to occur. One student said that none was most likely but they were all equally likely to occur, because "you can get a head or a tail either way". He indicated that the sequence HTHTHT was the least likely to occur because "it keeps getting the same pattern again and again - it's not very likely to happen". In the first instance the reasoning was the outcome approach but on the following situation the representativeness heuristic was used - the change in the type of reasoning between the 'most likely' question and the 'least likely' question is an example of the M-L switch.

Table 2 below gives some examples of the reasoning and misconceptions of the pre-service teacher education students, as revealed through their interviews.

Table 2
Reasoning and Misconceptions of some Teacher Education Students

Student no.	Types of reasoning and misconceptions	Dominant form of reasoning
T1	normative; representativeness (positive recency); anchoring; subjective reasoning.	representativeness
T2	outcome approach; representativeness (negative recency); normative; strong anchoring.	representativeness
T5	outcome approach; representativeness (incl. positive recency); normative; anchoring.	normative
T9	normative; strong anchoring; equiprobability bias (although followed up with reasoning to counter this).	normative

As with the younger school students, each teacher education student did not display only one type of reasoning throughout his or her interview. Even though each had a dominant form of reasoning or misconception, a variety of types of reasoning were used. It is significant that of the teacher education students, the majority (67%) had studied mathematics through to year 13 of secondary school yet in spite of the reasonably strong background of study of probability that they had had, key misconceptions were still exhibited.

From comparing the types of reasoning used by the two groups (school students and teacher education students), there is a difference in the proportion of each group who predominantly use normative reasoning: seven of the nine (78%) teacher education students compared with four of 16 school students (25%). However, in spite of the use of normative reasoning, there was a wide variety of other types of reasoning that were used concurrently with the normative reasoning in both groups of students. The teacher education students did not exhibit the use of causal reasoning as did the younger students (i.e., "I can control the outcome of the dice" type of reasoning), nor did they have belief systems as the younger students did (e.g., "I think the other team are unlucky because the dice is yellow, their counter is yellow, and the game board is yellow", or "Maybe God's giving them luck"). Otherwise, the major misconceptions were prevalent in both groups.

To further compare the two groups, their types of reasoning were classified according to the criteria used in the frameworks of Tarr and Jones (1997) and Jones, Langrall, Thornton, and Mogill (1997). The constructs used in the classification were: sample space (can students identify the complete set of outcomes in a one-stage or two-stage experiment); probability of an event (can students identify and justify which of two or three events are most likely or least likely to occur); probability comparisons (do students have the ability to determine and justify which probability situation is more likely to generate the target event in a random draw, and whether two probability situations offer the same chance for the target event); conditional probability (do students have the ability to recognise when the probability of an event is or is not changed by the occurrence of another event, and to determine probabilities on response to the occurrence of another event); and independence (do students have the ability to recognise and justify when the occurrence of one event had no influence on the occurrence of the other). Tarr and Jones (1997, p.44) recognise that with respect to the concept of independence, "even when students seemingly exhibit an understanding of the concept of independence, the representativeness heuristic may still prevail". One such example came from a teacher education student with a background of advanced study in mathematics who commented that "I realise the probability of getting the numbers 1 to 6 [in a Lotto ticket] is the same as the probability of getting those other numbers [5, 11, 19, 21, 24, 38] but it just sounds, having 6 different numbers, sort of sounds better... [The 1 to 6] just doesn't sound right". This illustrates that the intuitive understanding of probability can overrule the mathematics of the situation - the 'heart rules the head'.

Table 3 gives the levels that a sample of students appear to be at. In the table, each probability construct is abbreviated as follows: sample space - SS; probability of an event - PE; probability comparisons - PC; conditional probability - CP; independence - I. The number following the construct abbreviation identifies the level of that the student has been classified for that construct. The four levels defined by both Tarr and Jones (1997) and Jones, Langrall, Thornton, and Mogill (1997) are: 1 - subjective; 2 - transitional; 3 - informal quantitative; and 4 - numerical reasoning. For example, I3 indicates level 3 on the independence construct. The coding for the student indicates whether they are a teacher education student (T) or a school student (S).

Table 3
Examples of Framework Levels from Tarr & Jones (1997) and Jones et al (1997)

Student	Construct/level
T1	SS4, I2
T3	I3
T5	I4, PE4
T8	I3, although other indicators for I4 are strong, but representativeness used
T9	SS4, I4
S1	I3 although some indication of I4; SS3
S7	PE1, I2

Tarr and Jones (1997) posit that students' level of understanding would be stable across the constructs, and claim that their study shows this to be the case; in fact they found less instability in students' understanding than that found by Jones et al (1997) in their study of younger third grade students. Table 3 indicates some of the difficulties that exist in attempting to classify the levels of probability understanding because of conflicting evidence in relation to some of the descriptors of levels; the levels of students are not consistent across the constructs. There were cases where a student appeared to be at a certain level but responses to other questions indicated a different level. There were inconsistencies shown by both the teacher education students and the school students with regard to the level of one construct compared to the level of a related construct. Although the frameworks were useful for assessing some areas of probability understanding, they also appeared inadequate for situations such as identifying levels in relation to 2-stage experiments, where the outcomes are not equally-likely (such as throwing two dice and recording the sum or the product of the two numbers obtained).

Since probability in real-life ranges across various interpretations of probability (with the exception of formal probability), the contexts of the questions (including the practical experiments) covered the range of probability interpretations. The ones that involved an experiment or practical activity required the results to be linked to predictions of future outcomes – the frequentist interpretation. Also, aspects of the subjective nature of probability were investigated as were questions relating to the classical interpretation.

The context in which the concepts were examined may have affected the type of response because of some of the inconsistencies in the students' responses. The degree of influence is not known, but this possible link is worthy of further research. It appears that from the variety of types of reasoning exhibited by the students, it is important in the classroom to use a wide range of contexts and to ensure that the different interpretations of probability are linked, so that a thorough understanding of probability is developed by students. Teachers must be aware that the level of understanding demonstrated in one construct while using one context does not necessarily indicate that the same level would be demonstrated in a different context. Similarly, the level of understanding in one construct is not necessarily indicative of the level of understanding in a different construct.

Conclusion

Compared with the younger students, preservice teacher education students have had more real-life experiences involving probability and have been involved in the study of probability concepts at secondary school level (if not beyond). In spite of this, they still demonstrate many of the same misconceptions as the younger students. This indicates that textbook-type exercises involving a classical interpretation of probability are not sufficient to ensure that students develop a thorough understanding of probability. It appears clear that the probability understanding of preservice teachers is not significantly more sophisticated than that of students they may be teaching. This will need to be addressed in teacher education mathematics courses to ensure that the concept knowledge that teachers take into the classroom is appropriate for effective teaching of probability. A variety of suitable activities (across a number of different contexts) for overcoming misconceptions needs to be found or designed. For teachers to be able to effectively and easily assess the probability understanding of students, a more appropriate framework needs to be developed. The design of such a framework will depend upon further research into the field of probability concept development.

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Towards a Framework for Numeracy Assessment

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One approach to assessing numeracy is through performance assessment that requires students to create a response. Four specially designed performance assessment tasks, addressing different aspects of mathematics, were used to assess students' numeracy achievement. These tasks were considered for their educational measurement characteristics and the underlying variable interpreted from a numeracy perspective. An overall developmental sequence of numeracy was found, with associated 'productive elements'. From these, a framework for numeracy assessment is proposed and implications for assessment and classrooms are examined.

Current Australian definitions of numeracy all stress the practical and contextual nature of the concept (e.g., AAMT, 1997a; AAMT, 1997b). As well as the application of mathematical knowledge, numeracy often requires the communication of the mathematical ideas to other people (DEA, 1995). Assessing numeracy, therefore, requires demonstration of not just the mathematical skills, but also the communication of ideas and thinking within a grounded context. Performance assessment, defined as 'assessments in which pupils create an answer or product that demonstrates their knowledge and skill…' (Airasian, 1994 p. 228) is one way of achieving this.

Judging performance assessment is generally done by comparison against some pre-defined over-arching criteria (e.g., Beesey et al., 1998) or by using a scoring rubric developed by considering students' responses to the task during trialing or administration (e.g., Harmon et al., 1997). These scales may be termed holistic or analytical respectively. Rarely does performance assessment provide an analytical scale within an holistic sequence of development of thinking or skill (Moskal, 2000).

This paper reports the outcomes from a rigorous assessment program using four different performance assessment tasks. The assessment was intended to evaluate the effectiveness of a teacher professional development program aimed at enhancing numeracy outcomes of Indigenous students in lower secondary years—the INISSS program (Improving Numeracy for Indigenous Students in Secondary Schools) (Nicholson, 1999). The tasks were developed specifically for the program to match the teaching approaches being encouraged in the professional development sessions (Callingham, Griffin & Corneille, 1999). Each task involved students in a short, structured investigation around different mathematical concepts. The targeted ideas were: identifying relationships (*Street Party*); exploring number patterns (*Bean Counters*); variation and bias (*Come in Spinner*); and area (*Newspaper by the Metre*). Each task provided both holistic and analytical scales, the holistic scale being intended to define overall concept development for teaching purposes and the analytical intended to provide an easily used scoring system for consistent rating of students by teachers.

Research Questions

The four tasks used each addressed a different area of the mathematics curriculum. With this in mind, the first research question addressed was:

- Do the tasks measure the same variable in a consistent and reliable manner?

If this were established, then the second question became:

- How could the variable be interpreted in the context of students' demonstrated responses and classroom practice?

Methodology

Nearly 2000 year 8 students in the nineteen (19) INISSS project schools attempted the tasks. A pre-test/post-test model was used, with each student completing two tasks early in the 1999 school year and the other two in October 1999. Classroom teachers administered the tasks to their classes in mathematics lessons. Teachers presented each task as an assessment task, but using their normal teaching and learning activity such as scaffolding through open-ended questioning and reading the questions to poor readers. Any tools normally available in the classroom, including calculators, could be used except where restrictions were specified.

Each task was based around a story shell and provided a graded set of items that targeted a developmental sequence within a particular area of the mathematics curriculum. The text of *Street Party* is provided in the appendix, other tasks were similar in structure. Each question within each task was also provided with an analytical scoring scale that was used for marking student responses (Callingham, 1999). Thus each task provided teachers with an holistic scale of development that covered the whole task, and a detailed analytical scale for each item. An example of these scales is provided in Table 1 for question 6 from *Street Party*, a task addressing early algebra concepts. Students were asked to make a long table by placing small tables end-to-end, and to find out how many people they could seat for given numbers of tables. The analytical scale is placed approximately where the different stages would fall on the holistic scale that applied to the whole task.

Table 1
Holistic and Analytical Scales for Street Party Question 6:

How could you work out how many people they could seat for any number of tables? Write to Dean explaining your method in the space on your record card.

Holistic scale	Analytical scale	Score
	No explanation or irrelevant answer.	**0**
Students recognise patterns in pictures and diagrams, and continue these accurately. They use this ability to solve simple problems involving direct relationships only.	Explanation **based on patterns only** e.g., it always goes up by 2.	1
Students can describe straightforward relationships in words, and relate these to a real situation. They use this ability to solve problems involving direct relationships where the numbers involved are too large to allow just a continuation of the pattern.	Explanation based on a **generalised relationship** expressed in words or symbols.	2
Students can describe straightforward concrete relationships in words or symbols and manipulate these effectively. They use these skills to solve problems involving direct and inverse relationships.		

By providing both holistic and analytical scales it was hoped that marking the tasks would be clear and unambiguous and interpretation of the students' responses would be enhanced. The 41 teachers who participated in the professional development sessions received training in the use of the scoring rubrics; other teachers relied on the advice of these colleagues.

Following a feedback session with teachers after the first administration of the tasks, and initial data analysis, some modifications were made to some scoring rubrics, to make them easier for teachers to use and more explicit about the nature of the desirable response. For each task, at least four rubrics were left unchanged to provide an anchor for the second administration analysis. Some minor modifications were also made to the student answer sheets to allow students more room for their responses, by presenting these as folded A3 rather than A4 size. These changes were not expected to influence the substance of students' answers (Callingham, Griffin & Corneille, 1999).

Findings

Measurement Characteristics of the Tasks

Person ability and item difficulty. Rasch analyses using the QUEST computer program (Adams & Khoo, 1995) allowed the relative positions of person ability and item difficulty to be plotted on a single scale. This is shown in Figure 1. The variable map indicates that each task has items ranging from very low difficulty to very high difficulty, as shown by the positions on the scale. Further, it matched the range of ability in the year 8 students who completed the tasks, shown on the left-hand side of the figure. Thus the tasks allowed all students, from the weakest to the most able, to demonstrate achievement.

Fit to the model. To establish the uni-dimensionality of the variable, as required by the Rasch model, the fit of the data to the model was determined. After the first administration, one group of items in the *Newspaper by the Metre* task did not appear to fit the model. The scoring rubrics for these were changed to make them more explicit and understandable to the teachers who had to mark them. The fit map for the second administration is shown in Figure 2.

Although there is some misfit of data to the model, shown by the markers that fall outside the accepted limits, this can mostly be explained by students finding the items easier than expected at the second administration. The earlier misfit in the newspaper task items had disappeared, suggesting that the application of the revised scoring rubrics had improved the fit of these items to the model. Overall, the fit to the model is acceptable, and thus it can be concluded that the tasks were measuring the same variable, which was defined as numeracy.

Interpretation of the Variable

Since it was evident that the tasks were all measuring the same variable, it was important to identify common skills and understandings across all tasks. This was done by a qualitative content analysis of clusters of items having similar difficulty levels. For example, the easiest items required students to solve simple, one-step problems with the concrete materials provided if needed. *Bean Counter* 1 asked students to solve an addition square involving single-digit numbers, using beans to model this if they wished. *Street Party* 1 required students to make a long table from two small tables and work out how many people could sit at it. *Newspaper by the Metre* 1 asked students to construct a square metre out of newspaper. The cognitive processes involved appeared to require pattern recognition in straightforward contexts—the pattern in the beans in the squares, in the number of people and in a square shape. A similar procedure was undertaken for all items.

At the lowest level, the items required pattern recognition and use, further up they demanded identification and application of rules and, at the top level, generalisation to a range of situations. Thus a sequence of pattern → rule → generalisation was identified. This provided a hypothesised overall holistic scale that was derived from the clusters of students' responses, determined by application of the analytical item scales.

```
  4.0                               |
                                    |
                                    |
                                    |
                                    |     s5  .2 s6  .2
  3.0                               |     s4  .3 s8  .3
                                    |     s2b .3
                                    |
                               X    |     b6b .2 n9  .2 s7b .2
                               X    |     b7  .2
                               X    |     b4      b5b .2 s3  .2 p8b .3
  2.0                          X    |     n7b .2 n8b .2 s8  .2
                              XX    |      n4b .2 n6b .2 n7a .2 n8a .2 s2a .2 p4  .3
                            XXXX    |     n6a .2 s4  .2 p10 .2
                           XXXXX    |     p9
                         XXXXXXX    |     p8b .2
  1.0                   XXXXXXXX    |     b6b .1 n5  .2 s7b .1
                       XXXXXXXXX    |     n9  .1 s7a     s8  .1
                 XXXXXXXXXXXXXXX    |     b5b .1 b6a     n3  .2 n7b .1 n8b .1 s2b .2
                XXXXXXXXXXXXXXXX    |     b5a     n4b .1 n6b .1 p5b .3 p6  .3 p8a     p10 .1
               XXXXXXXXXXXXXXXXX    |     n4a .2 s4  .1 p4  .2 p8b .1
                  XXXXXXXXXXXXXX    |     b7  .1 n7a .1 n8a .1
   .0       XXXXXXXXXXXXXXXXXXXX    |     n2  .2 n5  .1 n6a .1
                  XXXXXXXXXXXXXX    |     p5b .2 p6  .2
               XXXXXXXXXXXXXXXXX    |     b3  .2 s2b .1 s5  .1
                   XXXXXXXXXXXXX    |     n3  .1
                    XXXXXXXXXXXX    |     p5a
 -1.0                XXXXXXXXXXX    |     s6  .1 p5b .1 p6  .1 p7
                       XXXXXXXXX    |
                           XXXXX    |     n2  .1
                           XXXXX    |     n4a .1
                            XXXX    |
                             XXX    |     s3  .1 p4  .1
 -2.0                       XXXX    |     p3c
                           XXXXX    |
                             XXX    |     b3  .1
                               X    |     s1      p3a     p3b
                               X    |     s2a .1
 -3.0                         XX    |     b2
                               X    |
                               X    |
                                    |     p2
                               X    |     n1
                                    |     b1
 -4.0                               |
                                    |
                                    |     p1
                                    |
                                    |
 -5.0                               |
-------------------------------------------------------------------------------------------------
 Each X represents    7 students
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Key to Items: **p** Street Party **n** Newspaper by the metre **s** Come in spinner **b** Bean counter

Figure 1. Variable map for INISSS numeracy tasks

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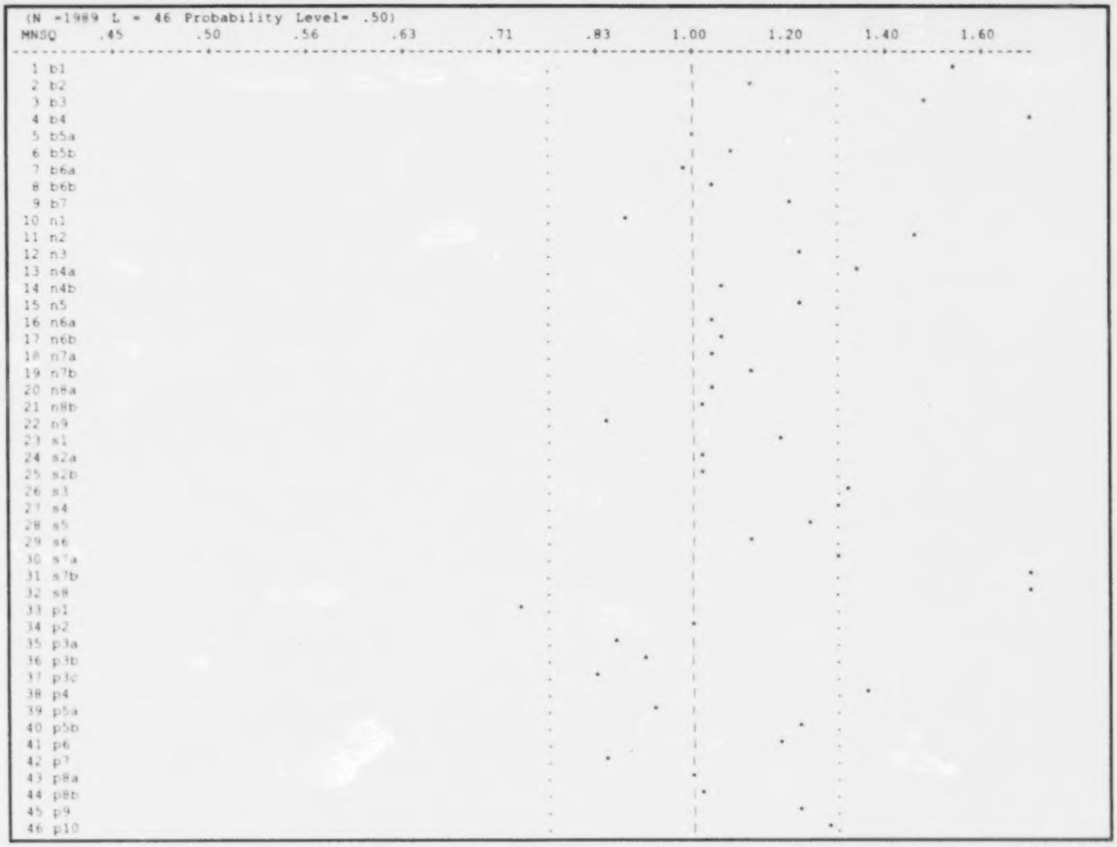

Figure 2. Fit map for INISSS items

Students' Responses

In order to verify this sequence, consideration was given to actual students' responses produced during the assessment process. While the responses fitted the overall hypothesised cognitive developmental sequence, it appeared that there were differences in the manner of responding to the tasks that were aligned to this sequence.

Students who were at the lower end of measured ability typically provided minimal written responses. Their written explanations were often irrelevant such as '...youseing my brans (*sic*)'. Some of these students had been videotaped while undertaking the tasks and they were revealed using strategies such as counting by twos or physical manipulation of the concrete materials supplied. Students operating at around the middle level of the ability range tended to write their explanations in natural language while those at the upper end could use symbolic notation and mathematical language. These response modes were termed 'productive elements'. A summary of the 'cognitive development' and 'productive elements', together with examples of typical students' responses, is provided in Table 2.

This combination of 'cognitive development' and 'productive elements' provided a framework for the interpretation of students' demonstrated outcomes. It described increasingly sophisticated mathematical concepts that were matched by a sequence of response modes and this formed the basis for reporting students' progress to teachers.

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Table 2.
Hypothesised Developmental Framework for Numeracy Assessment

Cognitive	Productive	Party	Newspaper	Bean	Spinner
Relationship/ generalisation use	Summary such as graphs or symbolic language.	Expresses rule in symbolic language e.g., 2n + 2 or similar and uses this to solve extended and inverse problems.	Estimation of area of larger and smaller objects in square metres based on understanding of relationship between units and personal benchmarks.	Maximum number of beans is 35 (with associated reasoning).	Two factors affecting spinner - physical such as pencil not being centred, introduces bias, and need for large sample - as gets bigger more likely to centre around expected value.
Rule extension			Estimation using personal benchmarks based on familiar objects e.g., exercise book.	"Because you must have at least one bean in each square the number of beans in other squares is limited."	
Rule application	Written explanation - natural language	"For any number of tables you double and add two for the ends."	Recognition of size by direct comparison with square metre.	Completes all possible solutions to a repetitive problem.	"It's a 50:50 spinner therefore for 1000 spins you'd expect 500 darks."
Rule recognition		"It goes up by 2's."	"You can cut up the square and stick it together again and it will still be a square metre."	"As one goes up the other goes down."	"It's half and half."
Pattern use	Verbal explanation in natural language.	Counts by 2 orally.	Measuring to determine size.		
Pattern recognition			Use of square.	"We juggled the beans, the numbers add up."	

Classroom practice. Considering the measured variable in the context of classroom practice provided further verification of the hypothesised framework. Since most of the professional development sessions had been videotaped, these records were examined to relate the teaching approaches that were being advocated to the numeracy assessment framework.

The teaching approaches involved open-ended investigations with concrete materials. Following the initial assessment, teachers reported being disappointed with the ways in which their students completed the tasks. In particular, they commented on the poor explanations provided by students in their responses, and undertook to develop these skills explicitly. Student talk and discussion was encouraged, and many teachers reported increasing the emphasis on literacy skills. Videotaped classroom sessions indicated that teachers were doing this through modelling and reinforcing the importance of providing explanations, especially in written form.

Feedback from teachers after the second assessment session showed how much classrooms had changed. Students not only undertook the assessment tasks with greater reported levels of skill and confidence, they also persisted for longer and wanted to spend more time on the tasks. Some teachers reported spending up to six lessons developing the ideas behind the tasks. This may well be a case of assessment driving practice. However, in this instance the assessment process was specially designed to match the teaching and learning strategies being advocated.

Discussion

The performance assessment tasks discussed here provided a sound measurement approach to the assessment of a single construct defined as numeracy. These tasks supplied a means of assessing underlying mathematical ideas and skills through an analytical scoring rubric for each item within an holistic cognitive developmental sequence. In addition, the manner in which students responded to the tasks, suggested that different 'productive elements' were involved—talk, writing in natural language and the use of symbols and mathematical language. Combination of the cognitive sequence and 'productive elements' provided a framework for assessing numeracy, based on quantitative data from Rasch modelling and a qualitative analysis of the item content and students' responses. This framework has the following potential for teachers and administrators.

Firstly, reporting against this framework indicates at what level students are working, measured using assessment tasks that match teaching practice. The close link between teaching and assessment provides information that can be used to plan appropriate teaching programs, for intervention or extension. Further, the 'productive elements' aspect suggests that effective teaching should both target the cognitive level and be encouraging work production through a cycle of talk, record, symbolise. The framework explicitly targets a level and describes how teachers might intervene.

Secondly, this framework could be used to develop assessment tasks. The overall sequence provides the scope of the task and the analytical scales for each item can be designed to address different levels within the holistic sequence.

Further work is needed to confirm the components of the proposed framework, both cognitive and 'productive elements'. In addition, the feasibility of using the framework in these ways needs to be established by additional work with teachers. However, these early findings are encouraging, and this appears to have the potential to be a useful, rigorous assessment tool for teachers.

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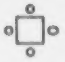
Appendix: Text of Street Party Task

Dean's community is planning a street party to celebrate the year 2000. They have lots of small square tables. Each table seats 4 like this:



The community decides to put the tables in an end-to-end line along the street to make one big table.

1. Make a line with 2 tables. How many people will be able to sit at it?
2. Make a line of 4 tables. How many people will be able to sit at it?
3. Make a line of tables that would seat
   (a.) 8 people. (b.) 12 people. (c.) 20 people.
4. Find two ways of showing **all** your results so far.
5. Dean says they can borrow 99 tables. How many people could they seat? Write a note to Dean explaining how you got your answer.
6. How could you work out how many people they could seat for any number of tables? Write to Dean explaining your method in the space on your record card.
7. Jen wants to use a small table that is not square. Make a different shape for a small table. Draw your small table for Jen showing the people sitting round it in the space on your record sheet.
8. Use your small table to make some big tables by putting small tables together. Draw three diagrams in the space on your record sheet showing how the big table grows, and the number of people that can sit around it as it grows. Explain to Jen what happens as your table grows by showing your findings in another way.
9. How many of your small tables would you need for 200 people?
10. Find a rule to work out how many of your small tables you would need for any number of people at the party. Write a note to Jen explaining your rule.

Students' Technical Difficulties in Operating a Graphics Calculator[1]

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We report on how students deal with some technical aspects of the operation of a graphics calculator. Clinical interviews were conducted with 25 Year 10-11 students as they used graphics calculators to study graphs of straight lines and parabolas. Three common student difficulties were identified: a tendency to be unduly influenced by the jagged appearance of graphs; a poor understanding of the zoom operation of the graphics calculator; and a limited grasp of the processes used by the calculator to display graphs. Implications for teaching are discussed.

Since graphics calculators (GCs) were first developed in the mid 1980's they have become steadily cheaper, more user-friendly, and more powerful. As a result, they are being increasingly used in mathematics teaching. Although there is considerable research on the use of GCs in the classroom, the majority of these studies simply describe the use of graphics calculators in teaching experiments without differentiating between the role of the GC and the context in which it is used (Dunham & Dick, 1994). In particular, research has yet to identify those aspects of GC use which best facilitate student learning (Penglase & Arnold, 1996). The research reported here is an attempt to rectify these shortcomings.

Background

In precalculus mathematics, a great deal of attention is paid to graphs of linear and quadratic functions. Students learn to recognise the important features of such graphs and link the symbolic form to their shape (line, parabola) and other details (intercepts, gradients of lines, and the direction, axis and vertex of a parabola). They do this by drawing and studying a number of linear and quadratic graphs.

When drawing graphs by hand, examples need to be carefully chosen to minimise complications. Thus, teachers and textbooks usually specify symmetric scales and choose functions whose important features lie near the origin. Learning takes place gradually and higher levels of sophistication are developed slowly over time. In the familiar paper and pencil environment, students are able to exercise direct control over the size and scaling of the graphs which they draw.

A number of papers have been written on how to use GCs in precalculus mathematics (e.g., Asp, Dowsey, & Stacey, 1993). Many writers suggest that a major advantage of the GC is that it allows more freedom to explore (Demana, Schoen, & Waits, 1993). However, such exploration inevitably means that students have to deal with unsymmetrical scales, blank screens or partial views, and non-integer coordinates. Because students are not plotting individual points, and there are no labeled axes or even grid lines, the whole process might appear magical (Dion & Fetta, 1993) and there is clearly a danger that fundamental misconceptions might arise (Mueller & Foster, 1999).

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[1] This project is supported by a Strategic Partnerships with Industry—Research and Training (SPIRT) grant from the Australian Research Council. The industry partner is Shriro Australia Pty. Ltd. (distributors of Casio calculators).

At the same time, students also have to deal with the non-trivial task of learning how to operate the GC itself. Giamati (1991) studied the use of graphics calculators on students' understanding of transformations of graphs and found that some students were adversely affected by also having to learn how to use the GC. She noted that the students' limited experience in using the GC and their lack of familiarity with its basic operations may have accounted for their lack of progress. Goldenberg (1988) identified several student difficulties related to the GC, including problems with the use of pixels as representations of points and misconceptions associated with scale changes when zooming.

Two further studies are relevant: Williams (1993) remarked on a number of difficulties students have when points of discontinuity are not visible due to the low resolution of the GC screen; while Vonder Embse and Engebretsen (1996) noted the confusion caused by the awkward-looking coordinates which often appear when tracing graphs on a GC. They demonstrated the advantages of being able to create so-called "friendly windows", in which each pixel has an x-coordinate with a small number of decimal places.

The present study was designed to investigate such technical difficulties, specifically those related to (a) pixels, (b) zooming, and (c) the determination of pixel x-coordinates. It is part of a larger study in progress. A report on students' conceptual difficulties is available elsewhere (Cavanagh & Mitchelmore, 2000).

Method

Sample

Clinical interviews were conducted with 25 students, 5 students from each of 5 Sydney metropolitan high schools (15 Year 10 students: 8 girls and 7 boys; and 10 Year 11 students: 5 girls and 5 boys). Students were drawn from higher ability mathematics classes because it was felt that they would be better able to respond to the challenge of the interview tasks and to articulate their ideas clearly. It was assumed that the difficulties which these students demonstrated would also be found in students of lower ability.

All the students who were interviewed had studied the graphs of straight lines and knew the gradient-intercept equation $y = mx + b$. They were able to sketch the graphs of parabolas given in general form and were familiar with the quadratic formula. They could use this formula to solve quadratic equations and locate the zeros of a parabolic graph.

The students had used a GC, the Casio *fx*-7400G, in mathematics lessons for between 6 and 12 months prior to the first interview. In one school, students owned their own graphic calculators and were able to use them in all lessons and examinations. However, the majority of students had limited access to a class set of GCs owned by the school. Broadly speaking, the students were novice users who had very little experience with GCs and had only used them to display graphs of linear and quadratic functions.

The Interviews

Each student was interviewed individually by the first author for fifty minutes on three separate occasions, each approximately two weeks apart. The students completed a variety of graphing tasks using a Casio *fx*-7400G and were asked to interpret the calculator's output. All of the interviews were videotaped and selected segments were later transcribed so that a more detailed analysis of the students' responses could be made.

The Tasks

The tasks which are relevant to this paper were as follows.

1. Use the graphics calculator to find the intersection of $y = 2x - 1.5$ and $y = 3x + 0.8$.

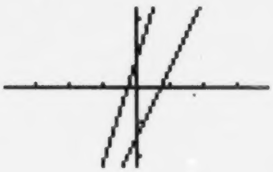

Figure 1. Graphs of $y = 2x - 1.5$ and $y = 3x + 0.8$.

Figure 1 shows the two functions as they are displayed in the initial viewing window. (On the *fx*-7400G, this window is friendly with tick marks at intervals of 1 and pixels at intervals of 0.1.) The two lines have a similar slope and appear to meld together over a number of pixels near their intersection point. Task 1 investigated how students might interpret this apparent contradiction.

2. Display the graph of $y = 0.75x^2 - 1.455x - 1$ on the graphics calculator. Find the intercept with the positive x-axis and the coordinates of the vertex of this parabola.

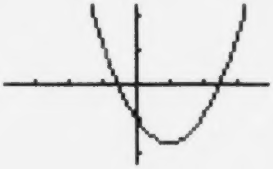

Figure 2. The graph of $y = 0.75x^2 - 1.455x - 1$.

Figure 2 shows the parabola in the initial window. The task was designed to investigate two questions: Firstly, how do students interpret the coordinates displayed on the GC screen when the x-intercept is irrational? Secondly, how do they deal with the situation where the lowest point of a graph is represented by a line of pixels rather than a single one?

3. Display the graph of $y = 2x - 1$ on each graphics calculator. Move the trace cursor to the points (0, -1) and (1, 1). What do you notice? Can you explain what has happened?

For this task, the student was given two calculators—one set to the usual initial window and one set to a window in which both the x- and y-axes were displayed from -10 to 10. (The students were shown these settings before they responded to the task.) On both calculators, it was possible to move the cursor to the point (0, -1). However, on the second calculator, it was not possible to move the cursor until the coordinates (1, 1) appeared on the screen. Because the window was not friendly, almost all coordinates appeared with 4 decimal places. Figure 3 shows the two calculator screens with the cursor on the pixel nearest to (1, 1).

Task 3 was designed to investigate whether the students were aware of the procedure used by the GC to assign x-coordinates to pixels. Knowledge of this procedure is essential if students are to understand how "friendly windows" might be created.

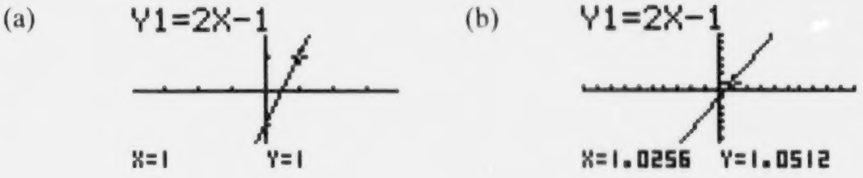

Figure 3. The graph of $y = 2x - 1$ in (a) the initial window, (b) a 10 by 10 window.

Results

Task 1. All 25 students were able to zoom out or scroll the window down until the region near the intersection of the two lines was displayed on the screen. However, only 7 students (28%) recognised that the low resolution of the screen and the fact that the two lines had very similar slopes was the reason why their images melded together over a number of pixels. The remaining 18 students (72%) expressed surprise at the image they saw and explained that they were expecting to see a distinct, easily identifiable pixel at the point of intersection. When this single pixel did not appear even after they had zoomed in, 5 students (20%) explained that the pair of lines briefly ran alongside each other and had more than a single intersection point. Of the others, 6 students (24%) continued zooming and tracing until they obtained coordinates which they felt were "nice" (integer or near-integer values), while 7 (28%) simply chose a point close to the centre of the group of pixels where the lines appeared to merge.

Task 2. All of the students zoomed in to find the intercept and vertex more accurately. They all expressed the belief that the relevant point could be found exactly provided one was prepared to zoom in a sufficient number of times. All of the students also expected that the horizontal line at the base of the parabola would become shorter each time they zoomed in until eventually one pixel would be seen at the vertex. Only 3 students (12%) correctly stated that the scale of the graph they saw displayed on the GC screen had changed after they had zoomed. The other students spoke of zooming as a distinctly different operation from any rescaling the graph.

Task 3. Explaining why it was not possible to locate (1, 1) on the second calculator proved to be an extremely difficult task. In fact, only 2 students (8%) gave a satisfactory explanation which included a reference to the relationship between the number of pixels across the screen and the window settings. Nine students (36%) were unable to answer at all, and the remaining students often gave the impression that they made a particular response because they could think of little else. For instance, 6 students (24%) regarded the unequal scaling of the axes as the source of the problem; 3 students (12%) thought that the problem was due to the linear function itself, and that if another line which contained the point (1,1) was graphed then it would be found; 3 students (12%) said that there was a pixel assigned to (1,1) but the low resolution of the screen meant that it could not be seen until they had zoomed in a number of times; and 2 students (8%) thought that the settings of both coordinate axes caused the problem rather than relating it to the x-axis alone.

Discussion

Students' responses to the tasks suggest three key areas of technical difficulty in using graphics calculators.

The Low Resolution of the Screen

Throughout the interviews, the students regularly commented on the jagged appearance of the graphs they saw on the GC and demonstrated that they were aware of the pixel approximations associated with the low resolution of the screen. For example, most students were able to explain why the calculator displayed the parabola shown in Figure 3 with a horizontal line of pixels near its vertex. However, even though they recognised the inconsistencies which arose from time to time between the position of the cursor and the coordinates, the students were still likely to base their answers exclusively on the visual representations of the highlighted pixels. Furthermore, despite the fact that the students claimed to have more confidence in the coordinates displayed on the screen, they needed constant reminders to consider these values before deciding on an answer.

A related issue concerns the students' ideas about the nature of points. In coordinate geometry, the concept of a point is essentially one of location or position on the number plane. It does not make sense, mathematically speaking, to regard a point as having other properties like size or shape. However, many of the students did attribute such characteristics to points on the GC screen. Goldenberg and Kliman (1988) commented on the tendency of students to speak about the size of points and described this misunderstanding as a "bead necklace metaphor" which "allows ideas of scale to be applied to points ... allows them to be conceptually magnified, lined up in a row, and so on" (p. 5). We noted similar responses from the students whom we interviewed.

One unexpected but positive aspect of the low screen resolution is the fact that students were able to contrast the patterns of pixels in the graphs of linear and quadratic functions to make inferences about their gradients. The majority of students concluded that, whereas the slope of a line remains constant, the slope of a parabola changes as you move along the curve and increases as you get further away from the vertex. The relatively small number of pixels on the GC screen might therefore provide a useful visual support for students in the early development of differential calculus concepts.

Zooming

The students did not have a good understanding of the zoom operation of the GC. As Goldenberg and Kliman (1988) also found, there was a marked tendency among the students to disassociate the zoom operation from any change in the scale of the graphs they saw. Instead, the students most often described zooming in terms similar to the use of a magnifying glass on a physical object when, as one continues to zoom in, previously obscured details are gradually revealed.

Students generally failed to recognise scale changes. We found that they had a tendency to regard scale in *absolute* rather than *relative* terms. That is, the students interpreted scale as either the distance between the tick marks on the coordinate axes or their value, but they did not consider scale as the ratio of distance to value (see Cavanagh & Mitchelmore, 2000).

The students' poor understanding of the zoom operation was evidenced in other ways as well. For example, when zooming in to locate the intercept of the parabola in Task 2, many students expressed surprise when they saw the same coordinate values repeated over and over again. This encouraged them to think that there must be a limit to the number of times one could zoom in before the GC screen would eventually "run out of points". The students also showed little appreciation of when it might be appropriate or helpful to use the zoom facility of the GC and when it was not. They invariably attempted to solve any problem they encountered by zooming and persisted with this approach long after it should have been clear to them that it was unproductive.

The Procedures used by the GC to Display Graphs

Students seemed to have little understanding of how a GC produces a graph. The results of Task 3 show that very few students knew that the x-coordinates a GC assigns to the pixels depends on the specified range of x-values to be displayed. Responses to Task 1 and other tasks not reported here indicate that most students did not realise that the GC displays the calculated y-coordinate (i.e., the value calculated from the given function and the x-coordinate of the pixel) and not the y-coordinate of the pixel where the cursor is positioned. The students also often failed to recognise when the y-coordinate was rounded and tended to regard the coordinate values as exact. Other, informal observations we have made suggest that many students also do not know that (in the default setting for Cartesian graphs) the GC calculates and displays one pixel in each column of pixels and then "joins up" these pixels to display the graph.

Students also did not know how to change the spacing of the tick marks displayed on the axes of a graph using the *scl* parameter in the window setting. In tasks not reported here, all the students we interviewed assumed that the *scl* value was connected with the scale of the axes, causing considerable confusion.

Implications and Conclusions

Kissane, Bradley, and Kemp (1994) argue that students should develop the technical skills required to operate a GC effectively. While it may not be necessary for all students to have a highly developed technical understanding, our work suggests that an appreciation of some basic aspects would be helpful. These include an understanding of the zoom operation of the GC and the link between zooming and scale; the methods used by the GC to assign values to the columns of pixels; the calculated nature of the y-coordinates displayed when tracing; and the fact that these y-values are often decimal approximations rather than exact values.

The discussions we had with students during the interviews indicate that they can gain insight into the technical operation of the GC but that this does not come naturally to the majority of students. We agree with Dick (1992) that students would benefit greatly from confronting the limitations of the technology and attempting to explain them, and feel that teachers should not structure their lessons or choose examples to avoid the kinds of technical difficulties we have described. However, we clearly need to take care in how we challenge students' misconceptions.

Students may benefit from understanding how the increment in the adjacent columns of pixels across the screen is calculated. This knowledge might help them recognise why a particular coordinate value is not represented by a pixel on the screen, and assist them in creating their own friendly windows rather than being constrained by the small number of default window settings that are available on the GC (Dowsey & Tynan, 1997).

The links between changing the window settings, rescaling the axes, and zooming needs to be made clearer to students. Our work suggests that students regard these operations as distinctly different and do not understand the effects of zooming on the range and scale of the axes.

Misconceptions may lessen with greater exposure to GCs (Ruthven, 1990). We found that the students who owned their own GCs more frequently exhibited a critical awareness of the calculator's output and were less likely to be confused by its technical limitations.

Finally, one of the things which struck us most during the interviews is that we should never assume too much about what students perceive when they look at the screen. What may seem obvious to the mathematically experienced may not be equally apparent to novice learners. For instance, more than one student struggled to explain how the coordinate axes like those in Figure 3 (b) could both range from -10 to 10 and yet look so different; they had simply failed to notice the rectangular shape of the viewing window.

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Curriculum Framework Implementation: Measuring the Impact on Secondary Mathematics Classroom Culture

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A quantitative investigation of 18 Western Australian secondary school mathematics classrooms was conducted prior to implementation of the *Curriculum Framework*. The *Classroom Cultural Elements Questionnaire* was administered to profile existing learning environments. Results revealed learning environments with varying levels of elements conducive to improved learning outcomes. This finding suggests *Curriculum Framework* implementation in mathematics requires attention to these elements of the classroom environment and development of a classroom culture focussed on the educative needs of students rather than traditional subject content.

Current curriculum initiatives in Western Australia centre upon the notion of outcomes-based education. This emphasis is consistent with national and international trends to ensure systemic, school and teacher accountability as a means of improving the educational outcomes of all students (Caldwell, 1993). It signifies a move away from emphasis upon subject centred curriculum delivery towards focus on the learning outcomes of individual students. The *Curriculum Framework* (Curriculum Council, 1998), is characterised by identification of sequential long-term educational goals. By comparison, previous curricula prescribed the objectives of specific courses of study, units of work or years of schooling. Five *Core Values* underpin the *Curriculum Framework*: pursuit of knowledge and achievement of potential; self-acceptance and self-respect; respect and concern for the rights of others; social and civic responsibility; and environmental responsibility. It also contains thirteen *Major Learning Outcomes, which* express the goals of kindergarten, primary and secondary education.

Current discrete subject-based curricula are to be restructured into eight generic learning areas: The Arts; English; Health and Physical Education; LOTE; Mathematics; Science; Society and Environment; and Technology and Enterprise. The *Curriculum Framework* document specifies seven *Key Principles* to guide schools in planning and curriculum development: an encompassing view of the curriculum; an explicit knowledge of the core values; flexibility; inclusivity; integration, breadth and balance; a developmental approach; and collaboration and partnerships. Implementation of the *Curriculum Framework* is anticipated to be particularly consequential for secondary schools with faculty based organisational and curriculum structures. At the classroom level, realisation of the major outcomes will probably necessitate student-centred learning and flexible modes of curriculum delivery. *Curriculum Framework* implementation is anticipated to cause changes in classrooms, which will be evidenced by a shift towards the psycho-sociological learning environment, rather than emphasis on subject-centred content, processes and skills, which characterise many traditional classroom environments.

Aims of the Study

The aim of the investigation described in this paper was to examine prior to *Curriculum Framework* implementation, the presence in secondary school mathematics classroom of attributes expected to be conducive for outcomes-based education implementation. This is the first stage in a longitudinal study of learning environments within the context of major curriculum reform. Although the focus of this inquiry is on mathematics classrooms, concurrent examination of classrooms in the other seven learning areas is also being undertaken. The overall study will utilise a developmental mixed-method approach. The data collected and analysed in this paper is from the quantitative first phase of the overall investigation.

Theoretical Background

The literature on Outcomes-based Education (OBE) frequently differentiates between objectives and outcomes. For example, Griffin (1998) viewed outcomes as having a behavioural orientation concerned with demonstration of student knowledge and skills in comparison to objectives not necessarily specifying student performance. Spady and Marshall (1991) considered OBE to be founded upon three premises: all students can learn and succeed, but do so in different ways and over different periods of time; successful prior learning is the precursor to successful future learning; and schools control the conditions conducive to successful learning. Spady and Marshall (1991) identified three approaches to the use of outcomes in curriculum development: traditional OBE has specification of learning outcomes preceded by decisions about curriculum content and structure; transitional OBE is characterised by de-emphasis of curriculum structure by integration of subject and content areas with focus on student learning; and transformational OBE commences with identification of broad educational outcomes, which then predicate curriculum structure, learning experiences and the organisation of the school.

Notwithstanding the strong local and international support for an outcomes-based approach to curriculum development, teaching and learning, OBE has been subject to criticism. This centres upon concern over the potential for outcomes-based curricula to be narrow, fragmented and behaviourist (Glatthorn, 1993; Towers, 1994).

The *Curriculum Framework* has been developed in cognisance of the potential benefits and disadvantages associated with previous outcomes-based curriculum reform initiatives. The *Curriculum Framework* acknowledges the *Core Values* of society, identifies thirteen *Major Learning Outcomes*, and provides guidelines for school-level implementation. Specification of a general framework rather than a curriculum *per se* is intended to provide overall direction and purpose for curriculum development. However, selection and sequencing of content, skills and learning experiences are to be decided at the school level based upon local circumstances. Adoption of the *Curriculum Framework* is predicated on the assumption school staff will have a clear understanding of the intent of the framework and effect the attainment of broad long term learning outcomes for students. The foundations for constructing the school curriculum are the five *Core Values* and the thirteen *Major Learning Outcomes*.

Researching the effects of the *Curriculum Framework* on schools and classrooms is potentially difficult due to the influence of the local context on school level curriculum planning and actualisation. Curricula across different schools may vary considerably. The approach taken in this study was to consider realisation of *Curriculum Framework* intentions from the perspective of their effect on classroom learning environments. It was assumed attainment of the cognitive and affective outcomes expressed in the *Major Learning Outcomes* and the *Core Values* by students would be evidenced by the presence of specific beliefs, values, attitudes, norms and resulting behaviour within classrooms. These attributes of a group of people characterise their culture. Donahoe (1993) provided a general definition of culture; "the values, beliefs, behaviours, rules, products, signs and symbols that bind us together" (p.302). The fidelity of *Curriculum Framework* implementation can be assessed by examination of classroom culture.

The *Classroom Cultural Elements Questionnaire (CCEQ)* (Cavanagh, Dellar, Ellett, & Rugutt, 2000) was developed to investigate the psycho-sociological aspects of the classroom learning environment. The theoretical background for instrument development was research literature on school culture, school effectiveness, school improvement, effective teaching and effective learning. Distillation of this literature resulted in identification of five dimensions of an effective classroom culture: educational values of the individual student; involvement of parent(s) in their child's schooling; classroom group; the teacher; and outcomes of learning.

Data from a sample of 682 students in English, Mathematics and Society and Environment classes were factor analysed. The original instrument was refined in consideration of the factor structure matrix and validity of constructs being measured. *CCEQ* refinement resulted in an instrument which profiles the presence of twelve classroom conditions; student educational values, learning outcomes, parental involvement, peer caring, peer discussion, peer emphasis on learning, peer support, student autonomy, teacher caring, teacher control, teacher expectations and teacher support. Many of the conditions examined by the *CCEQ* are consistent with those expected to result from attainment of *Curriculum Framework* outcomes. It is anticipated successful implementation of the *Curriculum Framework* will necessitate and be evidenced by an increased presence of these classroom conditions.

Methodology

The *Classroom Cultural Elements Questionnaire* is comprised of 87 items organised into twelve scales (Table 1) with a four-point Likert scale response facility. The *CCEQ* was administered in two Western Australian secondary schools to 682 Years 8-10 students, 334 students were from 18 mathematics classrooms (Table 2). The need to avoid multiple instrument administration to the same students in different subject areas resulted in the Mathematics sample from one school being entirely Year 8 students.

All of the students present at school on the day of administration were surveyed. The internal reliability of the twelve scales was assessed by Cronbach Alpha analysis of reliability. The capacity of the instrument to discriminate between different classroom groups for each of the twelve constructs was determined by One-way ANOVA analysis of variance with multiple range testing. The validity of the instrument scales to solicit data on classroom culture was examined by consideration of the statistical correlation (Spearman) between the twelve constructs. The differences between individual classroom groups for each of the twelve constructs were identified by One-way ANOVA analysis of variance with the Least Significant Differences Test applied.

Table 1

Classroom Cultural Elements Questionnaire Scales, Sample Items and Scale Size

Scale	Sample Item	No. of Items
Educational Values	My future will be improved by what I learn.	9
Learning Outcomes	I perform to the best of my ability.	13
Parental Involvement	My parent(s) help with my homework.	8
Peer Caring	Students are tolerant of one another.	5
Peer Discussion	We talk about our progress.	6
Peer Emphasis on Learning	We believe that everyone can learn.	6
Peer Support	Students encourage each other to accept challenges.	7
Student Autonomy	We are allowed to commence new work when we are ready.	6
Teacher Caring	We feel safe in this teacher's class.	7
Teacher Control	The teacher expects us to complete our work on time.	5
Teacher Expectations	The teacher expects us to perform to the best of our ability.	4
Teacher Support	The teacher provides us with encouragement.	11

Table 2

Sample Distribution by Group and Year

Group	School	Number of Students	Year Level	Group	School	Number of Students	Year Level
1	1	12	Year Eight	10	2	22	Year Eight
2	1	25	Year Eight	11	2	16	Year Eight
3	1	21	Year Eight	12	2	12	Year Nine
4	1	18	Year Eight	13	2	24	Year Nine
5	1	24	Year Eight	14	2	23	Year Nine
6	1	24	Year Eight	15	2	8	Year Ten
7	1	26	Year Eight	16	2	15	Year Ten
8	2	15	Year Eight	17	2	14	Year Ten
9	2	19	Year Eight	18	2	16	Year Ten

Results

Cronbach Alpha analysis of the internal reliability of data from the twelve scales (Table3) indicates individual respondents responded consistently to the items within each scale. This is evidence of the items within each scale soliciting data on a common construct as administered to this sample.

Table 3
Scale Internal Reliability

Scale	Internal Reliability (Alpha)	Scale	Internal Reliability (Alpha)
Educational Values	0.86	Peer Support	0.89
Learning Outcomes	0.92	Student Autonomy	0.82
Parental Involvement	0.84	Teacher Caring	0.88
Peer Caring	0.84	Teacher Control	0.89
Peer Discussion	0.83	Teacher Expectations	0.84
Peer Emphasis on Learning	0.78	Teacher Support	0.94

One-way ANOVA with multiple range testing was used to examine the capacity of the instrument to discriminate between the classroom learning environments for this sample of 18 class groups Table 4 presents the F ratio and probability value for each of the twelve scales. The results of this analysis indicate the instrument was able to discriminate between the 18 class groups for the twelve constructs under investigation. Spearman correlation coefficients were calculated to examine the relationships amongst the twelve constructs. The data presented in Table 5 is a summary of the inter-scale correlations. The positive correlations between the scales suggest the constructs are inter-related which is supportive of the proposition they collectively represent a common phenomenon. The culture of the classrooms examined are characterised by the twelve constructs or cultural elements investigated and also by inter-dependency between these elements.

Table 4
Cceq Capacity to Discriminate between Different Class Groups (One-way Anova)

Scale	F ratio	F probability	Scale	F ratio	F probability
Educational Values	2.27	0.003	Peer Support	2.34	0.002
Learning Outcomes	3.90	0.000	Student Autonomy	3.40	0.000
Parental Involvement	3.27	0.000	Teacher Caring	5.20	0.000
Peer Caring	4.09	0.000	Teacher Control	4.34	0.000
Peer Discussion	4.52	0.000	Teacher Expectations	4.82	0.000
Peer Emphasis on Learning	2.33	0.002	Teacher Support	5.49	0.000

Table 5

Range of Inter-Scale Corrleation Coefficients (Spearman)

Scale	Range of Inter-scale Correlation (Spearman)	Scale	Range of Inter-scale Correlation (Spearman)
Educational Values	0.59 - 0.15	Peer Support	0.74 - 0.34
Learning Outcomes	0.63 - 0.43	Student Autonomy	0.54 - 0.14
Parental Involvement	0.53 - 0.30	Teacher Caring	0.81 - 0.27
Peer Caring	0.74 - 0.33	Teacher Control	0.67 - 0.28
Peer Discussion	0.63 - 0.44	Teacher Expectations	0.71 - 0.31
Peer Emphasis on Learning	0.61 - 0.39	Teacher Support	0.81 - 0.36

Scale mean scores were adjusted by being dividing by the number of items within each scale so the range of means across the 18 class groups would correspond to the four-point Likert scale on the questionnaire. Adjusted scores below 2.0 indicate "disagree"/ "strongly disagree" and those above 3.0 indicate "agree"/ "strongly Agree". One-way ANOVA analysis of variance with the Least Significant Differences Test (LSD) was utilised to determine the number class groups with statistically significantly ($p<0.05$) different scale mean scores. This test provides only relative data since if one group is significantly lower, then the remainder of the groups are counted as being higher. Table 6 presents the range of adjusted scale mean scores and the number of significantly different groups for each scale. The data in Table 6 indicates differences in classroom culture for the twelve elements being investigated.

When the LSD test results are considered, the lower range adjusted scale mean scores for *Parental Involvement, Peer Discussion, Student Autonomy* and *Teacher Support* are probably from only one group for each scale respectively. In the majority of classes the scale mean scores are above 2.0 which is indicative of the presence of the twelve elements within the classrooms. However, affirmative responses, "agree"/ "strongly agree" would produce scale mean scores greater than 3.0. Only responses to *Educational Values* and *Teacher Expectations* are in this range. Although the other 10 classroom cultural elements are present in the majority of the mathematics classrooms investigated, students have not consistently affirmed their presence.

Table 6

Range of Adjusted Scale Mean Scores and Number of Classes with Statistically Significant Higher Mean Scale Scores

Scale	Range of Adjusted Scale Mean Scores	Number of Classes	Scale	Range of Adjusted Scale Mean Scores	Number of Classes
Educational Values	3.4- 4.0	11	Peer Support	2.3 - 3.0	7
Learning Outcomes	2.1 - 3.0	16	Student Autonomy	1.7 - 2.7	17
Parental Involvement	1.8 - 2.7	17	Teacher Caring	2.3 - 2.8	17
Peer Caring	2.2 - 3.2	11	Teacher Control	2.4 - 3.4	17
Peer Discussion	1.9 - 3.1	17	Teacher Expectations	3.0 - 4.0	15
Peer Emphasis on Learning	2.6 - 3.2	13	Teacher Support	1.9 - 3.3	17

Discussion and Significance of the Study

Analysis of the results of CCEQ administration in this study has revealed the constructs were reliably measured. The extent of inter-scale correlation supports the proposition of the twelve cultural elements being inter-related and collectively representing the higher order construct of classroom culture. This finding is consistent with previous school culture research and the consequent theoretical conceptions of school culture (Cavanagh & Dellar, 1997, 1998). The *Learning Outcomes* element is a significant component of classroom culture, and the range of inter-scale correlations indicates its relationship with the other eleven elements. Although it could be considered as the outcome of an effective classroom culture, this element is both a vehicle for classroom improvement and a component of an effective classroom culture. Successful prior learning provides motivation for future learning and strongly influences satisfaction with schooling (Samdal, Wold, & Bronis, 1999).

Apart from the specific applicability of the *CCEQ* in assessing *Curriculum Framework* implementation, it also has general application in the design and execution of teacher professional development activities to support curriculum improvement and enhance student learning outcomes. The notion of an effective classroom culture characterised by the twelve cultural elements provides teachers with an alternative perspective on the classroom environment. The data obtained from the *CCEQ* informs teachers about the nature of the prevailing cultural elements within traditional mathematics classrooms. It also emphasises that effective implementation of curriculum change (*Curriculum Frameworks)* is underpinned by and built upon the current classroom culture. Data on the prevailing classroom culture also profiles the readiness of the teacher and students to engage in classroom improvement.

The study has confirmed the appropriateness and utility of the *Classroom Cultural Elements Questionnaire* in examining mathematics learning environments through a classroom culture orientation. Consistency between the major outcomes of the *Curriculum Framework* and the constructs examined by the *CCEQ* has enabled the results of the quantitative investigation to profile elements of prevailing mathematics classroom culture of relevance to the intentions of the *Curriculum Framework*.

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Young Adults Making Sense of Data

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Young adults were presented with raw data involving a number of variables and asked to produce a report that highlighted any "interesting aspects" that they observed. The approaches taken varied from simple frequency information through to scatter plots and the use of ratios to consider the relationships between three variables. Although the use of sophisticated statistical techniques was not expected, the limited use of even basic techniques was surprising, given the increased emphasis on Chance and Data in school curricula.

In 1987 Frances Curcio argued the need for students to be given opportunities to describe the relationships and patterns that they observe in data (Curcio, 1987, p.391). Since then, Chance and Data together have been incorporated more fully into the mathematics curriculum, identified as a separate important strand of the curriculum in many Australian states. Although this may mean that more students are learning important statistical techniques, such as graphing and calculating measures of central tendency, it is not entirely clear to what extent Curcio's broader program has been implemented. In particular, students who can perform statistical processes when specifically requested may *not* be able to choose for themselves when such skills ought to be implemented. The work of Gal, Rothschild, and Wagner (1990) and Watson and Moritz (1999), for example, suggests that of those students who understand how to calculate the mean, many may not appreciate its significance or usefulness when dealing with data sets. Their research showed that when comparing two samples of different size very few students chose to determine means despite knowing the mechanics of the process. Li and Shen (1992) highlight that students presenting graphical data may do so inappropriately.

One tool that has been used to examine students' statistical decision-making abilities is the data cards protocol, first described in 1995 (Watson & Callingham, 1995; Watson, Collis, Callingham, & Moritz, 1995), and briefly outlined below. This protocol lets students choose which aspects of the data to analyse and decide on the best representation. Initially intended for primary and secondary students, in the current study it has been adapted for use with young adults in order to ascertain which data handling skills are brought to bear by those who have completed compulsory schooling. This work continues the author's earlier study of the statistical cognition of young adults, that examined the conclusions drawn from data by young adults, and also considered their understanding of the relationship between a sample and its corresponding population (Chick, 1999). The purpose of the current research is to examine what aspects of data young adults choose to consider, and what statistical techniques they apply in their analysis.

Method

There were 32 students (21 males and 11 females) involved in this study. In 1997 all were enrolled in a terminating first year university mathematics service course. intended for students from the science, arts, commerce, and medicine faculties undertaking subjects requiring basic calculus. This course had no statistics component. The students had completed Year 11 or Year 12 mathematics subjects that included the study of basic statistical ideas,.

including collecting, organising, displaying and interpreting numerical data together with calculating summary statistics. Most were also exposed to straightforward techniques associated with normally distributed data.

The *data cards protocol* has as its central focus a set of data about 16 individual children. For each child the data comprise information about name, weight, eye colour, favourite activity, number of fast food meals consumed per week, and age. In most implementations of the protocol the data are presented to subjects on 16 individual cards (hence the name of the protocol), but for the current study they were presented in a table, with the child's gender specifically identified as well. The data for the protocol, as developed by Watson et al. (1995), are included in the Appendix. Subjects were provided with this table and asked to identify and justify any interesting features of the data. For the current study, the protocol was implemented as an assignment question:

> Examine the data and produce a report which highlights any aspects of the data which you think are interesting. You are free to choose your approach to this problem and what to include in your report. If you make any claims about the data be sure to back them up with suitable evidence.

The students prepared written reports in response to this question; these responses provided the data for this research. It must be emphasised that the task was purposefully open-ended, so that students were free to choose both the particular aspects that appealed to them and the actual methods of analysis. Some students capitalised on this opportunity by considering a wide variety of relationships, while others were either unsure what actually constituted a satisfactory response, or did what they felt would be just enough to satisfy the assignment marker. Caution must therefore be applied to our consideration of quantitative aspects of the analysis.

For the purposes of this paper a number of aspects of the students' responses were considered. Every comment pertaining to one or more of the data card variables was noted, together with the number of variables involved. In addition, the nature of any statistical analysis associated with the comment was noted. For example, some students referred to graphs they had drawn, some calculated averages, while others just gave a written description presumably based on what they had observed in the original table of data. It must be noted that because the data set was so small, many students almost certainly observed its features from the actual data without utilising any formal statistical methods. They may well have regarded such observations as obvious and not needing any formal evidence; this factor must be borne in mind when considering the students' limited use of graphical and statistical techniques.

Results and Discussion

Variables Considered by Students when Analysing the Data

Analysing the responses from the 32 students identified 219 instances where the students made an observation concerning one or more of the variables from the data. In many cases, however, these observations referred to the variables of only a few of the children in the data set, rather than an appropriately selected sample. As an example, one student made a claim about weight and gender, but as supporting evidence referred to two specific individuals from the data set. This was still classified as a two variable observation; in addition its inappropriateness was noted. The number of variables considered by students and the number of instances of such observations are given in Table 1. No student restricted their analysis to examining just one variable at a time, in contrast to results achieved with primary school children (see Watson et al., 1995), although 18 students (39 instances) made at least one single variable observation. Many of these observations involved the gender variable, highlighting the fact that there were equal numbers of boys and girls in the data (10 students), or noting the age range or average age (11 students).

Table 1
Number of Variables Considered by Students (N = 32)

Number of variables considered	Number of students	Number of instances
1 variable	18	39
2 variables	32	123
3 variables	25	43
4 or more variables	11	14

All 32 students made at least one observation regarding two variables. One student *only* considered two variable relationships (three instances), with an additional student considering both one and two variable information but no more. All but seven students referred to three variable combinations, with five students considering three or more such combinations. Of the 11 students considering four or more variables, many did so by recognising the existence of other influencing variables in a two variable association. As an example, eight students (11 instances) acknowledged the interdependence of at least four of the five variables, age, fast food, favourite activity, gender, and weight. It should be noted, however, that not all of them did this by considering the actual data, relying instead on external knowledge.

For two variables, the most common pairs considered were fast food/favourite activity (considered by 19 students); favourite activity/gender (17 students); fast food/gender (16 students); age/fast food, fast food/weight (10 students each); and age/gender, eye colour/gender, and favourite activity/weight (9 students each). Not surprisingly, the only pairs that were not considered by any students were eye colour paired with weight or age, and pairs involving name. For three variables by far the most common triple was fast food/favourite activity/gender (11 students), with other combinations considered by five or fewer students. Name was never one of the three variables, and eye colour was considered only once, although it was one of the four variables considered by two additional students.

Statistical Techniques Utilised by Students

Many basic statistical techniques were implemented by students in support of their claims. These techniques included various graphical approaches; calculating means, modes, percentages and fractions; producing tables; and determining age/weight ratios for investigating the relationship between three variables. In contrast, for some claims many students did not present any formal statistical arguments. Instead, they wrote the claim and appeared to assume that it was clearly visible from the raw data. The following quote from one student serves as an example: "It is seen that males watch more TV than females do as a favourite activity, while females tend towards activities such as netball and swimming." At the very least an ordered table or some appropriate numerical values could have accompanied this claim. This student's whole report was in a similar vein.

Table 2 lists the techniques utilised by the students, linked to the number of variables considered. Sometimes a single table or graph was used to support a number of observations, so the values in Table 2 do not give the total number of tables or graphs produced. Most of the techniques listed are self-explanatory. Lists of data or tables without labelled columns were classified as *Quasi tables*; the use of ranking words, like "more" or "least," together with other worded claims was classified as *Worded description*; and *Other numerical techniques* includes the use of ratio. All but eight instances of *Sum or total* were simple frequencies (e.g., number of males and females), with the eight non-trivial examples involving total number of fast foods for certain categories of individuals. These often included an average calculation as well. Nobody used the median, presumably because the students did not think it was particularly appropriate for the data.

Table 2

Statistical Techniques Used by Students (N = 32) and Number of Variables Considered

	Single variable		Two variables		Three variables		More than three variables		Total occurrences	
Statistical technique used	S	I	S	I	S	I	S	I	S	I
Table	0	0	11	32	6	11	1	1	11	44
Quasi table	2	2	4	4	2	2	1	1	8	9
Graph	1	1	9	12	3	3	2	4	10	20
Worded description	10	20	28	72	18	28	11	13	32	133
Fraction	1	1	4	4	0	0	0	0	5	5
Percentage	2	2	7	11	2	2	1	1	8	16
Average (mean)	3	3	18	42	5	7	5	5	21	57
Mode	1	1	9	9	2	2	1	1	11	13
Sum or total	12	13	15	25	4	6	1	1	21	45
Other numerical technique	12	12	9	13	9	10	2	2	18	37

Note. S = number of students utilising the technique; I = number of instances of the technique.

Most of the 36 tables produced by the students, and referred to on 44 occasions, were sub-tables or rearrangements of the original table of data. One student based all her claims on sorted versions of the original table, sorted according to the criterion of interest. A few students produced 2 × 2 contingency tables, but these merely gave frequency information such as for the number of students in the four categories determined by gender and the activity or passivity of favourite activity. No student produced a more complex 2 × 2 contingency table involving three variables similar to that shown in Table 3. Claims involving three (or more) variables were dealt with using other techniques.

Table 3
Fast Food Consumption as a Function of Gender and Favourite Activity

Mean number of fast food meals consumed per week		Favourite Activity	
		Active (N=6)	Passive (N=10)
Gender	Male (N=8)	2.0	7.8
	Female (N=8)	0.7	2.6

Note: "Active" includes football, netball, and swimming; "Passive" includes boardgames, TV, and reading.

Given the emphasis on graphical techniques in early statistical study and the effectiveness of graphs as a means of illustrating characteristics of data, it was surprising to find that only ten students used this form of analysis, producing a total of 17 graphs (which were referred to on 20 occasions). Previous research with Grade 5/6 students (Chick & Watson, 1998) found that many students did not attempt a graphical representation until *after* prompting by the researchers. Graphs may not be seen as an automatic "tool of choice" for dealing with data; this may be exacerbated by the fact that producing a graph requires more effort than alternative statistical approaches. The availability of spreadsheet and graphic calculator technology may reduce this tendency, although this occurred only to a limited extent in the current study. Five students appeared to take advantage of computer spreadsheet programs to produce their graphs, but not always with success. Figure 1 provides a classic example of what can occur when students use a program's features without thinking about whether such features are appropriate. Li and Shen (1992) have previously reported students' similar difficulties with graphical representations.

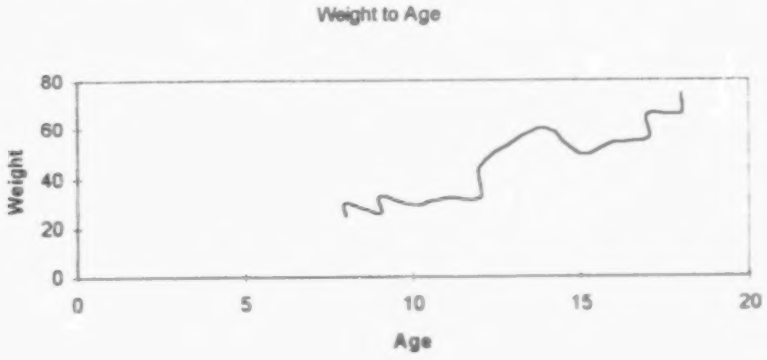

Figure 1. Weight to Age graph produced by a student using a spreadsheet program.

Hand drawn graphs were not totally free of problems either. Figure 2 shows a graph of weight against age, presented as a bar graph rather than a scatter plot and with a poorly labelled horizontal axis. Three students drew more appropriate scatter graphs, but only one drew a trend line or possible line of best fit. The student who plotted Figure 3 and its corresponding graph for males alluded to a trend line when he wrote that the two graphs show "that the males have a faster constant weight rate, while the females' weight is slow and inconsistent" [spelling and punctuation corrected]. Nobody calculated any correlation coefficients, but this may not have been part of their previous statistical experience.

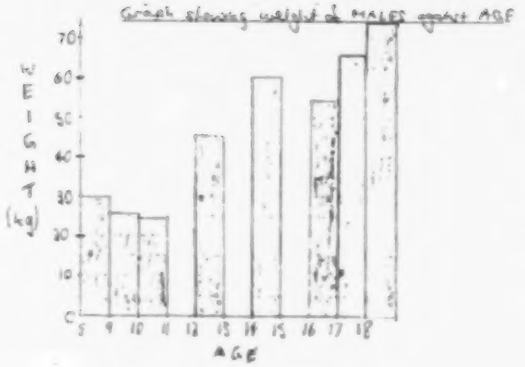

Figure 2. An example of an in appropriate bar graph format.

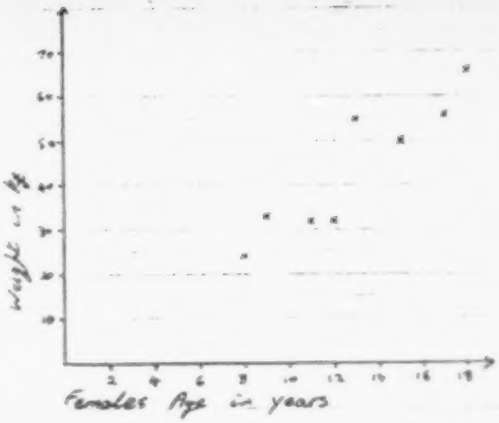

Figure 3. Scatter graph with no trend line.

Three students produced reports with no statistical or graphical information at all. They made claims such as "The females on average eat less fast food than the males" without actually calculating any statistical measures. One referred to individual data to support the points he discussed, as illustrated by: "[four names] are all male gender, have television watching as their favourite activity and consume 10, 7, 8 and 12 fast food meals respectively. They each consume many more fast food meals than any other school children presumably because it is more time consuming to prepare a meal than to purchase one … ." The prevalence of descriptive analyses is further illustrated by the fact that of the 219 claims made, 64 (29%) involved worded descriptions only, with no supporting statistics. A further seven students had no graphs or tables in their reports but at least calculated summary statistics such as proportions or means.

A total of 40 claims (in addition to the 64 worded descriptions) used a single statistical technique; 14 of these involved sums or totals (mostly trivial) and 11 involved means. The majority of the 115 claims involving two or more techniques included worded descriptions. These words did not just recount the values obtained but gave extra clarification and highlighted relationships. The following quote illustrates the use of words to give meaning to the statistical values: "The 'sporty' subjects [=children] consume far less fast food than 'other activities' subjects, 5.2 meals per week compared to 1.3 meals per week." Tables were involved in 41 of the multi-technique claims, and means were used in 46, with 19 claims using both. Students often prepared tables or partial tables and then calculated the means of variables of interest.

Two students dealt with three variables by determining the ratio of weight to age for all the children in the data set and then considered this in relation to a third variable (fast food consumption in one case and favourite activity in another). Both students added the ratios to the table of data, but neither completely capitalised on this quantitative information. One gave a worded description about the claimed relationship and used a statistically appropriate term: "There is for example, a high correlation between a large number of meals and a large age/weight ratio. This would be explained by the fact that fast food is generally unhealthy and fattening." No correlation was actually *demonstrated*, however; it is possible that she thinks it

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is clear from the tabulated data. The other student took the averages of the weight/age values for the 'active' children and the 'inactive' children, and claimed the 'inactive' group's average is higher. Unfortunately, the actual averages were not given.

Conclusion

Before drawing some tentative conclusions it must be reiterated that the open-ended nature of the task and the lack of guidance to students about the associated expectations certainly influenced what students did. Furthermore, the students may not have been particularly interested in the data set, as they had no say in its generation. One student, whose report was a litany of largely unrelated observations supported by worded descriptions, concluded his report by saying "The list of silly aspects goes on and on." This is, perhaps, not an unreasonable response. It would appear, nevertheless, that most students took the task seriously, and that they approached it in the way intended: as a data analysis task in which the object was to seek trends and summary information.

The students sought and found many relationships in the data, demonstrating them using a variety of techniques. They did not, however, always choose the best technique for the job, nor carry it out in the correct way. The students often used their previous experience to explain some of their observations, when, in fact, the data itself could have been used, particularly with connections among age, weight, fast food, favourite activity, and gender. Finally, despite the explicit request to present suitable evidence for claims, there were many unsubstantiated assertions.

The results of this study and the author's earlier work (Chick, 1999) suggest that despite the increased emphasis on chance and data there is still more to be done to ensure that students have the ability to call on appropriate skills and understanding in statistical contexts. Although it is conceivable that the limited use of graphical and statistical information may be a consequence of the small data set (with properties that can certainly be determined to some extent "by inspection"), many students do not seem to appreciate the need for and value of quantitative evidence. It would appear, then, that Curcio's 1987 call needs to be reiterated and emphasised for older students: they need to be provided with the opportunity to collect data and present it in meaningful and statistically valid ways.

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Appendix: Table of Data for the Data Cards Protocol

Table 4
Table of data supplied to students for interpretation

Name	Gender	Age	Favourite Activity	Eye Colour	Weight (kg)	Fast food meals consumed per week
Adam Henderson	M	12	Football	Blue	45	5
Andrew Williams	M	14	TV	Blue	60	10
Anna Smith	F	11	Boardgames	Brown	32	1
Brian Wong	M	9	Football	Green	26	1
David Jones	M	8	TV	Blue	30	7
Dorothy Myers	F	15	Swimming	Blue	50	2
JanelleMacDonald	F	18	Reading	Blue	66	4
Jennifer Rado	F	9	Boardgames	Green	33	4
John Smith	M	10	Football	Green	29	0
Kathy Roberts	F	12	Netball	Brown	32	0
Mary Minski	F	13	Reading	Green	55	3
Peter Cooper	M	16	Boardgames	Green	54	2
Rosemary Black	F	8	Netball	Brown	24	0
Sally Moore	F	17	Reading	Brown	56	1
Scott Williams	M	17	TV	Blue	66	8
Simon Khan	M	18	TV	Brown	74	12

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The Understanding and Use of Trigonometric/Algebraic Knowledge during Problem Solving

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An important feature of students' understanding of algebra and trigonometry is the relations and representations that are constructed between these two areas of school mathematics. In this study we report one students' understanding of the concepts before and after two lessons. The student developed knowledge about the topics that appears to be consistent with what the teacher was aiming at. However, in a problem-solving situation, the student activated knowledge that was different and incorrect. The results provide tentative support for the argument that students sometimes develop 'garbled knowledge'.

Introduction

There is substantial agreement that an important goal of mathematics teaching is to develop students' understanding of concepts and procedures associated with concepts. An important element of such understandings would be the establishment of deep links among concepts, facts and procedures (Hiebert & Carpenter, 1992). The need to examine connections that students construct has also been endorsed by major curriculum reform documents (National Council of Teachers of Mathematics, 1991). A number of studies have explored the type of understandings developed by students, and the quality of teaching provided by teachers. While these studies have generated a useful body of data, in the main, few have examined the student's understanding in relation to what has been presented by the teacher in the classroom and the quality of that understanding as revealed in problem solving. This is a report of a study which was aimed at providing a detailed analysis of one student's understanding of trigonometric knowledge after a series of two lessons.

There are many things to examine in a student's understanding of mathematics. Carpenter and Lehrer (1999) argue that different types of mental activity contribute to the growth in understanding: the relationships or connections made during knowledge construction, the application of knowledge, reflection about this knowledge use, and articulation of what is known. In this report we set out to seek evidence of each of these types of activity in a student whose lessons on gradient we had observed and analysed. Our involvement in this student's class gave us the opportunity to observe what Sierpinska (1994) note to be three key aspects to mathematics understanding: what is being understood, the context in which it is understood and the operations activated during the process of understanding. The 'what is being understood' could include concepts, facts, principles and procedures. An equally important component of this understanding is the relationships that students construct among the different components of their knowledge in the context of the classroom activities designed by the teacher. As the students' attain higher and more sophisticated levels of understanding, one could expect students to show evidence of more varied and extended schema of relationships that could be used to solve problems of increasing complexity. It is in the course of this problem solving that the quality of the knowledge schema is tested.

Teaching and the Construction of Connections

The above framework suggests that in order to improve mathematical understanding, classroom teaching needs to focus on helping students construct schemas that are rich in content and relations. A schema-based analysis of knowledge constructed by students implies that the quality of explanations employed by the teacher and the range of examples and metaphors used by teacher could be expected to have a major influence on the nature of schemas developed by students. This line of reasoning suggests that we could generate important data about students' understanding by examining the relationship between schemas that teachers activate during their teaching and those constructed by the students. In this project we examined this relationship by documenting in detail the knowledge schemas activated by a teacher during teaching about gradient and comparing one student's schema before and after the instruction and the use of that schema in a problem solving activity.

Method

Participants

Participants in this study were part of a larger study in which a group of experienced teachers who were required to plan and teach a series of two lessons to their Year 10 classes. The teachers came from a pool of participants within the states of Queensland and Western Australia. They were assessed to be exemplary by the Education Departments and Mathematical Associations. In this report we consider one teacher and a student from his class. The male high school teacher had twenty years experience in teaching of mathematics at a senior secondary level. He was also the head of the mathematics department of his school.

The student was considered to be an average student on the basis of his performance in a series of four tests and comments from the teacher. On the basis of classroom tests the teacher rated him as an average student in a class that was preparing students to take mathematics as a major subject in the final two years of schooling. The teacher and student will be referred to as Gary and Tom respectively in this paper.

Tasks and Procedure

The purpose of the study required that we assess the knowledge of the student before and after instruction by the teacher. Three tasks were developed for the purposes of assessing students' knowledge about geometry. The first task, Free Recall (FRT), required students to talk aloud about any idea that they could associate with the topic of plane geometry and about shapes and their properties. The second task, the Problem Solving task (PST), consisted of four problems. All the four problems were related either to plane and/or coordinate geometry. The first problem involved the use of Pythagoras's theorem in a rectangular coordinate system. Solution of the second and third problems required an understanding of the relationship between properties of squares and rectangles, segmentation of their areas, and knowledge of the gradient of straight lines in a rectangular coordinate system. The fourth problem, again necessitated an integration of knowledge of properties of square and right-angled triangle.

The final task, Geometry Probing (GPT), was designed to obtain further information about student's talked about during the Free Recall task. In the GPT the student was required to respond to a number of questions each of which aimed at probing what was said during the first session. The probes were designed to provide the student with opportunities to display his available knowledge of features of concepts relevant to this area of mathematics. The use of the above techniques has been argued to provide rich data about interconnections that exists between knowledge units, and their structure (Royer, Cisero, & Carlo, 1993).

The student met with us on three occasions and undertook one of the tasks in each session, in the order indicated above. In the first session he was invited show his knowledge of geometry by talking, drawing and writing in response to the FRT. During the second session he was asked to complete the PST which involved generating solutions to four problems. He was encouraged to talk aloud during the solution attempts. In the third session we sought the student's responses to the probe questions. All three sessions were video-taped and transcribed for subsequent analysis. Data from the above three tasks were used in the development of pre-teaching concept map for the student.

Gary made a decision to spread his introductory lessons on linear equations across two lessons. Both his lessons were video-taped and later analysed. Part of the analysis involved identification of the concepts, principles and procedures taught during the lessons and it is this that we will report on here. The components of the lesson that addressed our focus concept gradient and associated concepts are shown in the teacher's concept map (Figure 1). In other parts of the analysis we rated the lesson in terms of moves made by the teacher to establish and strengthen connections in the knowledge schema that were the focus of the lessons (Lee, 1998). For the purposes of the present study we focussed on Gary's discussion of the concept of 'gradient'. In this regard Gary's lessons allocated substantial amounts of time to both schema establishment and schema strengthening.

Following the lessons, the student was asked to participate in two further interviews. During the first of these post-teaching interviews, the student attempted to solve two problems. We asked him to 'think aloud' as he worked on these problems. The problems were based on the content discussed during the class lessons and were designed to elicit responses from the student about his understanding of gradient. During the second interview the student

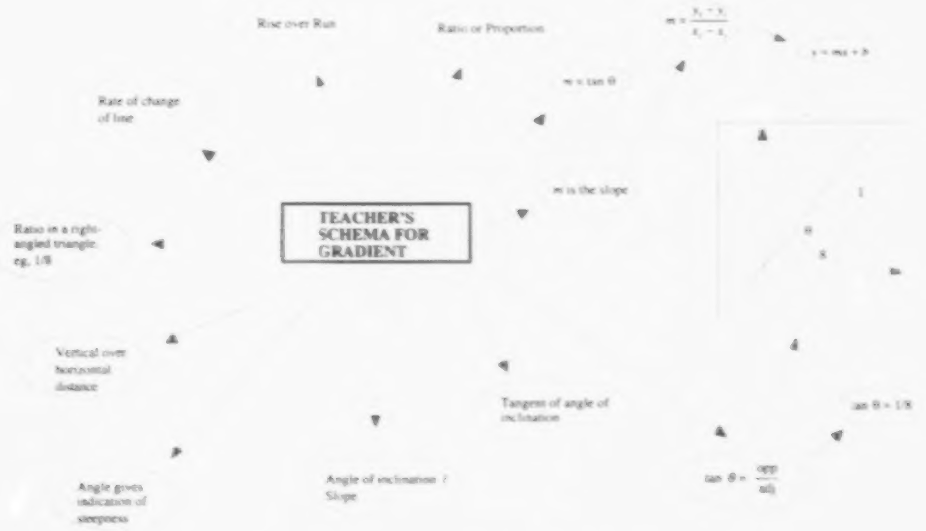

Rise over Run
Ratio or Proportion
$m = \frac{y_2 - y_1}{x_2 - x_1}$
$y = mx + b$
$m = \tan\theta$
Rate of change of line
m is the slope
TEACHER'S SCHEMA FOR GRADIENT
Ratio in a right-angled triangle eg. 1/8
1
θ
8
m
Vertical over horizontal distance
Tangent of angle of inclination
$\tan\theta = 1/8$
Angle gives indication of steepness
Angle of inclination / Slope
$\tan\theta = \frac{opp}{adj}$

Figure 1: Components of teacher's lesson.

responded to a series of focus questions. These focus questions were used to gather information about student's perceptions about what the teacher was aiming at and developments in his understanding of concept of gradient. In these interviews we gave Tom the opportunity to articulate and reflect on his understanding of the focus concepts.

Data from the pre- and post-teaching interviews formed the basis for the development of concept maps for the student. These concept maps were anchored by a number of subsidiary concepts for gradient. The subsidiary concepts included students' knowledge about right-angled triangles, the coordinate system and properties of lines and parallel lines. For the purpose of this paper we have included concepts maps for right-angled triangles (Figure 2) and the coordinate system (Figure 3). In both these figures, concepts that appear in oval-shaped (not shaded) figures were activated prior to and after teaching. Concepts that appear in rectangle-shaped figures (not shaded) were activated prior to teaching only. Concepts that appear in rectangle-shaped figures and shaded were only activated after teaching.

Results

Figure 1 shows that teacher has activated a range of ideas during his lessons. Analysis of the sequence of his presentation indicated that Gary spent considerable time discussing the idea of [illegible]ient informally before formalising it in terms of proportion/ratio, tangent of an angle a[illegible] coordinate geometry. This informal discussion drew explicit links between gradient and geographical features near the school. Gary's representation of gradient addressed included the fundamental notion of slope and its relation to the concept of function and it is clear in Figure 1 that there was an amount of redundancy associated with representation of the concept of gradient. The state of his knowledge network is rich and extensive which Stump (1999) argued to be an important element of a better of understanding.

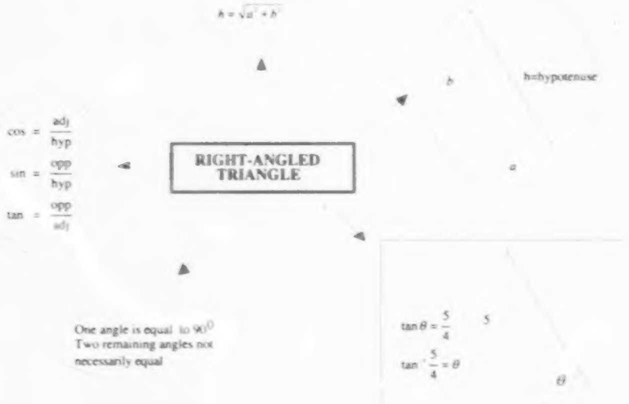


Figure 2. Concept map for right-angled triangles.

Figure 2 shows that prior to the lessons Tom had already built up a number of concepts about the properties of right-angled triangles including those related to gradient. This is indicated by concepts inside the unshaded oval shapes. The bottom right-hand corner shows that Tom appear to have acquired something new from the lessons about the relationship between gradient and tangent of angle. From Figure 3 we see that Tom's understanding of coordinates were reasonably complete even before the lesson. Taken together, Tom appears to have developed a level of understanding of gradient that might be expected to be useful in the problem solving that would follow. For example, on the basis of this understanding we would expect him to be able to calculate the gradient and equation of a line joining two points in a Cartesian system. We tested our expectations about his understanding in our post-teaching interviews.

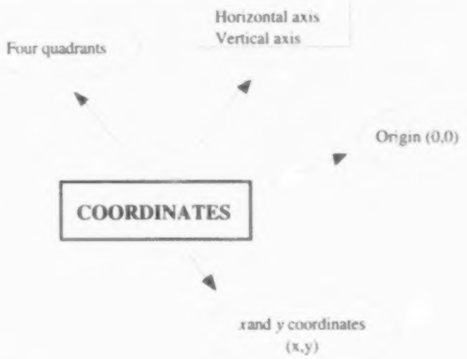
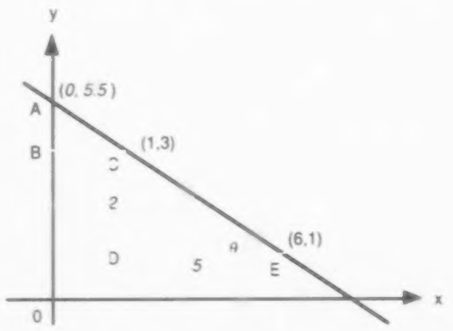


Figure 3. Concept map for coordinates.

Figure 4 shows Tom's attempt at completing the problem which required him to find the y-intercept and equation of the line ACE. Analysis of Tom's think aloud activity and transcripts showed that he followed the seven major steps set out below Figure 4. In Step 1, Tom found the slope by using the notion of vertical distance over the horizontal distance within the right-angled triangle CDE. The distances were found by looking for differences in the y and x-coordinates of points C and E. He repeated this in Step 2 but reversed the order of the coordinates. Both procedures yielded the same result, though Tom did not note this. He then found the tangent of the angle θ in Step 3. In Steps 4 and 5 he incorrectly



1. $\frac{3-1}{1-6} = \frac{2}{-5} = -0.4$
2. $\frac{1-3}{6-1} = \frac{-2}{5}$
3. $\tan\theta = \frac{2}{5} = 0.4$
4. Inv tan .4= slope
5. $\tan^{-1} .4 = m$
6. $y = mx + b$
7. $y = .4x + 5.5$

Figure 4. Problem and Tom's solution attempt.

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concluded that 'inv tan 0.4' is equal to the slope of the line. This is a new element in his currently activated schema. It was not mentioned in the pre-teaching interview and was not discussed in the lessons. We might reasonably expect it to be connection that he had madepreviously in learning about inverses of trigonometric ratios. In Step 6, Tom activated the general form of the equation for a straight line. His actions in Step 7 show that he understood the various components of the general equation and replaced the values for gradient (*m*) and y-intercept. The value of b is generated via his somewhat garbled reasoning that 'for every one across you go 2.5 up; so AO (the y-intercept) equals 3+2.5'.

Tom could identify the appropriate coordinates (steps 1 and 2), could establish the tangent of the angle (step 3), and could recall the correct symbolic representation of the linear equation (step 6). However, his problem-solving moves show that he was unable to set up an appropriate set of links between gradient, the coordinate system, tangent of an angle and the representations of the concept of gradient presented in the lessons. His schema for ratio/proportion used to calculate gradient was different from what the teacher attempted to establish as 'rise over run' in the lessons (Step 7). In this move he showed that he had difficulty interpreting the relationship captured by the expression $m = \frac{y_2 - y_1}{x_2 - x_1}$, despite the fact that he had earlier used part of this knowledge in steps 1 and 2. Thus at the end of this problem-solving attempt Tom shows clear evidence of confusion about the relationship between *m*, tan θ, and slope of the line.

Discussion

The expectations of problem-solving performance we had built up on the basis of our analysis of the student's pre-teaching interview and of our analysis of the lessons were not confirmed. Instead we were left with strong evidence that a garbled knowledge schema had been established by this student after two class lessons. What do we want to make of this?

First we do not want to make a final judgement about the quality of the teaching, which we earlier rated as good, or a final judgement about the student's knowledge state. We have a very restricted sample of both this teacher's teaching and this student's knowledge, a sample that would not support such judgements.

Yet consider what information this episode does provide. Here we attempted to briefly open a window on this student's schema knowledge. Prior to the teaching it appeared to be reasonably robust when we used one form of assessment. But it failed in the test of application. Some of the required links between knowledge components appeared to be incomplete or inappropriate. What would be expected (see Moscovitch, 1996) to be useful context provided by the teacher in situating the concept of gradient in the local geographical context was not reflected in the understanding of this student.

There is much in this student's schema to be sorted out by the teacher in future lessons. However, as teachers we would probably expect the 'average student in a top class' to have done much of this sorting out for himself as the lesson unfolded. A revisiting of the lessons did show that there was no provision in the lesson for the teacher to inspect the student's unguided understanding of what had been shown on the blackboard and in the computer simulations. The examples in the lesson had also included positive slopes.

It may have been that the student's analysis of lesson material was too superficial and that he was 'captured' by the process of calculation (Verschaffel & DeCorte (1997). Indeed there is no evidence in his post-lesson articulation of his understanding of use of synonyms such as 'steepness' for gradient, nor is there evidence of explicit reflection on the state of his understanding. Perhaps the student needs to use these types of activity more actively. At the end of this episode it is clear that the student needed to retreat rather than advance in his construction of knowledge about gradient. If he did not it is unlikely that he would construct a suitably powerful schema. If Tom is like many 'average' students we must consider whether we can allocate them enough attention and time to build such structures, perhaps they need to spend more time covering less mathematics.

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Function Representations and Technology-Enhanced Teaching

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In recent years calculators and computers have been used more often in mathematics teaching. While their potential advantages in terms of dynamic multiple representations has been recognised for some time, it has not always been clear how to integrate the technology into conceptually based teaching. While it is generally agreed that technology could play an influential role in enhancing mathematical understanding, there is little data about how students' cognition and knowledge building are affected by the instructional use of such tools. In particular, a theoretical perspective on how to teach in such a way as to encourage students to build inter-representational conceptual structures has been lacking. In this study we take up this issue by examining how two experienced teachers plan and incorporate calculator and computers in their teaching. The results suggest that these teachers aim to build knowledge schemas that are rich in connections and representations.

Background

It seems inevitable that not only the nature of the processes, objects and concepts of mathematics, but also the way teachers and students perceive them, will affect the way in which mathematical ideas are taught. These differences often may not make much impact but may be more important when the concept is one which is fundamental to mathematics, such as that of function. Vollrath (1994, p. 70) describes how "*global strategies* are needed for *leading concepts* that permeate the whole *curriculum*, for example the concept of function is a candidate for such a leading concept." What are some of the variables which influence the learning of this leading concept, and how can technology assist teachers in helping students to develop an understanding of it?

Functions can be seen in a number of different ways by students and teachers, and these have been described in terms of different theoretical perspectives. For example, they can be seen purely as an input–output process, or as an encapsulated object (Tall & Thomas, 1991; Cottrill *et al.*, 1996), or some stage in between. In contrast, Hitt (1998, p.125) describes five levels of understanding of the concept of function, each of which is related to the representation of a function, with the highest level being "Coherent articulation of different systems of representation in the solution of a problem." There are a number of different uses of the word representation in the literature (von Glasersfeld, 1987) and we shall follow the definition of Kaput (1987, p. 23) who proposed, following Palmer (1977), that "any concept of representation must involve two related but functionally separate entities. We call one entity the representing world and the other the represented world." This idea applied to the representation of function is particularly important, since functions can be represented in a wide variety of different ways, such as in algebraic symbol form, set diagrams, ordered pairs, graphs and tables of values, and so on.

Translating between these various forms is something which an experienced teacher might take for granted, but to do so one needs to have an overview of the way the definition of function relates to each representation, and how sub-concepts, such as independent and dependent variables, one-to-one, roots, discrete, continuous, etc. are manifest in each representation (Chinnappan & Thomas, 1999; Hong, Thomas, & Kwon, in press). A precise view of these, and the processes which apply to each representation can help one make judgements about processes which, unlike concepts, may not travel well between representations. For example, the process in the graphical domain corresponding to that of solving linear equations by adding multiples of the variables to both sides takes on a completely different form, involving rotations of straight lines.

In our view calculators and computers can assist in the building of links between the sub-concepts in each representation. This is not only because they intrinsically employ multiple representations (Kaput, 1992) or that they can be "dynamic interactive and recording representational media" (Goldin & Kaput, 1996, p. 411). The key property they possess is the fact that the representations can be dynamically linked, with the technology providing fast feedback on student input (Kaput, 1992), so that a change in one representation, such as a table, is immediately mirrored by a corresponding change in the graphical representation. It is true that computers and calculators may appear to present some cognitive obstacles to an understanding of function, such as the limitation of having only discrete variables in a table of values. However, in practice the zoom feature of most graphic calculators admits a variable table range which makes the function domain *potentially* infinite. Figure 1 shows the potential of technology to provide a central role in the linking of four key representations of function.

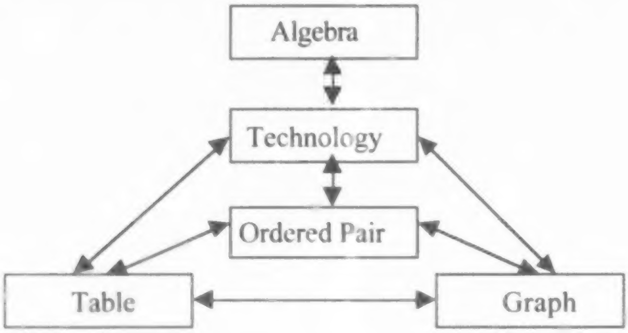


Figure 1. Technology as a catalyst in the linking of function representations

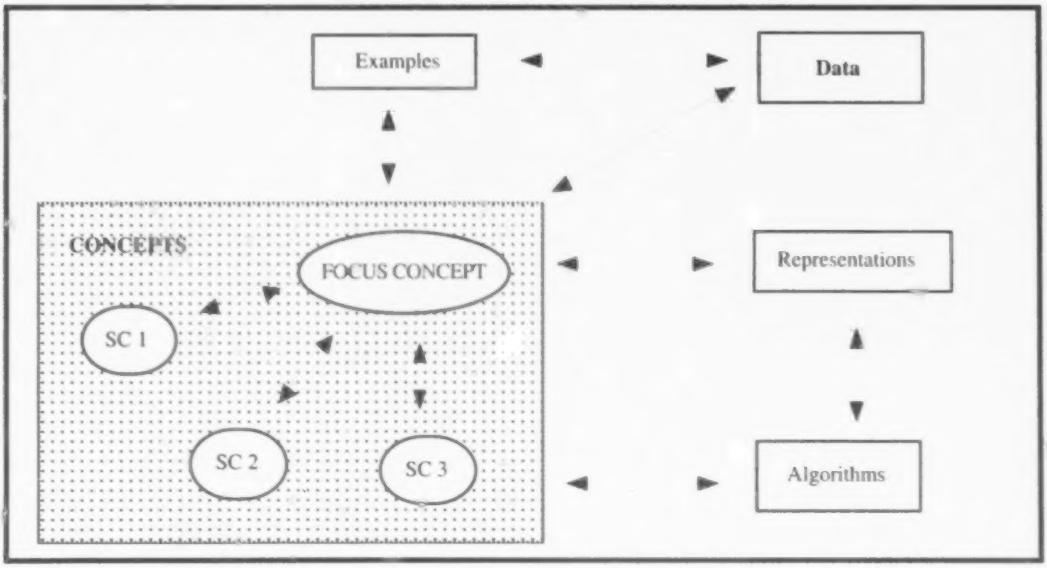
Previous research (Chinnappan & Thomas, 1999) has investigated the pedagogical value of teachers' rich conceptual function schemas, and Vollrath (1994) has suggested that mathematics teachers need to have these if they are to be able to teach function on the basis of conceptual relationships. Research by Williams (1998) shows that it is not always the case that students have such well developed conceptual schemas for function, and Even (1998, p. 119) concluded that the prospective secondary teachers in her study "had difficulties when they needed to flexibly link different representations of functions...". In addition, teachers' schemas have to include an understanding of the role of technology in facilitating inter-representational links in order to be able to use it effectively. This paper describes the kind of pedagogical schemas (Shulman, 1986) which two teachers have developed, focussing on the way they help them think across the representations, and how this has proven beneficial in their teaching.

Method

This research employed a case study methodology, considering the function schemas of three experienced teachers, A, B and C, with 17, 16 and 31 years teaching respectively, and each of whom had also made extensive use of calculators and computers in their teaching. The teachers were given a free response interview where they were encouraged to talk about 'functions' and how they would teach them. The interviews were recorded on audiotape and later transcribed for analysis. In addition, four lessons of teachers A and C were observed and recorded on video. The interview and lessons of teacher C, Margot, have been described elsewhere (Chinnappan & Thomas, 1999), and this paper discusses some of the data from teachers A and B.

Results

In our analysis of the data we were interested in identifying teachers' knowledge about functions and the use of technology in fostering such knowledge. Figure 2 shows a framework for the examination of conceptual understanding developed by teachers.



Note. SC 1 stands for subsidiary concept one, etc.

Figure 2. A macro model of the structure of teachers' mathematical knowledge and problem modelling.

Our model suggests that a conceptually-driven teacher or teaching will place emphasis on developing the knowledge components identified in Figure 2 and the constructions of links among them. For example, in teaching about probability, the teacher will consider not only subsidiary concepts such as fractions but also the many ways to represent the concept of probability. The question is: What is the role of technology in teaching the focus concept, and how could it be effectively used as an instructional tool?

We return to our model in seeking tentative answers to these questions. The effective utilisation of technology as an instructional tool by the teacher could be examined by the actions of the teacher in his/her attempts to foster the construction of understandings that are based on components in Figure 2. A feature of such teaching could be the sequencing of lessons and learning experiences such that students are provided with examples and various images or representations of the focus concept. Additionally, teachers also need to draw out the conceptual building blocks vis-à-vis the subsidiary concepts. Such an approach would enable students to locate the focus concept within their prior knowledge, and develop a better understanding of the focus concept and its relations to other related concepts. The success of the lessons would also be the influenced by the knowledge the teacher has about the students.

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The Interviews

In their interviews the teachers certainly did emphasise the links between different representations of function and the role of technology in moving between them. For example the 'obvious' link between the algebra and graphs was mentioned, but their comments on the linking also emphasised the secondary concepts.

> A: ...the roots of the equation how they can be found graphically and algebraically...so that we can move onto parabolas and I will get them to virtually plot the points using a calculator...they won't relate that to any roots of a parabola, that they will think that the graph is a completely separate operation and it's only after they've done it a great number of times that they will see a sort of relationship.

> B: I guess the things that interest me are where they cut the *x* and *y* axis. Obviously graphically it is very easy to tell where they are. Besides using a graphics calculator or even using Excel or WL plot, or something like that I can easily check where they cut, so if I was doing this with a class, I think that I would begin without looking at trying to factorise a polynomial and trying to show that where the factors equal 0, there are the roots. I would get the kids to explore where it cuts the $y=0$ line by getting closer and closer and magnifying, blowing up...I think are a lot clearer to students through using the calculator, graphics calculator or through using a computer program.

Here the teachers both highlight the need to relate algebraic aspects of function, including subsidiary concepts of factor and root, to graphs drawn using a computer or calculator. Other subsidiary concepts of function were similarly prominent in their comments, such as dependent and independent variables, and coefficients, relating the algebra to a graph.

> A: Yes, the idea of bringing in coefficients, the initial worksheet works with the coefficients of x^2 or even earlier back to the straight lines. It's really the idea of the coefficients and the line being to the coefficient of *x* being the gradient slope...I really think that it is an important thing to get across the formula for the *x* and *y* axis, and that actually comes out really nicely on the computer, when you've gone through all of the gradients and they can see the gradients getting closer and closer to 0...I think that's the first time that it becomes clear that as you vary this value, you get a corresponding change in the second variable and the relationship between the two.

> B: I stress the idea of an independent variable and a dependent variable and again I think that something like Excel makes that patently obvious. You know, its sort of reality that here we are choosing particular values and here is what comes out of that choice, so from that point of view I think that's really magic.

The algebraic and graphical representations were not the only ones they referred to, of course. Tables and ordered pairs (or co-ordinates) also featured strongly, with connections made to graphs, etc. Seeing ordered pairs as a separate representation requiring an explicit link to others may be overlooked some times. Comments relating to these included:

> A: ...showing the relationship in ordered pairs and then you can show these and how they relate on a graph...that actually has a formula or a relationship and then we can move on from there to the actual equation

> A: I introduce the difference between functions and relations using the mapping arrow diagrams showing how the value of the range is repeated which means that it's not a function. Similarly the ordered pairs, the member of the range affected and you can show on the graph that when the name's repeated you get points one above the other, and therefore a vertical line test will show that.

> B: I'm impressed by the fact that you can use, develop, a table on a graphics calculator or develop a table on Excel and then just plot those values on a graph. I think it's a very rich step and a step that is lost in using a graphic program.

> B: The other beautiful thing about something like Excel is that you can have, and a graphics calculator, is that you can have a function or a polynomial in different forms such as $x^3 + 6x^2$ + something, and then in a factored form, and by drawing a table of values you can see that they are in fact identical and you can in fact split the polynomial up into sections x+5, x+6 and multiply each of those various columns together so you can actually see the impact of those factors on the overall polynomial as well.

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Teacher A's second comment above actually takes a definition of a function and links this focus concept through three representations, set diagram, ordered pairs and graphs. Similarly, Teacher B's second comment shows that when he teaches using a spreadsheet he explicitly links algebraic factors of a polynomial function with a table of values representation. Comments like these demonstrate the flexibility and conceptual nature of the function schemas of these teachers, and the central role of the technology in passing from one representation to another.

It is also worth noting that both of the teachers, A and B, like the teacher Margot that we have previously described, have modelling of realistic situations as a strong focus of their teaching of function.

> A: ...say they had three pieces of wood, so they'll have three x's, two x^2 and an x^3 ...The most useful applications I've seen as far as graphs go...are things like the real life situations of graphs, of comparisons of graphs, like putting things, two axes tall or height...like, as an introduction to rates of change...different shaped vessels, how their rate varies if you continually put in a second amount that is the exact same height, varies to the volume, I know those sorts of graphs are good practical examples, it gives some realistic view of these.
>
> B: The whole curriculum is about going towards maths in the real world and within meaningful context, and I think the use of spreadsheet for example provides opportunities for students to explore real relationships between variables. Things like π and circumference and speed, distances, stopping distances, you know, I mean there's heaps of different relationships that can be looked at through a scatter graph for example, right, through, drawing a line through things, through modelling particuiar relationships... we are often hung-up on giving them a particular equation and asking students to plot points on it, rather than starting the other way and getting students to have points and then trying to find the equation through that.

Teacher A's Lessons

In analysing the actions of Teacher A, who used Excel to teach functions, we were interested to identify concepts and relations that he helped students experiment with prior to and during the lesson. During the first lesson, Teacher A got students to talk about some of the subsidiary concepts before introducing new ones. Table 1 shows the list of concepts that were discussed in the first lesson. All the concepts are clearly necessary and directly related to understandings about functions in general, and linear functions in particular.

In one of the activities, students were given a table with two columns for the x- and y-values. They were then required to look for a *pattern* that showed the relationship between x and y values. Such a relationship was labelled as *relation*, *equation* and *formula*. The teacher explained the how the terms were connected before introducing the notion of functions. The discussion also helped students use two methods to test for functions.

Table 1

Concepts Considered in the Lesson Prior to Using Excel

Focus Concept: Functions

Subsidiary Concepts	Comments
x- and y- coordinates	Symbols for showing a point within the Cartesian system (x,y).
Domain/Range	How does elements in domain affect values in the Range?, Restricted domain
Ordered pairs	Geometric meaning of the numbers that appear in as an ordered pair
Rule/Relation/Formula	Relations among these
Functions vs relations	Methods of deciding if a relation is a function

The activities of the teacher in lesson 2 were directed at a) reinforcing concepts from lesson 1, b) developing the concepts from lesson 1 further and c) generating alternative representations and procedures related to the focus concept by drawing on the various menus of Excel. In this lesson students were also encouraged to explore the linkages among the focus and subsidiary concepts more dynamically. They were given a prepared Excel sheet, showing three representations of a linear function: algebraic; tabular; and graphical.

Table 2

Concepts Considered in the Lesson Using Excel

Focus Concept: Functions	
Subsidiary Concepts	Comments
General form of a linear function	$y=mx+c$
Coefficient	The meaning of m , gradient
Graphs	Curves and lines
Meanings of letters in $y=mx+c$	Effect of replacing the letters with other letters
Family of functions	Square functions, square root functions
Rule/Relation/Formula/equations	Relations among these
Functions vs relations	Vertical line test; use of zoom menu to locate points of intersection

The reinforcing of ideas from lesson 1 involved attention to symbols and conventions that are used in writing equations for linear functions. This was achieved by asking students to type in the general equation for linear functions in the Excel worksheets ($y = mx + c$).

Teacher A then directed students to replace the available space on the sheet where the letter in the equation was with numbers and observe the effect of their actions on the table and graph that was generated by Excel. During this phase of the teaching, he alerted students to terms such as coefficient, gradient and constant. The focus of his explanation was on the relationship between the letters x and y, ordered pairs and functions. The discussion at this stage also centred on how to use the number in the ordered pairs as a way to test to see if a relation is a function. The above sequence of activities indicate that our teacher was helping the students construct symbolic, tabular and graphical representations of functions.

While the paper and pencil activities in lesson 1 provided an important starting point for students to explore some of the subsidiary concepts, the activities did not allow student visualise the consequences of their actions immediately nor let them test some of their conjectures. Excel permitted the instantaneous observation of such actions. In addition, Teacher A also used Excel to stimulate constructive dialogue among his students. Within the Excel environment mistakes were easily observed by students and corrective action were taken by students independent of the teacher.

Discussion

In this study we have analysed the knowledge base and skills of two experienced teachers in their use of technology. Data showed that the effective utilisation of technology by these teachers is buttressed by a well developed knowledge schema about the core concept and its relations to other concepts. An important character of this schema is how they were able to exploit the facilities offered by technology in promoting conceptual understanding among the students. This fostering of conceptual development and associated procedures about functions was evident in the way the teachers used computers and graphic calculators to construct alternative representation of the focus concept.

Teachers' use of analytic and graphical representations within a spreadsheet such as Excel can help students locate the secondary concepts and establish links to the focus concept. Teacher A, for example, used Excel to show the geometric effect on the line by altering m and c as variables in the linear equation. While this brought about spectacular changes to what students could see on the computer screen, his lessons did not deviate from highlighting the subconcepts that underpinned his actions, such as variables, ordered pairs and x and y coordinates. The flexibility with which the experienced teacher is able to introduce technology without losing sight of the associated concepts is a key to a successful lesson.

Underlying the effective use of technology by our teachers is the high level of knowledge they have developed about the technology they chose to use. Our results indicate that teachers need to build sound knowledge about the menus and procedures that are incorporated into the software or the calculator.

How does technology act as a catalyst for inter-representational thinking? One reason may be that technology facilitated the accessing of prior knowledge that may otherwise remain dormant. Lawson and Chinnappan (1994) showed that some students do not access and use knowledge that they have learnt before. This failure to access available knowledge, it was shown, impeded progress students made in solving problems and acquiring new knowledge. Lawson and Chinnappan (1994) suggested that there was a need to develop strategies that would help students search their memory and activate what has been learnt before. We argue that technology may act as a prompt to access what is stored in the long-term memory.

It is also plausible that the use of technology in learning may cause knowledge to be organised around the technology giving qualitatively better schemas and hence foster access to previously-acquired knowledge. This may be because links are better established and reinforced in an environment that is dynamic and allows students to visualise concepts in different ways, across a number of representations.

The results of this study provide tentative support to the argument that experienced teachers use technology with a view to building conceptual understanding among their students. In doing so, a teacher's focus is on disentangling the various components of the concept(s) that are in question. We suggest that future research need to provide more fine-grained analyses of this link between technology use and the construction of links among the concepts, and across the representations.

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The Early Numeracy Research Project[1]: Developing a Framework for Describing Early Numeracy Learning

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This paper describes the development of a framework of key aspects of early numeracy learning. The framework was developed as part of a project seeking to identify processes for supporting and enhancing numeracy learning in the early years of school. The framework built on work of others, through considering key research in numeracy learning and others' attempts in developing such frameworks. Data collected from interviews with over 5000 children led to modifications of the framework.

In the past ten years, there has been considerable community and political attention to measuring the outcomes of students' learning (Australian Education Council, 1994; Board of Studies, 2000). Increasingly, state and territory departments of education indicated that the early years of schooling were crucial in providing the kind of positive start to students' literacy and numeracy learning that was needed to develop confident and capable lifelong learners.

In Victoria, the Early Literacy Research Project (Hill & Crevola, 1998) worked with 27 disadvantaged primary schools to bring about substantial improvements in early literacy outcomes. Part of this research involved the development of models and guidelines for teaching, assessment and additional support for young children learning to read. As a result of the research, Hill and Crevola offered a "general design for improving learning outcomes" (p. 122), which they believed had application in literacy, numeracy, and other curriculum areas. The design was greatly influenced by research literature on educational effectiveness, including a finding that the impact of classroom effects on student learning greatly exceeds that of school effects (Creemers & Reezigt, 1996; Hill & Rowe, 1996). The nine elements of the design (Hill & Crevola, 1998, p. 123) included leadership and coordination; standards and targets, monitoring and assessment, classroom teaching programs; professional learning teams; school and class organisation; intervention and special assistance; home, school and community partnerships; and beliefs and understandings.

The Early Numeracy Research Project (ENRP) was established in 1999 by the (then) Victorian Department of Education, with similar aims to those of the Early Literacy Research Project, but with a numeracy focus. The ENRP is now a collaborative venture between Australian Catholic University, Monash University, the Victorian Department of Employment, Education and Training, the Catholic Education Office (Melbourne), and the Association of Independent Schools Victoria. The project is funded to early 2002 in 35 project ("trial") schools and 35 control ("reference") schools (for details see Clarke, 1999). The ENRP has a major professional development component, with teachers meeting with project staff for statewide, regional cluster, and local inservice programs.

Important differences from the literacy project included the need for development of a comprehensive and appropriate learning and assessment framework for early numeracy (such frameworks were well established for reading), and the need to address the personal confidence with and understanding of mathematics of many primary teachers. This paper

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[1] The Early Numeracy Research Project is supported by grants from the Victorian Department of Employment, Education and Training, the Catholic Education Office (Melbourne), and the Association of Independent Schools Victoria.

explains the need for the development of the learning and assessment framework, outlines the process for the development of the framework, and presents some results to illustrate the profiles of the students at these levels.

Measuring Numeracy Learning

The impetus for the project was a desire to improve numeracy learning and so it was necessary to quantify such improvement. It would not have been adequate to describe, for example, the effectiveness of the professional development in terms of teachers' professional growth, or the children's engagement, or even to produce some success stories. It was decided to create a framework of key "growth points" in numeracy learning. Students' movement through these growth points in trial schools could then be compared to that of students in the reference schools.

The project team studied available research on key "stages" or "levels" in young children's numeracy learning (e.g., Boulton-Lewis, 1996; Fuson, 1992, McIntosh, Bana, & Farrell, 1995; Mulligan & Mitchelmore, 1995, 1996; Pearn & Merrifield, 1992; Thomas, 1996), as well as some frameworks developed by other authors and groups to describe learning.

A major influence on the project design was the New South Wales Department of Education initiative *Count Me In Too* (Bobis & Gould, 1999; NSW Department of Education and Training, 1998) that developed a learning framework in number (Wright, 1998) that seemed to incorporate most of the desired elements in describing students' learning. It was soundly based on prior research and, in particular, on the Steffe counting stages (see, e.g., Steffe, Cobb, & von Glaserfeld, 1988; Steffe, von Glaserfeld, Richards, & Cobb, 1983), and it formed the basis of an individual interview designed to measure children's learning against the framework.

Discussions with Trish O'Toole and Greg Parker (personal communications, January 10-12, 1999) from the Catholic Education Office (South Australia) were also helpful in considering aspects of the measurement parts of the framework, as was the term "growth point" which they were using in their work.

The draft version of the Victorian *Curriculum and Standards Framework II* (Board of Studies, 1998) was examined but seemed on one hand too specific (in its outcome statements), and on the other hand too general (in its curriculum focus statements). The Numeracy Benchmarks were clear and comprehensive but were limited in scope and described only minimum achievement levels without which a student is unable to progress at school (Curriculum Corporation, 1997).

While the framework documents considered had much to offer, none served the needs of this project without substantial adaption and extension. It was decided to create a framework specifically for this project.

In developing the framework it was intended that the framework would

- reflect the findings of relevant research in mathematics education from Australia and overseas;
- emphasise the "big ideas" of early numeracy in a form and language readily understood and, in time, retained by teachers;
- reflect, where possible, the structure of mathematics;
- allow the description of the mathematical knowledge and understanding of individuals and groups;
- form the basis of planning and teaching;
- provide a basis for task construction for interviews, and the recording and coding process that would follow;
- allow the identification and description of improvement where it exists;

- enable a consideration of those at risk students who may benefit from additional assistance;
- have sufficient "ceiling" to describe the knowledge and understanding of all children in the first three years of school; and
- build on the work of other successful, similar projects such as *Count Me in Too*.

The principles informed the process of developing and refining the framework as is outlined in the next section.

The Development of the Framework

Having assembled relevant research and frameworks for Number and Measurement, the research team conducted a number of "think tank" sessions during which the initial framework was developed. During this time, colleagues with expertise in specific areas were consulted, and their advice incorporated into the development process. For 1999, the decision was taken to focus upon the strands of Number (incorporating the domains of Counting, Place Value, Addition and Subtraction Strategies, and Multiplication and Division Strategies) and Measurement (incorporating the domains of Length, Mass and Time). In 2000, the strand of Space has been added to the framework, but this will not be discussed here.

Within each mathematical domain, growth points were stated with brief descriptors in each case. There were typically five or six growth points in each domain. To illustrate the notion of a growth point, consider the child who is asked to find the total of two collections of objects (say nine objects and another four objects). Many young children will "count all" to find the total ("1, 2, 3, . . . , 11, 12, 13"), even once they are aware that there are nine objects in one set and four in the other. Other children will realise that by starting at 9 and counting on ("10, 11, 12, 13"), they can solve the problem in an easier way. *Counting All* and *Counting On* are therefore two important growth points in children's developing understanding of Addition.

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1. Rote counting
   Rote counts the number sequence to at least 20, but is unable to reliably count a collection of that size.
2. Counting collections
   Confidently counts a collection of around 20 objects.
3. Counting by 1s (forward/backward, including variable starting points; before/after)
   Counts forwards and backwards from various starting points between 1 and 100; knows numbers before and after a given number.
4. Counting from 0 by 2s, 5s, and 10s
   Can count from 0 by 2s, 5s, and 10s to a given target.
5. Counting from x ($x > 0$) by 2s, 5s, and 10s
   Given a non-zero starting point, can count by 2s, 5s, and 10s to a given target.
6. Extending and applying counting skills
   Can count from a non-zero starting point by any single digit number, and can apply counting skills in practical tasks.

Figure 1. ENRP growth points for counting.

For clarity this paper presents only some results from the *Counting* domain of the framework. The six growth points for counting are shown in Figure 1. Note that growth point 6 was not added until this year, and will therefore not appear in the discussions of the *Counting* data. These growth points informed the creation of assessment items, and the recording, scoring and subsequent analysis, as is discussed below.

Growth Points, Levels and Stages

The growth points are clearly a key element of this framework. In discussions with teachers, we have come to describe them as key "stepping stones" along paths to mathematical understanding. However, we do not claim that all growth points are passed by every student along the way. For example, one of our growth points in Addition and Subtraction involves "counting back", "counting down to" and "counting up from" in subtraction situations, as appropriate. But there appears to be a number of children who view a subtraction situation (say, 12-9) as "what do I need to add to 9 to give 12?" and do not appear to ever use one of those three strategies.

The interpretation of these growth points reflects the description by Owens and Gould (1999) in the *Count Me In Too* project: "the order is more or less the order in which strategies are likely to emerge and be used by children. . . . intuitive and incidental learning can influence these strategies in unexpected ways" (p. 4). In discussing "higher" level growth points in a given domain, the comments of Clements, Swaminathan, Hannibal, and Sarama (1999) in a geometrical context are helpful: "the adjective *higher* should be understood as a higher level of abstraction and generality, without implying either inherent superiority or the abandonment of lower levels as a consequence of the development of higher levels of thinking" (p. 208).

Also, the growth points should not be regarded as necessarily discrete. As with Wright's (1998) framework, the extent of the overlap is likely to vary widely across young children, and "it is insufficient to think that all children's early arithmetical knowledge develops along a common developmental path" (p. 702).

The Development of the Interview

Once the early drafts of the framework were developed, assessment tasks were created to match the framework. A major feature of the project is a one-to-one interview with every child in trial schools and a random sample of around 40 children in each reference school at the beginning and end of the school year (February/March and November respectively), over a 30- to 40-minute period.

Although the full text of the interview involves around 50 tasks (with several sub-tasks in many cases), no child moves through all of these. The interview is of the form "choose your own ending", in that the interviewer makes one of three decisions after each task. Given success with the task, the interviewer continues with the next task in the given mathematical domain as far as the child can go with success. Given difficulty with the task, the interviewer either abandons that section of the interview and moves on to the next domain or moves into a detour, designed to elaborate more clearly the difficulty a child might be having with a particular content area.

All tasks were piloted with children of ages five to eight in non-project schools, in order to gain a sense of their clarity and their capacity to reveal a wide range of levels of understanding in children. This was followed by a process of refining tasks, further piloting and refinement, and where necessary, adjusting the framework, as shown in Figure 2.

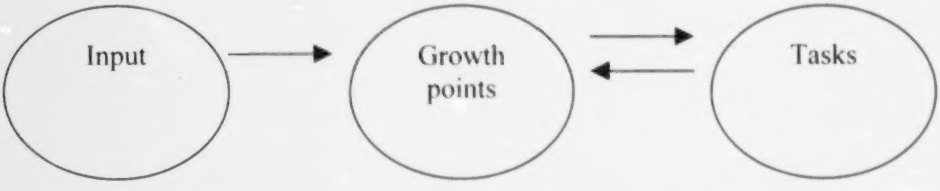

Figure 2. The process of developing the ENRP framework and the interview.

The form and wording of the tasks are influenced by the growth points for which they are intended to provide evidence, while at the same time the consideration of the data provided by a given task can lead to a refining of the wording of a given growth point.

The interview provides information about those growth points achieved by a child in each of the seven domains. Our aim in the interview is to gather information on the most sophisticated strategies that a child accesses in a particular domain. However, depending upon the context and the complexity of the numbers in a given task, a child (or an adult) may use a less sophisticated strategy than they actually possess, as the simpler strategy may do the job quite nicely in that situation.

Wright (1998) warns of the challenge of determining the actual strategy used by a child in solving a problem, as "a child may unwittingly or intentionally describe a strategy different from the one used" (p. 703).

It is important to stress that the growth points are "big mathematical ideas or concepts", with many possible "interim" growth points between them. As a result, a child may have learned several important ideas or skills *necessary* for moving to the next growth point, but perhaps not of themselves *sufficient* to move there. Also, to achieve many of the growth points requires success on several tasks, not just one or some.

Of course, decisions on assigning particular growth points to children are based on a *single* interview on a *single* day, and a teacher's knowledge of a child's learning is informed by a wider range of information, including observations during everyday interactions in classrooms. However, teachers agree that the data from the interviews are revealing of student mathematical understanding and development, in a way that would not be possible without that special opportunity for one-to-one interaction. It appeared that the children also enjoy that special time having the teacher "all to themselves". Teachers report that children appreciated the opportunity to show what they knew and could do.

Data Collection and Results

As well as moving carefully through the 16-page interview schedule, the interviewer completed a four-page Student Record Sheet. The information on this record sheet is then used by a trained team of coders together with a scoring algorithm to assign "achieved growth points" to each child for each content area. The rating of an individual child at a particular growth point is based on his or her responses to a number of different interview tasks. These raters demonstrated extremely high levels (all greater than 90%) of inter-rater reliability, as detailed in Rowley (in preparation).

A key criterion for the framework to be successful is the extent to which it reports on the spread and development of children's learning. Table 1 presents the percentages of children at each growth point in the first interview (March, 1999) for *Counting*, for each grade level.

Table 1

Percentage of Children at Each Counting Growth point, by Grade Level (%)

	Prep *n*=1702	Grade 1 *n*=1658	Grade 2 *n*=1498	Total *n*=4858
0. Not apparent	43.6	5.0	1.1	17.3
1. Rote counting	11.8	7.4	2.1	7.3
2. Counting collections	42.4	65.6	40.4	49.7
3. Forwards/backwards	1.6	12.5	15.6	9.6
4. Skip count from 0	0.5	8.4	32.4	13.1
5. Skip count from x	0.2	1.0	8.3	3.0

There is a clear development from Prep to Grade 1 to Grade 2. This part of the framework seems to allow description of a spread of development within the one grade level, and illustrates development across the grades.

It also seems that these results indicate possible directions or emphases for teachers at this level. For example, at each level, and overall, there is a large group of students who have achieved growth point 2 for *Counting*, but not growth point 3. It is possible that the development to growth point 2 happens naturally; it may be that development to growth point 3 may require some prompting. This possibly has different curriculum implications for each level. Perhaps the imperative at Prep is to assist children at growth point 0 and 1 to progress to growth point 2. In Grade 2, the emphasis may be on providing the experiences to assist the students to move beyond this point. Project teachers will be able to offer insights on this over the course of the project.

The growth points identified are meaningful and seem suitable for describing the learning of the children over time. For example, Table 2 compares the rating of the children at Grade 1 in the trial schools (only) from March to November.

Table 2
Grade 1 Children at Each Counting Growth Point in March and November (%)

	Grade 1 March *n*=1233	Grade 1 November *n*=1223
0. Not apparent	5.4	0.8
1. Rote counting	6.2	1.1
2. Counting collections	64.5	31.2
3. Forwards/backwards	13.2	12.4
4. Skip count from 0	9.8	41.9
5. Skip count from x	0.8	12.5

There is a clear movement in growth points evident in the Grade 1 data. Other grade levels show similar growth. Given the importance of the principle of having an appropriate ceiling for all children, an additional growth point was added (with related interview tasks) for 2000, as shown in Figure 1. Interestingly, 0.1% of Grade 1 children and 0.4% of Grade 2 children achieved this new growth point in the March 2000 interviews. Such data enable a comparison between growth in understanding in trial and reference schools, thereby providing a measure of the effectiveness of the professional development program. These comparisons will not be discussed here.

The framework is designed to allow the quantification of the learning of the children. However, we are more interested in identifying factors that may contribute to such learning. To complement the data on the children's learning a range of other data are being collected, including detailed questionnaires on teachers' beliefs and understandings about numeracy learning, regular journals kept by Early Numeracy Coordinators (the leaders of the professional learning teams in each school), as well as teacher and principal data on the effect of the project on teaching practice and student attitudes to mathematics.

In the third year of the project (2001), major emphasis will be given to studying those teachers and schools who have been shown to be particularly effective in building numeracy understanding.

The Refinement of the Framework

Data collected in March and November 1999 from approximately 5000 children informed the refinement of the framework and interview in preparation for the assessment period in

March 2000. Changes were also made in light of the perceived need to increase the focus on applying understandings in "practical" contexts. The major change to the framework was the incorporation of two domains for the Space strand (Classification of shape and Visualisation and orientation). There were also a number of word and phrase changes, to increase consistency and clarity for teachers and interested others.

Discussion

When the framework was first developed, a major purpose for its creation was to enable a measure of the effectiveness of the professional development part of the project. However, the framework is proving powerful in a variety of other ways.

Teachers are increasingly "owning" the framework, and using it to enhance their own understanding of children's mathematical learning. In the same way as the development of the framework and interview continue alongside each other (as in Figure 1), teachers' understanding of the framework is enhanced by their familiarity with the interview. Similarly, as the framework becomes better known, teachers view student responses during the interview in the light of their understanding of the growth points. Most importantly, the growth points provide a kind of "lens" through which children's mathematical thinking can be viewed, in all individual, small group and whole class interactions.

The framework in conjunction with data from interviews provides a basis for teacher planning and decision making. Teachers, coordinators, principals and the research team are increasingly reporting more focused teaching, in response to information gained in both interview and classroom situations.

The framework provides a way of reporting to parents on what children know and are able to do, in a relatively easily understood way. Parent information evenings at all 35 trial schools have contributed to growing goodwill towards the project.

The framework and interview must still be regarded as "work in progress". However, data to hand indicate the power of this approach in terms of both teacher professional growth and student mathematical learning.

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Challenges and Constraints in Researching Values[1]

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Values are taught in every lesson. However in mathematics classes this seems to be implicit rather than explicit. This paper outlines methodological difficulties encountered in researching the values teachers teach. One crucial area that has emerged is finding a common language with which meaningful dialogue can occur. We also reflect on differences, due to the different cultures, that have emerged between this Australian project, and a parallel project in Taiwan, including the influence of personal value systems.

Why Research Values?

Values in mathematics education are the deep affective qualities which education aims to foster through the school subject of mathematics (Bishop, FitzSimons, Seah, & Clarkson, 1999). They are a crucial component of the classrooms affective environment. Although values teaching and learning inevitably happen in all mathematics classrooms, the teaching of values appears to be mostly implicit. Thus it is likely that teachers have only a limited understanding of what values are being taught and encouraged. Values are rarely considered in any discussions about mathematics teaching. A casual question to teachers about the values they are teaching in mathematics lessons often produces an answer to the effect that they do not believe they are teaching any values.

This widespread belief that mathematics is the most value-free of all school subjects (Bishop, 1988) is not just held by teachers, but is also strong among parents, university mathematicians and employers (Ellerton & Clements, 1989). However mathematics is just as much human and cultural knowledge as is any other field of knowledge, and adults certainly express feelings, beliefs and values about mathematics which clearly relate to the mathematics teaching they experienced at school (Brew, 1999; FitzSimons, 1994). Hence it is clear that values teaching and learning does go on in mathematics classes, as it does in all classrooms. Furthermore we believe that the quality of mathematics teaching would be improved if we understood more about this phenomenon.

Values arise from different but overlapping influences on teaching. In this project we have identified three influences; general societal values often reflected in what we expect schools in general to be about, mathematical values that arise from the discipline of mathematics, and mathematics educational values that arise from the situation of teaching mathematics in school classrooms (Bishop, 1996). We have chosen to focus on those values that are associated more with mathematics and mathematics education rather than general societal values.

Thus we set out to answer three questions: (a) what are mathematics teachers' understandings of their own intended and implemented values? (b) to what extent can mathematics teachers gain control over their own values teaching? and (c) is it possible to increase the possibilities for more effective mathematics teaching through values education of teachers, and of teachers in training?

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How Do You Research Values?

The challenge in this project was not only to decide what were key questions to ask, but to develop an appropriate mix of investigative strategies that would help us gain some insight into this area of teaching. The remainder of this section will outline the methodology we have developed; the following section will then reflect on how a team of colleagues in Taiwan have developed a different mix of procedures as they investigate these issues.

The methodology we have used is rather traditional in some aspects. Essentially we have opted for an interlocking approach at two levels - micro and macro. At the micro level the approach is to work with individual teachers using a cycle of preliminary interview, a classroom observation, and a post-observation debriefing interview. This cycle is repeated with the same teacher two or three times. The classroom observations are video taped and the interviews audio taped. We plan to analyse the audio tapes but not the video tapes which will be used solely to capture episodes from the classroom to stimulate discussion with the teacher during the debriefing interview.

In attempting to elucidate the first aim of this project, we are first looking to see whether teachers can articulate their own intended values, and whether they can implement these in their classroom. Hence in the preliminary interview we will be asking teachers to nominate values that they suspect will arise during the lesson we will be observing. In other words the teacher will have the opportunity to plan for the teaching of particular values. We will be entering the observation lesson and looking specifically for these values nominated by the teacher. We are not nominating particular values and asking teachers to teach those. Nor are we choosing a methodology that has us observing lessons in a rather random way, hoping to observe values that happen to be taught. We are keen to make the process open for the teachers and to see whether they actually teach the values which they have nominated. We believe this approach is rather novel in education research. It not only asks teachers to reflect on their teaching behaviour and to say what values they are teaching; it also asks for authentication of the teacher's analysis by seeking to observe the stated behaviour in a classroom situation devised by the teacher.

In the observation lesson we will be taking particular interest in the critical decision points during the teaching. These critical decision points are times in the flow of the lesson when the teacher needs to make a decision that will influence the direction the lesson takes. It seems to us that it is in those decision times that the influence of values that the teacher is teaching may be most clearly evident. Clearly at these decision points, values are not the only influence. School policy, the physical situation within which the lesson takes place and so forth will also play at times a more dominant role than the values that the teacher may be teaching (Bishop et al., 1999). Nevertheless, the implementations of values will also play a role in the decision making.

In the post observation interview the videos will become the central prompting device for both the researcher and the teacher. We hope that the teacher will be able to remember points in the lesson at which they thought they were teaching the identified values. They will be able to use the video tape to help prompt their memory and elaborate on these episodes for the researcher. As well the researcher will also have noted points at which values teaching seemed to be occurring and the use of the tape will help the teacher recall these episodes. The aim of the debriefing interview is for the teacher and researcher to come to a shared agreement on some particular examples of when and how values teaching occurred in a particular lesson.

We plan to use a similar methodology to pursue the second aim of the investigation. The major change to the methodology will occur in the preliminary interview. We will no longer be asking teachers to nominate values that they normally teach, and from this broad set specify some which they suspect will be given particular emphasis in the coming observation lesson. Rather we will be asking teacher to implement some value(s) in the observation lesson that they do not normally teach, but on reflection they think they should be teaching. This change of emphasis will give further insight into how much control teachers may have over their teaching of values.

In piloting these techniques, and gathering some support from interested teachers, we have run a series of professional development sessions that incorporate some video clips of a teacher working with a grade 7 class, which were stopped at what seemed to be critical decision points. The participants were asked to nominate the options that the teacher had at these points. Subsequent discussions developed on influences which could have impinged on the various options available, including possible values. As well as the video, we presented a number of written episodes which were used to provoke teachers' thinking regarding the options they might have in classroom situations, and the underlying constraints that might be present. In particular, each discussion was finally directed to trying to decide what influence the teacher's values might have. Both these approaches worked well giving us confidence in the use of videos to stimulate recall of critical decision points, and the use of written scenarios in preliminary talks with teachers.

It has been instructive to reflect on the discussions we have had with six groups of teachers which indicate other issues for consideration. Perhaps the crucial finding from an analysis of our field notes taken during these sessions is the lack of an appropriate and shared vocabulary to discuss the types of values in which we are interested; that is values based in mathematics and mathematics education. The language involved in this investigation, and indeed in the transmission of values implicitly or explicitly in the classroom, is crucial. Indeed, this project essentially revolves around finding ways to make values linguistically explicit. This of itself will not lead to explicit values teaching. Rather, it will lead to a shared understanding between teachers and researchers. Because of the nature of language, a cultural artifact itself, one of course can never be sure that certain words do capture a shared meaning, or value in this case. If a shared understanding is accomplished, then we may be in a position to move to a further project that will involve examining the transmission of values from teachers to students. Both teachers and ourselves have struggled to find appropriate language so that these ideas - which are still being formed, reformed and refined - can be communicated in a positive manner.

As a research team we have always been conscious of this problem. In the research literature there have been many attempts at linking teachers' beliefs to their teaching of mathematics (McLeod, 1992; Southwell, 1995; Thompson, 1992; Tirta Gondoseputro, 1999). However the results of this research are equivocal (Bishop, 1999). Although some studies purport to find clear linkages, others do not. As Neuman (1997) suggests, subsequent actions need not necessarily correspond with stated intentions. A number of studies on beliefs were in the nature of self-reports, but there appears to have been few attempts to follow up these self-reports to see whether the teachers actually act upon their beliefs. For a variety of reasons we do not always act on our beliefs in certain situations. We wondered whether this explained some of the confusion in the literature. Hence one of the meanings we bring to 'values' in this study is the notion that 'values are beliefs in action'. That is, the values that teachers are teaching in the mathematics classroom are not only beliefs the teacher holds, but their behaviour in the classroom actually point to these beliefs. These are what we call 'values'.

But this in itself is too simplistic, even if it gives us a touchstone to work from. In our own discussions, and in the professional development sessions we have conducted, various notions are clearly embedded in this notion of 'value'. We summarise some of these in Table 1.

Table 1

Aspects of Meaning Either Used in or Arising from Discussions with Teachers

General meanings of 'value'	Mathematical values (after Bishop, 1988)	Mathematics educational values
To value:	Openness	Clarity
to command	Mystery	Flexibility
to praise	Rationalism	Consistency
to heed	Objectivism	Open mindedness
to regard	Control	Persistence
A value is:	Progress	Accuracy
a standard		Efficient working
a thing regarded to have worth		Systematic working
a principle by which we live/act		Enjoyment
a standard by which we judge what is important		Effective organization
		Creativity
something we aim for		Conjecturing
qualities to which we conform		

In a very real sense, this problem of language was inescapable. As noted above, a central feature of this project is to explore together the linguistic framework that we as researchers and teachers will use to try and share our understanding of the values that they teach in their mathematics classrooms. Thus it was decided that a set lexicon to be made available to teachers involved in the project was neither possible nor practicable.

'Conflicting values' was another issue that has arisen in our discussions with teachers. When contemplating the different situations it became clear that teachers are in difficult situations at times. For example, a teacher wishing students to develop an investigative stance to a project, and the students themselves, may wish to achieve closure at different points. In resolving this issue it may be that the teacher will need to draw on another set of more deep-seated values to resolve the conflict. On the other hand, the situation may be resolved from other sources, for example the submission dates set by an external examining body over which the teacher has no control.

As well as the micro investigation with individual teachers, we have developed a macro approach consisting of a survey that also gathers teachers' ideas on the values they teach. The survey has clusters of items built around the themes of teachers' understanding of values -

specifically, (a) values teaching in the mathematics classroom, (b) institutional and socio-cultural influences, (c) mathematical values, and (d) mathematics educational values - as well as teacher control over these values.

The issue of language also arose when piloting the survey form. For the items using a forced response format, a certain amount of rewording had to be undertaken to clarify the ideas we wished to interrogate. However the two major results from the piloting of the survey involved other issues. The teachers' responses showed that they are without doubt interested in the ideas of values, and their teaching of them; they recognise the importance of these notions and the need to investigate them. However associated comments indicate that these are new ideas for the teachers who clearly have not hitherto recognised that their teaching of mathematics does involve teaching values.

Lastly, a common theme we have detected running through responses from piloting the survey form and during the professional development sessions has been the presence of a certain amount of apprehension from the teachers. The subject of 'values' seems to immediately provoke in many teacher notions of judgement and finding fault. This may be a comment on our society, but it is an aspect of this project that needs to be taken very seriously. This in part is a language issue, but also means that we need to be scrupulous in respecting the teachers' personal value systems. We have become even more conscious of the role a teacher's personal value system may play when reflecting on a parallel project in Taiwan.

Taiwan Research

It was always planned and hoped that parallel projects would evolve overseas; it was expected that different cultures might well have an important influence on how and what values are taught in mathematics classrooms. Colleagues in Taiwan, led by Professor Fou-Lai Lin, have developed one such project (Lin & Chin, 1998) arising from our initial contact in 1996 and from continuing communications and meetings since then. In this paper we wish to reflect on the different methodology that our colleagues have developed as they try to gain some insight into how and what values are taught in Taiwan's mathematics classrooms.

We think it is true to say that they like us intended to conduct interviews and classroom observations with a small group of teachers, before moving to a larger scale investigation using survey techniques and more observations on a wider sample of teachers. They did indeed start with a small group of three teachers conducting interviews, followed by classroom observations, and then another interview. However our colleagues have developed in Taiwan what they described as a 'dynamic interview technique' (Wu & Lin, 1999). These are interviews that are stimulated by using anecdotes from observations of the teacher's classroom, and feeding back to the teacher pertinent analyses of these observations. This sequence then moves to in-depth dialogue with the teacher. Rather than having a range of teachers, the project has worked with just the three original teachers. The dialogues are conducted over twelve or more interviews with a teacher in a year, with one, two or three researchers present in an interview with one teacher. The researchers range from an older, very experienced, well known researcher through to a younger colleague who not long before was teaching in a school.

The background of the researchers proved to be of interest with regards the methodology of the project. It seemed that two teachers in particular responded in different ways because of the background of two of the researchers. One experienced male teacher responded in a more open manner to an interviewer who was more prepared to listen, to learn and to be able to share meanings (Chin & Lin, 1999a; Chin & Lin, 1999b). This may be a reflection of a Confucian trait in which a more experienced master plays "an active role in directing conversation while the researcher acted as a listener" (Chin & Lin, 1999b, p. 317) and learner.

In a second instance, a woman teacher who had been approached to participate in the project was at first reluctant to do so. She changed her mind after visiting one of the researchers in his office and noticing various artefacts that clearly indicated the researcher was a practitioner of Buddhism (Leu, 1999). In subsequent interviews it was noticed how the teacher responded differently to this researcher in comparison to the other researchers. As the interviews with the female teacher developed, it became clear that this teacher's overall life value system based on Buddhism was playing a prominent role in her teaching. She believed that one of the crucial aims of her teaching derived directly from these beliefs (Leu, 1999). It was easy for her to make direct links between this system of values she held and her teaching actions in the classroom. She also felt more at ease discussing such aspects of her teaching with the researcher who also practices Buddhism. For one thing she used a shared language derived from Buddhism in describing why and how she was dealing with values.

What do you do? In the 'classical' western situation that researchers have aimed for, there is supposed to be some notion of sameness operating from one interview situation to the next. This means that with appropriate protocols devised and followed, the results from all interviews can be combined because there is an assurance that the situations from which they were derived do not greatly influence the results differentially. In these instances from Taiwan, the interview situations turned out to be clearly different, and the differences did influence the responses of the teachers. It seems highly inappropriate to dismiss the results of these interviews because of the situations described. However to recognise these aspects of the situations and to include them as part of the data reporting seem to be essential. In these two cases it is clear that the intended curriculum was 'bent' in different ways to conform to some degree to each of the teacher's personal value set. These teachers held their personal beliefs so highly that they pervaded their acts of teaching and their self-perceptions as teachers. This seems to have been just as much an unexpected result for the Taiwanese team as it was for us.

We return now to the methodology that was used by the Taiwan group; multiple, intensive interviews with one teacher that spread over some twelve months. Why did our Taiwanese colleagues choose to use this approach? It seems that the major question that they are now asking is quite different from that with which they and us started. It appears that they are now trying to map the consistency of teaching values by a few teachers. One speculative reason for this alteration of the aim could be that the Taiwan research group is also driven by a cultural imperative. It may be that it seems more important to them that they are able to identify 'exemplary' teaching behaviour that can be held up as an example for other teachers to follow. Another formulation is that they are trying to depict what a 'model' teacher should be, so that other teachers have the opportunity to follow this model. We suspect the innate Confucianism that runs deep in the Taiwan culture may well influence such thinking.

Apart from reflecting on the changes in the two projects our different cultures seem to have promoted over time, another issue has been raised for our own project in contemplating our Taiwanese colleagues' work. They have used the fact that some of the teachers responded differently with particular researchers. This was unexpected. They have however recognised the special type of interaction that grew up and exploited it to gain a deeper insight into the consistency over time of the values that these teachers portrayed in their classrooms. In the Australian situation, we had intended to pair researchers and teacher participants on a basis of time convenience. However the Taiwanese experience has prompted us to think that if there is a relationship between the teacher and researcher, then we should consider exploiting this. It so happens that some of the researchers and teachers have known each other for a long time. It may be that a teacher will feel more able to discuss values with a friend than someone they have only met recently. Although we have been careful to try and depict this investigation as one that is trying to understand values arising from mathematics education and mathematics, as noted above, one of the most common uses in society of the word 'values' contains the notion of right and wrong, and of judgement. Hence some teachers have been quite reluctant to join the project irrespective of how and what we say. This misinterpretation may lessen if the teacher and researcher know each other and a certain trust between them is already present.

However in following this line, there is a cost as our Taiwanese colleagues have pointed out. We noted above the difficulties already encountered concerning the lack of a shared vocabulary between researchers and teachers. This is likely to diminish if the researcher and teacher know each other. However another difficulty may take its place. Because there is a shared vocabulary, the teacher and/or researcher may not feel the need to explore in the same depth their understanding of a classroom episode because they feel that the other will understand fully 'what they are getting at'. We need to guard against this. As well, reflection on past, shared experiences will need to be interrogated carefully since it again is so easy to assume the other will fully understand linkages made between such experiences and a classroom episode. But we are staying with the notion of working with the teachers for three lessons at the most. Hence, the extended time frame that our Taiwanese colleagues have to revisit instances when they realise in their analysis of an interview that ideas had been assumed because of say the shared Buddhism belief, will not be available to us.

Conclusions

In this paper attention has been drawn to the importance of methodology in researching values. Some approaches, which give insights into these ideas, have been offered. However difficulties still remain such as the development of a shared language. The influence of culture has also been highlighted, both at the societal and personal level. It may be better not to try and wash away such effects with methodological approaches, but recognise them as legitimate and look within them to gain greater insight into how values are taught.

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Mathematics Software Design in Mathematics Education: Factors that Promote Higher-Level Thinking

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This paper presents the results of a qualitative study involving the internet-based software LOGAL Tangible Math. The study was carried out in a secondary school classroom over a period of ten weeks and focused on identifying factors in the software design involved in promoting higher level thinking. Specifically, the roles of cognitive tools, question types, format of student answers, manipulable objects and navigational issues were examined. The authors conclude that higher-level thinking is encouraged by the provision of a rich set of tools, and the presence of manipulable objects that allow students to generate exemplars of a new concept.

Introduction

Imagine a secondary mathematics classroom. Near the window a group of students gathered around a computer monitor examining charts and tables chatter enthusiastically. A number of other students are working alone, intent on the task at hand, tapping at their keyboards. Meanwhile, a facilitator discusses with some other students their progress and helps them with any problems they have encountered. This classroom is fortunate to have a truly student centered learning environment. The computer software enables students to study mathematics at their own pace from high school to beyond early university level. Peer collaboration is encouraged through the software which is also cleverly designed to enhance the development of problem solving skills. The mathematical contexts are realistic and relevant for students aged from 12 through to 17.

It seems we are still an extremely long way from the idealised environment described in the above paragraph. Many questions are yet to be answered regarding the computer's ability as a 'teacher'. We know that computer software can be useful in drill and practice situations, but the focus of education has changed, and teachers' energies are now directed towards teaching students to think, to solve problems, and to be able to learn independently (Gipps, 1994). What is the computer's role in this type of education? Grabinger and Dunlap (1996) verbalised the question this way:

> ...So, how can we design machines to help people learn and think? Does this mean machines need to replicate human processes or that machines support processes? Can we use machines to help make the learning processes visible and more accessible? (p. 688)

Jonassen and Reeves (1996), discussing learning *with* technology, have pointed out the new focus for learners is interpretation of computer output:

> The learner should be responsible for recognising and judging patterns of information and organising it, while the computer should perform calculations, store information, and retrieve it at the learner's command. (pp. 697-698)

With this background in mind, the current study set out to examine the elements of educational computer software which would encourage the use and development of higher-level thinking by students in mathematics classrooms.

Definition

Prior to commencing the study, an operational definition of higher-level thinking was required. This proved to be a difficult task. A review of the literature revealed major differences in the way that higher-level thinking skills are defined. These differences exist not only between major fields of study, such as psychology and philosophy, but also between disciplines taught in secondary school such as English, social science, and mathematics. As a result, the authors searched for a common thread amongst the various views and also for an understanding of higher-level thinking relevant to the subject area in consideration, that is, mathematics. Lauren Resnick's work has been well published and appears to be generally accepted in the mathematics community. Resnick (1987) believes that 'higher-order thinking is difficult to define but easy to recognise when it occurs' (p. 44). In order to be able to recognise higher level thinking when it occurred Resnick produced a list of indicators. These indicators needed modification for the current study so that they more directly addressed the type of data generated. Table 1 contains Resnick's (1987) indicators and the modified version adopted for use in this study.

Table 1
Indicators Used to Recognise Higher-Level Thinking

Resnick's Indicator	Modified Indicator
Higher order thinking is *nonalgorithmic*.	Students make statements that do not obviously rely upon previously learned factual or algorithmic knowledge.
Higher order thinking tends to be *complex*.	Students make statements which associate two or more variables.
Higher order thinking often yields *multiple solutions*, each with costs and benefits.	Students produce a number of potential answers or a list of properties.
Higher order thinking involves *nuanced judgment* and interpretation.	Students provide a statement that demonstrates a thoughtful choice has been made between possible answers.
Higher order thinking involves the application of *multiple criteria*, which sometimes conflict with one another.	Students indicate they have applied two or more criteria in order to arrive at an answer.
Higher order thinking often involves *uncertainty*.	Students indicate uncertainty by taking time to review their solution and experiment with alternatives.
Higher order thinking involves *self-regulation* of the thinking process.	Students make their own decisions regarding which variables to examine, measure or compare.
Higher order thinking involves *imposing meaning*, finding structure in apparent disorder.	Students' responses indicate a search for significant features. Particularly if there is a choice involving which features to report.
Higher order thinking is *effortful*. There is considerable mental work involved in the elaborations and judgements required	This indicator was not used as it appeared to the authors that it always occurred along with one of the other indicators above.

It should be noted that a given instance of higher-level thinking does not require all of these indicators to be present (Newmann, 1990). On the other hand these indicators should not be viewed as discrete as they may occur in a wide variety of combinations.

Method

An interpretive, naturalistic methodology (Erickson, 1998; Denzin & Lincoln, 1998) was utilised. The first author acted as a participant observer and the second author collaborated as a cooperative colleague, observing sessions and participating in interpretation of the data. In working this way results were strengthened through researcher triangulation. The sample consisted of a class of 22 Year 9 students from a private girls school in Perth Western Australia working in pairs and interacting with the software over a series of 10 one hour lessons. During this period their answer files were logged and field notes taken. The operational definition of higher-level thinking was then applied to the data and instances of higher-level thinking identified. Subsequently, a database was created. This data base contained 112 records, one for each screen of the software encountered by the students. The database contained the following information:

- A screen capture from the software
- A description of the student action required from that screen
- A description of any processes required on that screen
- A record of the researcher observations resulting from that screen
- A list of the characterisations of higher-level thinking observed from that screen, either from direct observation or from student answer files
- A list of design elements utilised on that that screen

The screens on which higher-level thinking occurred were then examined further to investigate possible links between instances of higher-level thinking and the screen designs. The following design elements were included in the analysis:

- Tools: Whether or not the students had access to tools (angle measurement, length measurement etc).
- Answer box: Was a space provided for the students to answer a question and, if so, was the size of the space appropriate?
- Question type: what kind of question was asked of the students? Types considered were proof, conjecture, open, closed, simple and complex.
- Manipulable objects: Were there manipulable objects present on the screen?
- Text: What was the nature of the text on the screen? Did it provide instructions or information?
- Navigation: Did students have the ability for free navigation? Were there any hyperlinks?

The Learning Environment

The study was carried out within a broadly constructivist framework. That is, an environment in which the student is viewed as an active constructor of his or her knowledge as opposed to a receiver of knowledge transmitted by the teacher (Noddings, 1990). The reason for selecting this framework was that constructivist approaches to teaching and learning promote student's involvement in their own learning and enhance the likelihood of the development of higher-level thinking skills (Grabinger & Dunlap, 1996; Maor, 1995).

Specifically, a learning environment (within many constraints) similar to the 'Rich Environment for Active Learning' (REAL) as posited by Grabinger and Dunlap (1996) was created. These authors stated:

> One of the main assertions of REAL developers is that REALs improve problem-solving skills and enhance the likelihood that learning transfers to new situations. (p. 682)

It was felt, therefore, that the utilisation of a REAL would be most effective in encouraging higher level thinking, as well as providing a generally beneficial environment for the students. The attributes of the REAL which were implemented are described in Table 2.

Table 2
Attributes of the Rich Environment for Active Learning (REAL)

REAL Attribute	Explanation
Generative Learning Activities	Students learn through active involvement and use the tools available to them to develop and refine their understandings of the concepts being investigated. These type of activities encourage argumentation and reflection.
Cooperative Support	By working in groups the students can support each others learning. The discussion that goes on in the group can help the students in social negotiation of meaning. In addition, students working in groups may feel more confident to engage in the risks associated with tackling complex problems.
Authentic Learning Tasks	Students are provided with learning experiences which are as realistic as possible.
Student Responsibility and Initiative	The environment is student centred, placing a major emphasis on intentional learning. I wanted to encourage student questioning, self-reflection and self-monitoring.
Authentic Assessment Tasks	Students undertake contextualized, complex intellectual challenges (Wiggins, 1989) to assess their progress

The Software

The software selected for this study was the Geometry Inventor (LOGAL, 1997), from LOGAL Software Inc (http://www.riverdeep.net). This software was selected principally because it was designed with constructivist principles in mind. Following is a quote from correspondence with LOGAL Software Inc.:

> All of LOGAL's software is based on the constructivist philosophy that learning is the active construction of knowledge, and that knowledge comes to life when software helps students ask questions and learn through exploration. (Correspondence, 1997).

The Geometry Inventor is part of a suite of mathematics programs from LOGAL Software Inc. The entire suite is titled 'Tangible Math' and consists of components dealing with algebra, geometry, functions, probability and matrices. The geometry module was selected in consultation with the class teacher because it had a rich set of tools for the students to use and it was felt it would complement the students' existing mathematics course well.

Each lesson within the Geometry Inventor is broken into a number of stages. Lessons begin with a warm up activity that is followed by a number of experiments. The results of these experiments are then synthesised by the student in an analysis section. Finally the lessons conclude with a follow-up activity. Individual screens typically consist of some information to the student, a place for students to write their ideas, tools the students can utilise and some form of graphics which are usually manipulable (i.e., they are dynamic rather than static).

Results

Higher-level thinking was observed, according to the operational definition, on 22 of the 112 screens from the Geometry Inventor. However, on most of these screens multiple indicators were observed. In all there were 94 indicators of higher level thinking observed. The following table summarises the occurrences of each indicator as a percentage of the total observations.

Table 3
Indicators of Higher-Level Thinking

Indicator	% Frequency
Non-algorithmic	19
Complex	7
Multiple solutions	3
Nuanced Judgement	14
Multiple Criteria	4
Uncertainty	18
Self Regulation	15
Imposing Meaning	20

The next stage of the analysis was to attempt to match the instances of higher-level thinking with the design elements of the software available on the screen on which the higher-level thinking was observed. Each design element was considered separately and its role with regard to the observed instance of higher-level thinking was analysed. To help the reader to understand this process an example involving the indicator 'higher-level thinking is non-algorithmic' follows.

The indicator that was used for this category was that higher-level thinking was indicated when a student makes a statement that does not obviously rely upon previously learned factual or algorithmic knowledge.

An example of an answer indicating this type of thinking was given by a pair of students who defined a line TR with a positive gradient as follows:

> A line TR has a positive gradient if T is in the top right or bottom left of a square centred at R. (students' answer 14, PLT 23)

The students concerned were previously unfamiliar with the concept of the gradient of a line. The construction of their definition indicates that this answer was not obtained from the teacher or from a book. In the researcher's opinion, the students have constructed this definition themselves as a similar definition has not been seen before. This may be constrasted with a response such as, "The line slopes down to the left." (Student's answer 21, PLT 23), which is a commonly found way of defining a line with a positive gradient. As a result this was not classified as an indicator of higher-level thinking although these students *may* have indulged in higher-level thinking.

Having ascertained an instance of higher-level thinking the attributes of the particular screen stimulating this response were analysed. What were the students doing on this screen to produce this response? In this case students had to manipulate a line and, by viewing a dynamic table, investigate the conditions relating to positive, negative and zero gradients. When they had finally completed their investigation they were to enter their answer in an appropriate section on the screen. Hence, possible significant elements of design were manipulable objects, provision of tools with which to investigate, and the requirement of a student response in an answer box.

It is likely that all three of these played their role in encouraging the students to think beyond a surface level about the question put to them.

The remainder of this paper, due to limitations on publication space, will concentrate on the important design elements we have referred to as *tools* and *manipulable objects*.

Discussion

The Geometry Inventor Tools

The Geometry Inventor provided students with a rich set of tools with which they could measure properties of and/or construct geometric diagrams. For example, students could create any polygon, measure angles and sides, bisect angles, subdivide sides, measure area, create tables, create graphs of relationships and so on. Figures and diagrams created were dynamic, and hence, could be manipulated. All of the concepts developed were done so through the use of the tools, so it could be said that they were a vital element in producing any higher-level thinking in which the students engaged. These tools gave the students much freedom and the opportunity to test and experiment with their own ideas. However, it was observed that this freedom was not generally utilised by the students as they tended to follow the directions given in the software and perform only the investigations that were required of them. Furthermore, the geometry tools were not always the most significant of the design elements in producing the instances of higher-level thinking. In fact, on some occasions the presence of the tools actually inhibited the use of higher-level thinking. This occurred because students tended to immediately use the tools to measure attributes of the geometrical shapes rather than to look for patterns or generalisations. They thus turned lessons designed to produce higher level thinking into routine measurement tasks.

The above also raises another important issue. The authors believe generalisation and mathematical reasoning are higher-level thinking skills. However, in computerised mathematics lessons students tend to quickly generalise correctly because they *know* there is a pattern to find These student generalisations are usually made from too few experimental cases. Knowing something is one thing, knowing why is another, and being able to prove it still another. Lessons that use powerful measuring tools need to be devised to cover the latter two issues. The Geometry Inventor does indeed have some lessons which do this and these proved challenging and thought provoking for the students, and were certainly numbered among those lessons producing evidence of higher-level thinking. Perhaps the answer, in situations where tools are available to students, is to restrict their use at certain critical stages of the reasoning process to ensure the student thinks and doesn't simply measure.

Manipulable Objects

The Geometry Inventor could be classified as dynamic geometry software. That is, the emphasis of the software is on the use of tools to create geometric objects that can be manipulated. Thus, most screens in the lessons the students covered utilised this feature. 64% of the screens on which indicators of higher-level thinking were observed contained manipulable objects. The mere presence of these objects however, does not mean they contributed to the higher-level thinking. So what was their role, and is there evidence to suggest that they are significant in the encouragement of higher-level thinking? Skemp (1987), discussing the principles of learning mathematics states:

> Concepts of a higher-order than those which people already have cannot be communicated to them by a definition, but only by arranging for them to encounter a suitable collection of examples. (p. 18)

Before proceeding, the authors emphasise that they do not wish to equate higher-level thinking with the higher-order concepts that Skemp refers to here. He is talking about students learning a new concept which they didn't previously possess. For example, the Pythagorean theorem. It is the second part of this statement which is particularly relevant to the Geometry Inventor. In order for the students to make the step to a new concept, to generalise or abstract, Skemp suggests they need to be exposed to a suitable collection of examples. This is where the power of the Geometry Inventor is really seen. In manipulating the geometric objects the students are generating for themselves a group of suitable examples and they can generate as many as they like very quickly. This group of examples becomes the foundation for their thinking. We believe this is how these manipulable objects should be viewed, as a potentially infinite set of exemplars to a new concept.

For example, on a particular screen the students have a manipulable object, namely a triangle. They can use this triangle to generate examples of other triangles with all external angles greater than 90 degrees. Following this generation they are required to make a generalisation. It is at this point that the higher-level thinking becomes apparent. This thinking occurs during the search for relationships or patterns, either in the numerical data or in the visual data generated as a result of altering the shape of the triangle. To what extent higher-level thinking occurs during the data generation phase is difficult to say. The students observed generated the data and then examined the table they had produced in a search for patterns. However, it is not inconceivable that a student may manipulate the diagram and use the visual data to produce a conclusion (e.g. deciding visually that a triangle satisfying certain conditions is equilateral), although this was not observed. In such a case, the student may be utilising higher-level thinking skills while actually manipulating the diagram. Speculation aside, it appears from this study that the manipulation of the objects tends to be used to generate data. It is during the analysis of this data, the search for patterns and relationships, that higher-level thinking was observed.

Conclusion

The above discussion can appear to imply that design elements occur individually, or in isolation. In practice this is not the case. They are united in the software to produce a product, a series of lessons, which has effectively engaged the students in higher-level thinking. This fact is supported by the statistic that approximately 1 in 5 screens produced indicators of higher-level thinking, as characterised by Resnick (1987). The two design elements discussed in this paper that appear to significantly encourage higher-level thinking are:

- The provision of a rich set of tools and;
- The presence of manipulable objects allowing students to generate a set of exemplars of a new concept.

Although this paper has concentrated on the above two design elements, future research could focus, not only on these two elements of educational software, but also on the other design elements identified in the 'Method' section of this paper (answer box, question type, and navigation). This research would further our understanding of the link between these elements and higher-level thinking.

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First Experiences with a Computer Algebra System

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The results of a survey of 120 first year mathematics students confirm earlier findings that student approaches to learning mathematics are related to their conceptions of the nature of mathematics. In addition, a positive initial experience with the computer algebra system *Mathematica* is associated with a deep approach to study, a cohesive view of mathematics, and a more extensive background in computing. Features of students' initial experiences with *Mathematica* are described, based on qualitative and quantitative data from the survey.

Research concerning the integration of computer algebra systems (CAS) into teaching and learning at the tertiary level has mainly been of two kinds, the first being descriptions and evaluations of innovative teaching programs (e.g., Houston, 1994). The second kind consists of reports of teaching experiments in which comparisons are made between groups taught in a CAS environment, and groups taught in a traditional mode with content, and in contexts, appropriate to the requirements of the latter, (e.g., Heid, 1988; Meel, 1998). These and other papers indicate that students taught in a CAS environment achieve at least as well as those taught in traditional classes on measures of pen and paper skills at the first year level, and in many cases achieve a higher level of conceptual understanding. Direct comparisons are not appropriate, of course, in cases where the use of CAS changes the type of questions that might be set as assessment tasks and how they might be approached, for example in advanced calculus subjects.

An area in which there has been only preliminary research is the set of interactions between CAS experiences and students' approaches to learning mathematics, and indeed their conceptions of the subject itself. This paper reports some early results of a survey and interview study designed to investigate the experiences of first-year mathematics students, and their lecturers, at the University of Technology, Sydney (UTS), during the incorporation into teaching and learning programs of a particular CAS, *Mathematica*.

Background

Recent work at the University of Sydney (Crawford, Gordon, Nicholas and Prosser (1994, 1998a and b), and conducted without a CAS perspective, identified relationships between students' approaches to learning mathematics and their beliefs about the nature of mathematics. Two questionnaires were developed. One is based on the SPQ (Study Process Questionnaire) developed by John Biggs (1987). Among the items retained and revised are:

- "I think browsing around is a waste of time, so I only study seriously the mathematics that's been given our in class or in the course outline." *Surface approach subscale.*
- "I believe strongly that my aim in studying mathematics is to understand it for my own satisfaction." *Deep approach subscale.*

The Conceptions of Mathematics Questionnaire is based on an earlier phenomenographic study (Crawford et al, 1994), and has two subscales indicating a Fragmented Conception (Mathematics is seen as a collection of rules and formulas), and a Cohesive Conception (Mathematics is seen as a logical system for generating new knowledge). Examples of the items are:

- "Mathematics is a lot of rules and equations." *Fragmented subscale.*

- "Mathematics is a set of logical systems which have been developed to explain the world and relationships in it." *Cohesive subscale.*

Crawford et al (1998b) reported statistically significant correlations between these two sets of subscales, namely a positive correlation between a surface approach to study and a fragmented conception of mathematics, and a positive correlation between a deep study approach and a cohesive conception. When other variables which reflected students' perceptions of the teaching and learning environment, and prior academic ranking were included, two clusters of students were identified. These were described as follows:

> This analysis suggests that students holding cohesive conceptions of mathematics adopt deep approaches to learning mathematics, and have very different interpretations of learning mathematics at university. They perceive the learning environment as more satisfactory and fulfilling than do students reporting fragmented conceptions. Moreover, these students achieve at a higher level in their university study of mathematics than those students holding fragmented conceptions of mathematics and adopting surface approaches to learning. (Crawford et al, 1998a, p. 465)

The authors of that study emphasised that their model of student learning is not causal and deterministic, but instead indicates ongoing interrelationships between prior and post experiences and understandings, perceptions and study approaches, and the teaching and learning context.

This move away from causal models towards a descriptive framework is also seen in the development by Biggs (1991, 1993, 1999) of his 3P (Presage, Process, Product) model of student learning. The Presage components are Student (prior knowledge, abilities, preferred ways of learning...) and Teaching Context (curriculum, teaching method, classroom climate, assessment...). In this model, the introduction of a CAS is part of the Teaching Context, and will both influence, and be influenced by, the other components.

An important aspect of the model is that learning approaches are not fixed properties of an individual and are not necessarily transituational. This is where the role of *metalearning* is vital. Metalearning involves awareness of and executive control over one's learning processes (Biggs, 1987). This highlights the difference between the developed 3P model, and models based on an information processing metaphor that regard learning styles as fixed properties of the individual. Having to learn about a CAS may make a difference, then, to an individual who usually adopts a surface approach to learning mathematics, but is able to make use of CAS to carry out the personally directed investigations that usually distinguish a deep approach to mathematics.

Method

The Survey

In order to identify some of the interactions between students, the teaching context, and the kinds of task processing (approaches to study) adopted by students a survey was designed to collect information about students' conceptions of mathematics, their approaches to studying mathematics, and their experiences of learning *Mathematica.* It was expected that the relationships reported by Crawford and others for students at the University of Sydney would be identified, because of similarities in cohort, pedagogy, and context. What was not known was how students at UTS who scored differently on the various subscales of the Conceptions of Mathematics Questionnaire (CMQ) and the Study Process Questionnaire (SPQ) would vary in their reports of their CAS experience. Biographical data was also requested: age, sex, language spoken most of the time at home, prior mathematics studied, and a self-assessment of computing background.

First Experiences of Mathematica Questionnaire (MEQ)

The source of most of these 56 questions was the set of anonymous comments made a year earlier on subject evaluations by students responding to the open-ended question, "Overall, how would you describe your initial experiences with *Mathematica?*". Other questions were written in relation to some of the metaphors for graphing calculators that Kissane, Kemp and Bradley (1995) identified in a study of first year mathematics students' attitudes towards new technology. Two examples are "Mathematica was an invaluable aid to my learning of mathematical concepts", (the *Laboratory* metaphor), and "I resent the time taken on Mathematica assignments", (the *Nuisance* metaphor). Other items reflected practical experiences with the software and hardware, and opinions about the assessment. To record their responses to each item on the MEQ, students were requested to mark a position on a line that was about 4 cm long, with Disagree on the left and Agree on the right. The location of the student response on the scale was interpreted as a numerical measure of strength of agreement with each proposition. This gave a measure for each item from zero to 41 on a notionally continuous scale.

The Students

The target population for the survey consisted of all students enrolled in three different first year, first semester mathematics subjects. They will be referred to here as Subject 1 (for students majoring in mathematics, about 80 in this class), Subject 2 (for students in physical science courses, about 110), and Subject 3 (for students in engineering courses, about 400). The surveys were distributed in classes at the beginning of the subsequent semester but only for Subject 1 were the students allowed class time to complete the surveys: 60 were returned. The low response rates for Subjects 2 and 3 mean that comparisons across the three subjects must be made with caution, if at all. The responses as a whole do give information about the range of experiences, if not about the distribution of those experiences.

Because of choices made by the coordinating lecturers, the planned learning experiences with *Mathematica* for students in the three subjects were quite different. In Subject 1, students attended ten one-hour laboratory sessions, and were given worksheets to read and work through on the computers. These described the application of CAS to the solution of real world problems from a variety of contexts from finance to the design of theme park rides. An assignment on a different topic, models of population growth, requiring many of the commands introduced in the labs was given towards the end of semester and students spent many hours out of class working on it. In subject 2, students again attended weekly labs but had worksheets designed to introduce only those *Mathematica* commands that were relevant to the work covered in lectures that week. The worksheets contained several questions to be done with pen and paper as well as on the computer. Discussion and working in pairs was actively encouraged. The engineering students in Subject 3 had only three scheduled one-hour labs and were expected to use that time, and time out of class, to complete an assignment that reflected a main topic in the subject: mathematical modelling.

Results

Items from the MEQ with High and Low Means

The means and standard deviations of the five items with which students showed most agreement are listed below in Table 1. Also shown are the five items with the lowest means, that is the items with which students disagreed most strongly.

Table 1

Items with High and Low Scores on the MEQ

Item (Agreement is a high score on a scale of 0-41)	Mean	Std Dev'n.
48. It was frustrating when you forgot capital letters or used the wrong brackets and it wouldn't work.	30.4	10.0
44. I need more practice with *Mathematica* before I can use it as a tool.	29.3	10.4
30. *Mathematica* is too expensive to buy for myself so I did not have the benefit of working on it at home.	29.1	13.3
45. *Mathematica* is useful as a checking tool.	28.1	9.5
11. When I get an unexpected output or error message from *Mathematica* I try to work out the reasons why before changing my input.	28.0	10.3

Item (Disagreement is a low score on a scale of 0-41)	Mean	Std Dev'n.
9. I had difficulty using the printer.	8.8	11.3
23. I looked for books in the library on *Mathematica.*	8.9	11.8
15. I had difficulty saving my work onto floppy disks.	9.0	10.4
35. I have used *Mathematica* to investigate mathematical questions of my own that arose in other subjects.	9.7	10.3
17. I often used *Mathematica* to explore my own questions about mathematics.	10.3	10.2

Items Reflecting the Overall Experience

At the end of the questionnaire were two free response questions. The first was: "Overall, how would you describe your experiences with *Mathematica* last semester?" The purpose of this question was to gather information about the students' experiences in their own words, and to provide a check that issues that were important to students had indeed been reflected in the questionnaire items. The replies to the first question were transcribed and then sorted into three categories indicating a negative (47 comments), neutral (27), or positive (25) overall experience. Within each category, responses with a similar focus were grouped together and summarised. These groups are reported below with illustrative comments. Minor spelling and grammatical errors have been corrected. The numbers in each of the smaller groups are not considered important in this stage of the analysis where there is an attempt to capture the range of experiences rather than their distribution. It should be noted that many of the comments could have been placed in more than one group, and it is acknowledged that this grouping is subjective.

Comments Revealing a Poor or Negative Overall Experience

Time. "Very frustrating at times. I could see how it would be very useful in helping with some maths but with the little time we had with it I wasn't able to get a hang of it very well."

Inadequate teaching. "I felt that they didn't really teach us how to use the program, that they expected us to be able to use the whole program after giving us only a couple of commands."

Difficulty of Mathematica itself. "I found it hard to understand how *Mathematica* works." "It was difficult to learn and remember all the commands."

Assignments were too hard. "*Mathematica* was very hard to get the hang of. Assignment was so hard that I didn't and couldn't hand anything in . I find the language very hard to understand." "The tutes were good, but the assignment was a HORROR!"

Overall a waste of time. "Time could be spent on more useful learning in maths or other subjects."

Own lack of computing experience. "Frustrating because of my lack of computing experience."

Took too long to learn the syntax. "Painful. A lot of effort went into learning the syntax and in the assignments this meant trying to get it to work instead of trying to get the right answer."

Comments Revealing that the Overall Experience was Neutral or Satisfactory

Frustrating but useful: "It was hard to use and required a lot of work to operate. It was however useful for basic and tedious calculations in other assignments, but I had trouble applying it to complex problems."

Frustrating at first, but a useful/worthwhile experience later after student's effort or because of the group work: "Frustrating. But, ultimately after all the hours I put in to it, it was rewarding completing the assignment." "Neutral. I understand that *Mathematica* is a powerful tool ... I was however, unable to see the potential of

Mathematica – especially when applied to other subjects. This was mainly due to a lack of knowledge on commands, syntax, etc. This lack of knowledge, on the other hand, encouraged me to research on data regarding *Mathematica* and talk with my friends about maths..."

Comments Revealing that the Overall Experience was Worthwhile

Interesting: "A challenge, which made it interesting." "... Overall my first experience was interesting. It was good to see things that you usually can not." "It made maths a lot more interesting."

Useful: "I think it is a quite positive experience, because now I know there is software package that helps us to solve mathematics." "Good. I can see its use as a tool in industry."

Rewarding: "Big effort, but rewarding."

Helped with maths: "It was a new experience as before last semester I didn't even know *Mathematica* existed. Over all it tends to help one's understanding of mathematics."

In the light of this range of reported experiences, the 56 items in the MEQ were again reviewed and a shorter list of 26 items was constructed to try to capture the features of an overall positive experience with *Mathematica*. Again this was a subjective judgement although the checking against the students' written comments lends support to its being a useful choice. The average of these 26 items was then labelled *Overall Mathematica Experience*. For the whole sample the mean of this variable was 19.2 and the standard deviation was 5.7. The means for various subgroups were obtained and no statistically significant differences were found for groupings by Subject, Gender, Home Language, and Age (recent school leavers or mature age) across the whole sample. Computing Experience did make a difference, however, when the sample was divided into "low" and "high" levels of computing background.

Relationships with the SPQ and CMQ

One of the aims was to investigate the relationships, if any, between scores on the subscales of the CMQ (Fragmented/Cohesive), the SPQ (Surface/Deep), and the items on the MEQ. Those which yielded statistically significant correlations greater than 0.25 are reported below in Table 2.

Table 2

Correlations of MEQ Items with the Four Subscales of the CMQ and the SPQ

Item from MEQ	Fragmented Conception	Cohesive Conception	Surface Approach	Deep Approach
5. In general, the Mathematica assignments were too difficult for me.	0.28**	–0.26**	0.22*	
7. It was hard for me to understand what the Mathematica assignments were asking me to do.	0.29**		0.24**	
8. The Mathematica assignments were interesting.	–0.22*	0.32**		0.25**
11. When I get an unexpected output or error message from Mathematica I try to work out the reasons why before changing my input.	–0.30**	0.20*	–0.19*	
17. I often used Mathematica to explore my own questions about mathematics.		0.25**		0.28**
18. The only time I used Mathematica was when we had to hand something in.	0.26**	–0.25**		
22. Getting the right output from Mathematica is largely a matter of trial and error.	0.20*	–0.36**	0.21*	–0.25**
28. I enjoyed discussing Mathematica with other students.		0.29**		0.33**
29. Because of Mathematica, I had interesting conversations with others about mathematics.				0.42**
31. Mathematica just added to the amount we already had to learn.	0.28**	–0.27**	0.20*	–0.25**
47. Mathematica was fun.		0.37**		0.27**
53. Creativity was adequately rewarded in the Mathematica assignments.		0.26**		0.28**

Note. *Correlation is significant at the 0.05 level; **correlation is significant at the 0.01 level

The correlations between the subscales themselves are given below in Table 3, with the corresponding results from Crawford et al (1998b) shown for comparison. The correlations are in the expected directions and of comparable significance overall.

Table 3

Correlations Between Study Process Variables and Conceptions of Mathematics Variables.

	Conceptions of Mathematics Variables			
	Fragmented		Cohesive	
Study Process Variables				
Surface Approach	*0.36***	0.20*	*–0.04*	–0.22*
Deep Approach	*–0.12**	–0.17	*0.43***	0.58**

Note. *Correlation is significant at the 0.05 level; **correlation is significant at the 0.01 level. Figures in italics from Crawford et al (1998b), N=300; figures in normal font from current study, N=120.

The correlations between Overall Mathematica Experience and the four subscales of the CMQ and the SPQ are also in the expected directions, although none of them are very large. This is shown in Table 4 below.

Table 4

Correlations between Overall Mathematica Experience and the CMQ and SPQ Subscales

	Fragmented view	Cohesive view	Surface approach	Deep approach
Overall *Mathematica* Experience	–0.22*	0.32**	–0.19*	0.29**

Note. *Correlation is significant at the 0.05 level; **correlation is significant at the 0.01 level.

Discussion

There are messages in the results of this research for those designing introductory learning and assessment tasks for *Mathematica* or any CAS. A reading of Table 1 indicates that one semester's work in one hour weekly labs (or less in the case of Subject 3) was not enough time to convince most students in this sample that *Mathematica* is a productive tool for their own use. It could well be that Lecturers who have learned several "mathematical" programming languages in their careers (for example, BASIC, FORTRAN) simply underestimate the amount of time it takes to become familiar with new syntax, let alone new concepts such as assigning values to variables and defining functions. While some students were happy to teach themselves and make the necessary investments of time and reading, many expected more help from the teaching staff and from the materials provided. A possible direction for change is a complete revision of the kind of assignment (if any) that is set to be completed with *Mathematica*. Some kind of graded choice in topics might meet the need to provide challenge to students already confident in using computers, while allowing a less demanding task with more support from tutorial staff for those who need it.

In this sample, the students who have a positive first experience with *Mathematica* are not more likely to be male or female, from English or Non-English home language backgrounds, or to be older or younger. Previous computer experience does seem to help, and this echoes a finding by Galbraith, Haines, and Pemberton (1999, p. 219) that "computer attitudes are more influential than mathematical attitudes in facilitating the active engagement of computer related activities in mathematical learning."

The relationships between Conceptions of Mathematics, Approaches to Learning, and Overall *Mathematica* Experience are not strong but warrant further investigation. In this phase of the study, it is not possible of course to suggest directions for these relationships. We cannot say, for example, whether students who already had a cohesive view of mathematics found the assignments interesting because of that, or whether the nature of the assignments contributed towards that cohesive view. Crawford et al (1998a) found little change in either Approaches to learning or Conceptions of mathematics over students' first semester at university. (See Note). Whether or not a successful experience with a Computer Algebra System might encourage Surface Learners to adopt a Deeper approach, or perhaps contribute to a change in conceptions of mathematics from fragmented to cohesive, remains unanswered.

Future Directions.

Interviews with students and staff will provide more information about the issues that have been raised. In particular, it will be useful to investigate the differences and similarities between approaches (surface/deep) to *learning mathematics*, approaches (surface/deep) to *learning Mathematica*, and approaches (surface/deep) to *learning mathematics with Mathematica*. Further refinement of the items in the MEQ would lead towards the development of a shorter questionnaire that could be used in future investigations.

Conclusion

Among the advantages claimed for the introduction of technology (graphing calculators and computer algebra systems) into upper secondary and tertiary mathematics education are: preparing for further study and work, (Leigh-Lancaster, 1996); reducing time spent on routine calculations, thus allowing teachers and students to concentrate on concepts and the interpretation of results, (Mueller, Pedler, Anderson, & Bloom, 1998); reducing conceptual misunderstandings in mathematics by linking visual, symbolic, and numerical representations (Eisenberg & Dreyfus, 1991); stimulating a new approach to teaching mathematics through guided discovery (Leigh-Lancaster & Stevens, 1999).

Some of the students in the current study would agree with these claims, in particular those who spent time learning the syntax of the program and becoming familiar with its operating paradigm, and those who enjoyed the discussions about mathematics that arose while they worked on challenging tasks. However many are yet to realise these benefits and it is a challenge to design learning tasks, perhaps more integrated with or even replacing lectures, which will assist more students to benefit from the new technology.

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[1] *Note*. A typographical error appears in Table 3 of the reference Crawford et al 1998a. In the first column of figures, -0.25** should appear as 0.25**. (Personal communication with one of the authors of that study).

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Tertiary Students' Rankings of the Factors Behind their Choice to Study Mathematics and Computing: A Gender and Regional Study

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This paper describes an investigation into the factors which influence students in their choice of tertiary fields of study with major focus on mathematics and computing. The primary research questions were: what are the factors behind their choice, how are they ranked, and are there gender and regional differences underpinning these choices? 541 first year students were surveyed at four Queensland universities in 1998. Notable findings are reported, and the implications for promoting these disciplines are discussed.

Our student cohort is different to a decade ago (Tobias, 1990), and still changing. A sound understanding of current factors and influences underpinning students' choice has the potential to increase our understanding of our student cohort and to heighten our sensitivity to their expectations and disappointments. It has implications for the way in which degrees and courses are advertised and promoted in the community, and for the manner in which early undergraduate students are counselled. We need to be aware of changing trends in influences and aspirations, and develop sound practice based thereon.

There has always been interest in the factors that influence students entering tertiary study. Recent studies on secondary to tertiary transition difficulties (for example, Pargetter, McInnes, James, Evans, Peel, & Dobson, 1998), revealed that a factor like *strong interest in the field* is related to course satisfaction, and a factor like *parental influence* is related to first-year students' dissatisfaction with their chosen field of study. Significant attrition surveys, like that reported in Ramsay, Tranter, Sumner, & Barrett (1996), have pointed at the nature of relationships between factors of influence and persistence with, or withdrawal from, tertiary study.

There is little in recent Australian literature that focuses specifically on the factors that draw first-year students into either mathematics or computing. A recent investigation by Brinkworth and Truman (1998), in ten South Australian schools, revealed that the major influence behind scholars' choice to do Year 12 mathematics was the perception that mathematics was "*needed for further study*". A further strong influence was their "*ability to do mathematics*". Their espoused reason for not doing Year 12 mathematics was "*lack of appeal of the subject*", rather than "*inability to do it*". In the University of South Australia flexible admissions study (see Ramsay *et al.*, 1996), reasons for enrolment were sought generally. Of eight given reasons, those surveyed ranked "*to study in a field that really interests me*" the highest, followed by "*to gain entrance to an attractive career*". "*To be with my friends*" ranked lowest.

Gender issues in mathematics and computing remain a source of concern, with enrolment in most countries indicating that fields that require a major focus of study on mathematics and computing are still male-dominated, and likely to remain so in the near future. Increasing numbers of women are entering tertiary study in Australia: females comprised 55% of commencing higher education students in Australia in 1999 (see DETYA, 1999). Yet the

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number of non-overseas women commencing study in non-traditional areas in Australia remains low at 20%, and 1999 figures show a rise in female enrolment in non-traditional fields of under 3% when measured against those for 1998. In 1997, the year prior to the gathering of the data for this study, females comprised 18% of commencing computing students in Queensland, and 35% of those entering mathematics.

Significant attempts have been made to identify the difficulties, and to support and encourage women. MERGA and PME Proceedings reveal many strong mathematics studies, and initiatives are well documented elsewhere too. Some Australian computing initiatives are overviewed in Craig, Fisher, Scollary, and Singh (1998), and Bae and Smith (1997) reviewed some in the US. Growth has been slow, however, and even the goals of Australian federal efforts like the *New National Agenda for Women* have not been realised.

The gender imbalance is particularly evident in regional universities, where first-year mathematics and computing classes attract very few women students. Is it possible there might be gender or regional differences in the factors that attract or influence prospective students in the choice of these disciplines? The literature offers little to illuminate regional effects on such factors. A notable 1980's longitudinal study of rural youth in Ohio (McCracken & Fails, 1991) investigated the influences on career and curriculum choices of high-school students in a range of disciplines as they matured and entered college. The influence of *self* was consistently ranked higher than that of *parents, friends* or *counsellors*. No regional comparisons were reported, however, nor any for gender. A gender study by Stables & Sian (1995) investigated career aspirations and the reasons for choosing A-level subjects, and reported that though the girls were significantly better qualified overall, than the boys, girls lacked confidence and felt the need for more advice.

There are perceived cognitive and affective associations between the disciplines of mathematics and computing. Many studies (for example Shashaani, 1995) point at links between the two, and Markert (1996) found common reasons why women do not choose careers in science, technology, engineering and mathematics. Mathematics comprises a substantial portion of the tertiary computing curriculum, and the two disciplines are often grouped in academic structuring. To investigate the field of influence specifically within the areas of mathematics and computing, therefore, and to illuminate any gender and regional trends, the primary questions posed for this investigation were:

- What factors lie behind the choice to study tertiary mathematics and computing?
- Are these factors ranked differently by first-year mathematics students and by computing students?
- Are there gender differences in these students' ranking of the factors?
- Are there differences in the rankings given by those students from homes and schools progressing from more urban areas to the more rural?

Research Design and Methodology

The decision was made to survey a large sample of first-year students at both regional and urban universities in Australia in Semester 2, 1998. Two urban and two regional universities in Queensland were targeted as appropriate, the urban universities selected so as to offer a balance of type: one traditional, the other a university of technology.

To facilitate good design of a survey instrument, ten quite different students were interviewed about the reasons for, and influences behind, their choice of studies in mathematics or computing. Their responses were synthesised into eight major factors. The three factors that surfaced as dominant in these interviews, were *strength in the subject*, *interest in the subject*, and *employment prospects*. A fourth factor raised often enough to be listed, was a *significant occasion or event*. The other factors that emerged reflected the influence of a significant other person: *teacher*, *parent*, *older sibling*, and *friend/other*.

Probably because the research aims were different, these eight factors classified influences on students somewhat differently to those used in some other studies. For example, Ramsay *et al.* (1996) did not specify or separate the influence of *teacher* from other "reasons for enrolment", nor distinguish between *parent* and *sibling*, listing *family* instead. Nor did that study investigate the influence of *strength* in a particular discipline.

The eight espoused factors were incorporated into a questionnaire which asked students to rate how highly each had influenced their decision to study mathematics or computing: either *strongly*, *moderately*, *weakly*, or *not at all*. Students were invited to list any other factors of influence too. They were also asked to order their top four factors of influence, to provide a different type of ranking. The rating of each factor by all or most students offered breadth of views across the cohort for that factor as an influence, whereas the ranking or ordering of the factors of major influence provided a relative measure of how strong was the perceived influence of each.

Questions also invited response on gender, location of home and school, country of origin for high schooling, type of school attended (government/non-government, single sex/mixed gender), higher education and occupation of parents, position in family, course entry (school leaver/mature-age), degree and major. To facilitate understanding of responses on employment prospects, open-ended questions on career aspirations and perceptions of future home commitments were included. The questionnaire was tested in a pilot study, minor changes made, and the refined instrument then administered to first-year, second-semester students at lectures in all of the primary mathematics and computing programs at the selected universities.

Responses were not accepted if students did not indicate significant commitment to a choice of major within the disciplines of mathematics or computing. They were also screened to remove any unreliable surveys, and to eliminate any repetition from respondents being surveyed in more than one of the classes surveyed. The data was then analysed using SPSS where appropriate.

Results

A total of 541 students' responses were accepted, the majority from the large metropolitan universities: 50% at the traditional one, and 45% at the university of technology. The representation from the two regional universities was very much lower, only 5% of the sample, but the data nevertheless revealed a good spread of home and school locations, which was important for the research (see Table 5).

General Rankings

For simplicity in this report, students' responses have been combined into those indicating relatively high levels of influence (*moderate* or *strong*) and those indicating low influence (*weak* or *not at all*). Though response rates were generally very high, close to the full group for most factors, not all respondents rated every factor. Table 1 gives the percentage of respondents on each factor, who rated it at that level.

This data confirmed and quantified the feedback obtained from the interviews done in the preliminary stages, that the factors *interest in the subject*, *strength in the subject*, and *employment prospects* are generally most influential. Students' ranking of their major factors of influence confirmed the Table 1 order of these three factors: 46% chose *interest in subject* as the strongest influence, 18% chose *employment prospects*, 17% chose *strength in subject*.

Table 1
Percentage of Respondents on that Factor who rated it at that Level (overall N=541)

	Parent	Sibling	Teacher	Friend	Occasion	Interest	Strength	Employment
Weak or Not at all	64	79	67	69	76	5	11	18
Moderate or Strong	36	21	33	31	24	95	89	82

Though the ratings of the influence of significant people were generally lower, over 30% of students rated highly the role played by *parent*, *teacher* or *friend*, and 20% of students conceded that an influential role was played by an *older sibling*. Of note too, is that a significant *occasion or event* was rated strong or moderate by 24% of students.

A comparison with factors of influence ranked in other studies reveals evidence of support for some of the findings, but also some notable differences. In the Ohio rural youth study (McCracken *et al.*, 1991), *self* (without further breakdown of the influences therein) was consistently ranked highest as a source of influence, followed by *parents*, *friends* and *counsellors*. *School personnel* and *counsellors*, however, were ranked lower than *parents* and *friends*.

These results lend support to the University of South Australia attrition study (Ramsay *et al.*, 1996) where "*To study in a field that really interests me*" was rated highest out of the eight proffered reasons for enrolment, followed by "*entrance to an attractive career*". *Strength in a subject* was not specified separately there, and the reason "*To be with my friends*" rated lowest, but is different to the factor of influence of *friend* investigated here.

A Comparison of the Ratings by Mathematics and Computing Students

The degree course indicated by each respondent was used to distinguish between students studying mathematics and those studying computing, and the two groups were analysed separately. See Table 2. For simplicity, only the percentage of students that rated each factor *moderate* or *strong* is tabulated here. The percentage that rated each factor *weak* or *not at all*, simply covers all remaining respondents.

Table 2
Percentage of Respondents who rated the Factor as Strong or Moderate (overall N = 413)

	Parent	Sibling	Teacher	Friend	Occasion	Interest	Strength	Employment
Comp N=413	33	21	26	33	25	96	88	88
Maths N=101	47	19	61	23	23	92	94	66

Computing students comprised 413 out of 541 in the group, so it is not surprising that Table 2 reflected the same trends as those for the full group. Their votes for the factor of strongest influence also confirmed the top ranking of *interest in subject (*51% of all votes), and a higher ranking of *employment prospects (21%)* than *strength in subject (*11%).

Mathematics students rated *interest in subject* and *strength in subject* as most influential. *Employment prospects* were not rated nearly as strongly as by computing students. Votes for the strongest influence confirmed the relative ranking of the top factors: 32% chose *interest in subject*, and 32% chose *strength in subject*. Thereafter, 11% chose *teacher*, and only 9% chose *employment prospects*. It is notable that the influence of *teacher* on mathematics students was considerable, and that they also rated the influence of *parent* more highly than computing students did. The influence of *friend* was lower.

A Comparison of Ratings by Gender

The classes surveyed reflected an attendance of 27% females and 73% males. Women comprised 23% of those surveyed in computing courses, and 40% of those surveyed in mathematics courses, reassuringly higher than the Australia-wide figures for women commencing in non-traditional areas. For the group generally, the gender breakdown of ratings was much like that for computing students in Table 3 below, and there were statistically significant gender differences.

Males rated the factors *interest in subject* ($p = 0.003$, Male M = 382, Female F = 142), *employment prospects* ($p = 0.019$, M = 380, F = 142), and *strength in subject* more highly, on average, than did females. Both the t-test and non-parametric Mann-Whitney test gave similar p-values. Females, on average, rated the factors *parent, sibling, teacher, friend/other person* and *occasion/event* more highly, but with *sibling* ($p = 0.028$, M = 203, F = 84) yielding the only significant difference.

Table 3 offers a comparison of male and female computing students. Male computing students rated the factors *strength in subject* (not significant), *interest in subject* ($p = 0.000$, M = 312, F = 89) and *employment prospects* (n s), more highly than did females.

Table 3

Percentage of Computing Male/Female Respondents who rated the Factor as Strong or Moderate (Male Computing = 318, Female Computing = 95)

	Parent	Sibling	Teacher	Friend	Occasion	Interest	Strength	Employment
Male Comp	30	15	25	32	25	97	88	89
Female Comp	42	38	27	35	26	93	87	87

Females rated all the factors *parent, sibling, teacher, friend/other person* and *occasion/event* more highly than did males, but with only one significant difference, that of *sibling* ($p = 0.001$, M = 165, F = 58). (On average, however, both males and females judged *sibling* influence quite low.) Though similar percentages of males and females (97% of males, 93% of females) gave moderate/strong ratings of the factor *interest in subject*, the statistically significant difference between males and females is a result of the degree of positive influence. 68% of males chose strong, rather than moderate, compared with only 44% of females. More males (56%) voted *interest* as the strongest influence than did females (35%). There were no statistically significant differences, on average, between male and female mathematics students' ratings, but Table 4 shows interesting results.

Table 4

Percentage of Mathematics Male/Female Respondents who rated the Factor as Strong or Moderate (Male Maths = 60, Female Math s= 41)

	Parent	Sibling	Teacher	Friend	Occasion	Interest	Strength	Employment
Males Maths	50	21	63	18	25	86	93	62
Female Maths	42	14	59	30	21	100	95	73

A Comparison of Ratings by Regions

The data offered a good spread of urban and regional home and school locations: see Table 5. Because there was little difference between the distributions of the respondents' home and school locations for the majority of their high school years, results will be reported on home location. Differences for school location were minor.

Table 5

Distribution (in %) of Home and High School Locations
Missing data accounts for discrepancies from a total of 100%.

	major city	large inland town	large coastal town	small inland town	small coastal town	rural property
Home	58	10	9	9	7	4
School	59	11	9	9	5	1

Students' ratings of the factors of influence were analysed first in two groups: the *urban* group: students from cities and large towns (77% of respondents) and the *country district* group: those from rural areas and small towns (20%). *Teacher* influence was rated more highly, on average, by students from country districts (p = 0.009) for home location, and p = 0.041 for school location). There were no significant differences noted for ratings on any of the other factors.

Urban students rated *interest in subject* as the strongest influence, with a vote of 46%. *Employment prospects* were chosen by 20%, and *strength in subject* by 17%. Country district students rated *employment prospects* (11%) lower than *strength in subject* (18%), but still rated *interest in subject* highest (49%).

For greater clarity, respondents were divided further, into *three* regional groups: *major city, large town*, and *small town/rural*. Table 6 gives the regional breakdown of ratings for the whole group, which were similar to those for the computing group alone.

Table 6

Percentage of Major City, Large town and Smalltown/Rural Respondents who rated the Factor as Strong or Moderate (City = 314, Large = 106, Small = 107)

	Parent	Sibling	Teacher	Friend	Occasion	Interest	Strength	Employment
city/ large /small	34 / 38 / 35	23 / 17 / 20	30 / 33 / 42	28 / 48 / 27	24 / 30 / 21	95 / 92 / 98	89 / 88 / 93	81 / 86 / 82

Mathematics students, however, rated the influence of *teacher* highly, and more so in smaller regions. See Table 7. Computing students may not have had a specific teacher in mind: their ratings of *teacher* were lower, and progressed from city to rural as follows: 25%, 24%, 31%. In the large town group, generally, *friend* had a surprisingly strong rating, 48% of all students admitting to moderate or strong influence.

Table 7

Percentage of Major City, Large town, and Smalltown/Rural Mathematics Respondents who rated the Factor as Strong or Moderate (City = 51, Large = 20, Small = 27)

	Parent	Sibling	Teacher	Friend	Occasion	Interest	Strength	Employment
city/ large /small	44 / 38 /56	40 / 31 / 63	55 / 62 / 70	19/ 46 / 16	20 / 40 / 19	94 / 81 / 94	94 / 95 / 96	59 / 71 /74

Mathematics students from major cities voted for the strongest influence as follows: 35% chose *interest in subject*, 28% chose *strength in subject*. Only 12% chose *employment prospects*, and 10% chose *teacher*. Small town and rural mathematics students voted as follows: 48% chose *strength in subject*, 19% chose *interest in subject* 11% chose *teacher*, and 11% chose *employment prospects*.

Computing students ranked their strongest influences differently: those from major cities voted as follows: 48% chose *interest in subject*, 24% chose *employment prospects*, and 12% chose *strength in subject*. Only 11% of computing students from small towns and rural regions rated *employment prospects* as their greatest influence: 62% chose *interest in subject*. Influence of *friend* followed with 9%, a much bigger vote than *parent* or *teacher*.

Summary and Discussion

The investigation revealed that the factors *interest in subject, strength in subject* and *employment prospects*, are most widely felt by first-year students to be the strongest influences underpinning their choice to study mathematics and computing. Considerable numbers of students also rated *parent, teacher, friend* and *sibling* as moderate to strong influences, and some admitted to the strong influence of a significant *occasion or event*. Interview data validated and illuminated these findings.

Interest in subject is easily the most influential factor for computing students, followed by *strength in subject* and *employment prospects*, which rate similarly, and much higher than all other factors. Mathematics students rate *strength* and *interest in subject* equally strongly, but they rate *employment prospects* a bit lower, roughly on a par with the influence of *teacher*. *Parents*, too, are conceded to have considerable influence, nearly half the mathematics students admitting to strong/moderate influence. Results also indicate that *friends* exert a lesser influence on mathematics students than on computing students.

Gender differences revealed that males rated the factors *interest in subject*, *employment prospects*, and *strength in subject*, significantly more highly than did females. Females generally rated the influence of *significant people* in their lives more highly than did males, supporting claims in the literature that socialisation experience (Reinen & Plomp, 1997) is a more dominant influence in the lives and choices of females. It is important also to note that 17% of computing females ranked *strength in subject* as their strongest influence. Are there signs, at last, that women are beginning to bury the widely acknowledged "we can, I can't" paradox? Is pervading technology itself the catalyst for that?

Many of the regional differences were slight, which is reassuring considering the perceived disadvantage of fewer opportunities for scholars. Is this a result of access to technology bringing information to the more remote areas? Even if this is the case, one very important finding of this study is that the personal influence of *teacher* is espoused by mathematics students to be felt strongly, and increases towards more rural areas.

Comparisons with ratings given by computing students clearly indicate that mathematics students, especially those away from the cities, should have more information on prospects for employment. While it may be the case that some students are drawn to the study of mathematics without the additional motivation of career prospects, ways should also be sought to make this information more accessible to parents and teachers, whose influence should not be underestimated. Current efforts are commendable, but professional mathematicians should increase their efforts to advertise career opportunities.

There is evidence in the data also, of the positive influence of *occasions or events* like mathematics camps, competitions, and career expos. These are some of the activities listed by students when asked to specify the nature of that influence. Clearly these results offer support for the promotion of such activities, and for increased efforts to create more opportunities of that kind.

These findings have valuable implications for educators, and for those who promote the study of mathematics and computing, or support and counsel students. Mathematics teachers and educators in particular, should be aware of the influence they have on the career and study decisions students make, especially in more isolated areas. They should build a knowledge base that supports this appropriately, and should be involved in the planning and organising of information sessions and activities for scholars.

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Mathematics Enrolments in Australian Upper Secondary Schools: Trends and Implications

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Coping with and planning for developments in school mathematics requires informed debate based on factual information. A knowledge and understanding of school mathematics enrolment trends can facilitate planning and identify issues that need to be addressed. The trends in upper school mathematics up to 1990 have been well documented by the authors (Dekkers, de Laeter, & Malone, 1991). This paper presents highlights from that earlier research and reports on year 12 student numbers and mathematics enrolment trends for the period 1980 to 1998, based on the most recent statistics from State authorities. Implications of the trends are also discussed.

In recent times in Australia, concerns have been expressed at both State and Federal levels and within the community regarding the level of mathematical competence of school leavers. A particular concern is that of the decline in the number of students enrolling in the more academically challenging upper secondary school mathematics subjects (Brennan, 1994; The Herald Sun, 1996; The Age, 1997; The Australian, 1997, *Campus Review*, 1998). Clearly, as a developed country, Australia's future will increasingly depend upon its ability to compete in a competitive, technological world. It is equally important that schools are in a position to develop the mathematical potential of students who are mathematically able and talented. Coping with and planning for developments in school mathematics requires informed debate based on factual information. A knowledge and understanding of school mathematics enrolment trends can facilitate planning and identify issues that need to be addressed. The authors' data base, extending back to 1970, serves the purpose of contributing to that knowledge and understanding.

National School Enrolments and Mathematics Enrolments

The interpretation of upper secondary school mathematics enrolment trends needs to be considered in terms of the overall Year 12 enrolment trends. These national enrolments rapidly increased over the period 1980 to 1992 from 89 038 to 192 511. Thereafter they declined steadily before showing small increases in 1997 and 1998, when they reached 177 234. Hence there has been an increase of nearly 100% in Year 12 enrolments in the past 20 years. The number of females remaining at school to Year 12 is one of the major factors contributing to this increase as retention rates confirm. For example, in 1980 the female retention from Year 8 to Year 12 was 37% compared to 32% for males. In 1998 however, the retention rate for females was 78% compared to 66% for males.

Total mathematics enrolments for the period 1980 to1998 are shown in Figure 1. It can be seen from this Figure that increases in Year 12 mathematics enrolments have essentially 'paralleled' the growth in Year 12 student numbers. In interpreting Figure 1 it needs to be noted that many students do more than one mathematics subject, hence the graph is a comparison between mathematics subject enrolments and Year 12 student numbers.

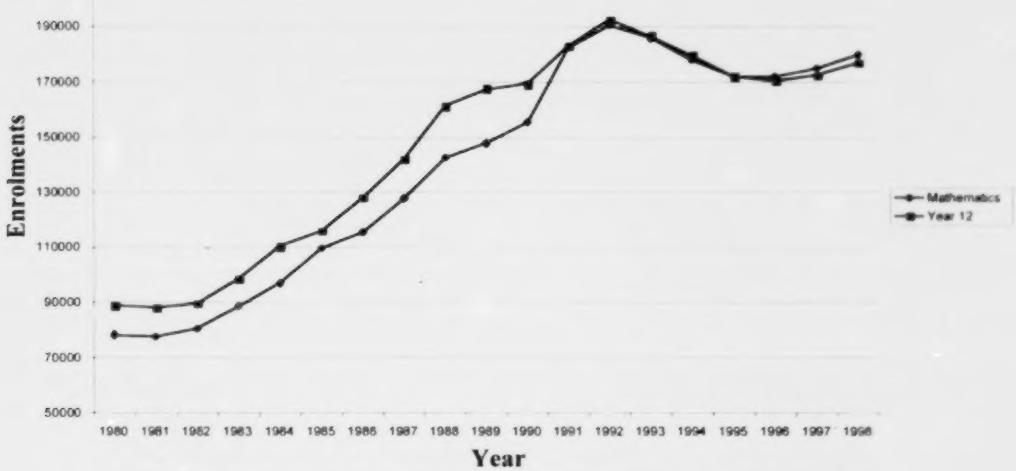

Figure 1. Year 12 student numbers and mathematics enrolments in Australia.

It can also be seen from the graph that since 1996, mathematics enrolments have exceeded Year 12 student numbers, and this gap has continued to widen each year since. Table 1 below is illustrative here. A comparison of male and female mathematics enrolments and Year 12 student numbers over the period 1980 to 1998 in the Table indicates a number of trends:

- In 1980 there were approximately the same number of male mathematics enrolments and male Year 12 students, while for females there were approximately 11 000 fewer mathematics enrolments than Year 12 students. Male mathematics enrolments exceeded female mathematics enrolments by approximately 8 000.
- During the 1980s these trends continued in general, but the difference between male and female mathematics enrolments declined, whereas the number of female Year 12 enrolments increased more rapidly than for males.
- In 1992 enrolments peaked for both male and female mathematics enrolments and Year 12 student numbers.
- Since 1992 male mathematics enrolments and male Year 12 students have declined. For females, mathematics enrolments have declined only slightly, while enrolment numbers have declined significantly.
- In 1998, male and female enrolments were approximately equal while the mathematics enrolments exceeded the Year 12 student numbers.

Table 1
Year 12 Student and Mathematics Enrolments in Australia

Year	Mathematics Enrolments			Year 12 Students		
	Male	Female	Total	Male	Female	Total
1980	43302	35104	78406	42221	46817	89038
1982	43543	37118	80661	41704	47941	89645
1984	52567	44432	96999	52728	57766	110494
1986	61314	54521	115835	61038	67074	128112
1988	74524	68189	142713	76322	85147	161469
1990	79678	76020	155698	79150	90321	169471
1992	99483	91381	190864	92944	99567	192511
1994	91266	87022	178288	85973	93890	179863
1996	87057	84972	172029	80682	90047	170729
1998	90573	89441	179978	83738	93496	177234

The data from Table 1 indicate the overall increase in mathematics enrolments resulting from increasing levels of enrolments in mathematics by females when compared to males. Whilst the total number of mathematics enrolments for males and females were approximately equal in 1998, there were some 10 000 fewer male Year 12 students than females.

The State and Territory Scene

Upper school subjects (Years 11 and 12) in each State and Territory can be categorised as either a Public Examination Subject (PES) or as a School Assessed Subject (SAS). A Public Examination Subject is defined as one that meets entry requirements and/or selection criteria for university level study. A School Assessed Subject is defined as one designed for students who are not intending to pursue university study.

The PES and SAS combined trend for (total) mathematics enrolments for the period 1980 to 1998 in each of the States and the Territories is presented in Figure 2. Each State trend reflects more or less the National trend provided in Figure 1.

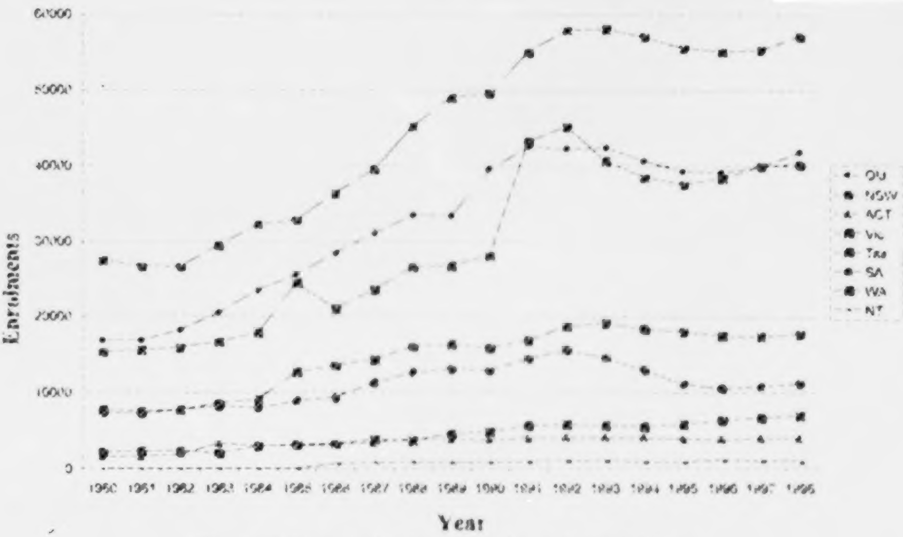

Figure 2: State territory total mathematics enrolments.

Total enrolments in mathematics and the Year 12 student populations in each State and the Northern Territory are shown in Table 2. For Queensland, New South Wales and Tasmania there are more mathematics enrolments than students. In the case of

Table2

A Comparison of Mathematics Enrolments and Year 12 Student Numbers - 1998

	Total Maths			% SAS			Year 12			Maths Participation Ratio		
	M	F	Total	M	F	Total	M	F	Total	M	F	Total
Qld	21115	20707	41822	8.5	6.9	7.7	17577	18786	36363	1.20	1.10	1.15
NSW	27258	29919	57177	0.1	0.1	0.1	25868	29087	54955	1.05	1.03	1.04
ACT	1916	2013	3929	20.6	13.9	17.2	2189	2081	4270	0.88	0.97	0.92
VIC	21382	18868	40250	0.0	0.0	0.0	20935	24429	45364	1.02	0.77	0.89
Tas	3670	3285	6955	57.1	57.6	57.4	2132	2356	4488	1.72	1.39	1.55
SA	5710	5459	11169	37.5	42.9	40.1	5820	6630	12450	0.98	0.82	0.90
WA	9021	8705	17726	21.8	24.1	22.9	8726	9537	18263	1.03	0.91	0.97
NT	465	485	950	66.2	74.8	70.6	491	590	1081	0.95	0.82	0.88
Nation Total	90537	89441	179978	9.6	9.4	9.5	83738	93496	177234	1.08	0.96	1.02

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Tasmania, students can opt to do mathematics in either Year 11 or Year 12 or in both years, and no distinction is made in the statistical data available from that State. Hence, in this analysis, it is not valid to compare the Tasmanian data with that for the other States and the Territories.

The data presented in Table 2 reflect higher levels of mathematics participation in Qld and NSW than for the other States and the Territories. The very considerable difference in the extent each State and Territory provides PES and SAS mathematics offerings in Year 12 is significant. There is no SAS mathematics available in Year 12 in Victoria whereas, in the Northern Territory and Tasmania, these subjects attract 70.6% and 57.4% of the total mathematics enrolment respectively. The data in Table 2 provides tentative support for the notion that extensive availability of SAS subject offerings (as is the case in Tasmania, South Australia and the Northern Territory) does encourage a greater level of participation in mathematics. In 1998, SAS enrolments comprised 10% of the total mathematics enrolments in Australia.

Enrolment in Mathematics Subjects by Level of Complexity

In addition to categorising mathematics subjects as either PES or SAS, a classification in terms of their level of difficulty is informative:

High Level: Those subjects involving specialized or advanced mathematics leading to tertiary studies in which mathematics is an integral part of the discipline, as in physical science, engineering or mathematics (e.g. Specialist Mathematics in Victoria; Mathematics 4U in NSW; Calculus in WA; Applied Mathematics in Tasmania).

Intermediate Level: Those subjects which involve a level of mathematical competence which provides a satisfactory background for tertiary studies in which mathematics content is minimal – for example in architecture, pharmacy, economics (e.g. Mathematics B in Qld.; Discrete Mathematics in WA; Mathematics 2U in NSW, Mathematics 1D in South Australia).

Low Level: Those subjects, which do not provide a suitable mathematical foundation for any tertiary studies (e.g. Mathematics A in Qld; Mathematics in Practice in NSW; Quantitative Methods 2U in South Australia).

Figure 3 below presents mathematics enrolment trends within Australia for the period 1990 to 1998 based on level of course (High to Low) as described above. The most apparent feature is the decline in the number of enrolments for high-level mathematics courses, although this trend has levelled off between 1996 and 1998. Concomitantly there have been increases in mathematics enrolments for the low and the intermediate-level mathematics enrolments.

In 1998, within Australia high-level mathematics enrolments comprised approximately 18% of the total mathematics enrolment whereas the intermediate and low level enrolments comprised approximately 44% and 37% of the total mathematics enrolments. In 1990, the percentage enrolments for high to intermediate to low-level mathematics enrolments were 24%, 42 % and 33% respectively .

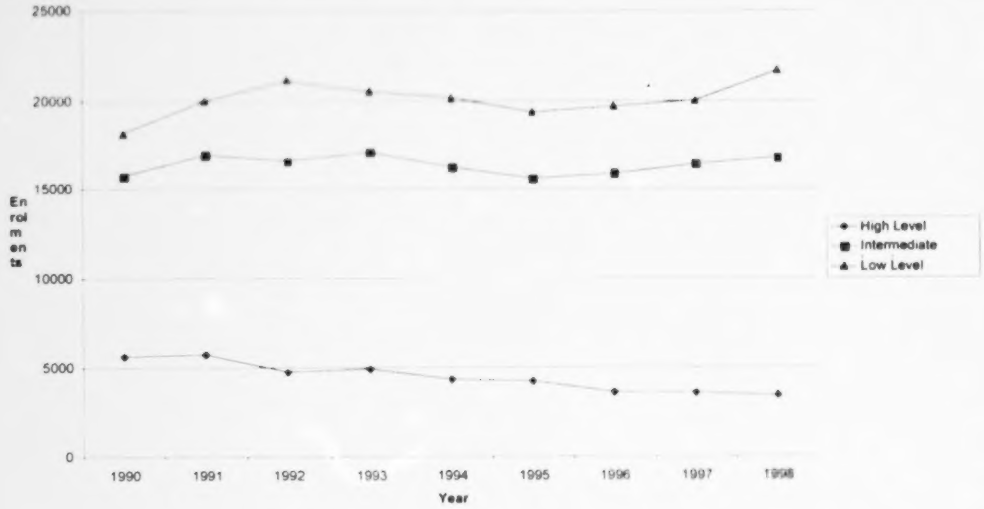

Figure 3: National mathematics enrolments by level of course.

It is well documented that a considerable proportion of students who take mathematics in Year 12 often also enroll in science subjects and more recently, in computing subjects at the tertiary level. This is particularly the case for students who take PES mathematics subjects for tertiary entrance purposes. Table 3 provides a

Table 3
Year 12 Science, Mathematics and Computing Trends in Australia (1990 - 1998)

	Year			90/98 Diff
	1990	1995	1998	
National				
Male	79150	81751	83738	4588 (5.8%)
Female	90321	90606	93496	3175 (3.5%)
Total	169471	172357	177234	7763 (4.5%)
Science				
Male	87148	79937	78362	-8786 (100%)
Female	80191	74563	74208	-5983 (7.5%)
Total	167339	154500	152570	-14769 (8.8%)
Mathematics				
Male	79678	87520	90537	10859 (12.6%)
Female	76020	84566	89441	13421 (17.6%)
Total	155698	172086	179978	24280 (15.1%)
Computing				
Male	10879	25426	29525	18646 (170%)
Female	10296	22339	20659	10363 (100%)
Total	21175	47765	50184	29009 (140%)

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comparison between National Year 12 mathematics, science and computing enrolments between 1990 and 1998. In examining Table 3 for 1990 it can be seen that:

- There were more science enrolments than mathematics enrolments for both males and females;
- The number of Year 12 students was comparable to the total mathematics and science enrolments;
- The number of computing enrolments represented only approximately one eighth of the total Year 12 enrolment; and
- The number of female Year 12 students was greater than that for female mathematics and science enrolments.

This scenario changed quite dramatically during the 1990's to the extent that in 1998:

- The number of mathematics enrolments for both males and females exceeded those for science;
- The number of computing enrolments has increased by nearly 150%, representing approximately one quarter of the total Year 12 enrolments. Whilst in 1990 the percentage of males and females was nearly equal, this had changed to approximately 60% for males and 40% for females in 1998;
- The total number of mathematics enrolments exceeded both Year 12 students' enrolments and science enrolments; and
- Male and female science enrolments both decreased after 1990, whereas for mathematics they both increased.

Implications of Trends

Three interrelated enrolment trends have clearly emerged within Australia with respect to mathematics enrolments and Year 12 student numbers. These are the:

- Increased retention of students from Year 8 to Year 12.
- Higher levels of retention of females to Year 12 than for males.
- Increased participation of females in mathematics courses.
- Overall drift away by students from taking higher-level mathematics subjects.

Thus the Year 12 school population in more recent times is comprised of students with a wider range of academic ability, career aspirations and interest than those in previous years.

It also emerges from the examination of the mathematics enrolment data that the level of student participation in school mathematics varies considerably between the States and the Territories, as does the range of subject offerings in mathematics, particularly for SAS mathematics. Queensland has a higher participation rate in mathematics (and science) than elsewhere in Australia when compared to Year 12 populations. This is evidenced by the extent SAS mathematics subjects are available in the States and the Territories. Nevertheless it would appear that the provision of SAS mathematics subjects has not altogether been a solution to attracting more students to do mathematics in Year 11 and 12. Indeed the tradition of offering conventional school mathematics curricula in preparation for university study has prevailed and remains relatively popular.

In order to achieve a greater level of participation in school mathematics for both males and females, a number of previously identified problems and issues in the provision of mathematics at Year 11 and 12 needs to be addressed. These are as follows:

- The recruitment and retention of qualified and well trained mathematics teachers.

The provision of incentives and opportunities for the ongoing professional development of mathematics teachers, both for context and method.

- The development and introduction of more creative approaches to the teaching of mathematics in schools. The use of the Internet is an obvious teaching resource which remains to be widely utilized for mathematics education.
- The development and implementation of mathematics curricula that challenge and motivate students in the study of mathematics.
- The breaking down of the influence of the university sector in mathematics syllabus content both for PES and SAS subjects. Currently there remains a tendency to "load" curricula with too much content, facts and figures, whereas what is required is a more conceptual, enquiring mode of mathematics education.
- The greater involvement of industry, commerce and "end-users" of the products of secondary education to help ensure that curricula choices available to students are relevant to careers and preparation for the workforce.

Until problems such as the above are addressed, it can be anticipated that the current status and appeal of mathematics education to students will remain unchanged.

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Making Sense with Diagrams:
Students' Difficulties with Feature-Similar Problems

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Students experience a range of difficulties in generating effective diagrams. Hence, it is important to explore these difficulties so they can be addressed during instruction on diagram generation. A cross-study comparison of the results of two network tasks revealed that students experience similar difficulties on feature-similar but non-isomorphic tasks. Students' difficulties on these tasks appeared to be due to a lack of sense-making in mathematics rather than a difficulty with the problem structure or the generation of a particular type of diagram.

The use of the strategy *draw a diagram* is strongly advocated by mathematics educators as a tool for problem solving (Australian Education Council, 1991; National Council of Teachers of Mathematics [NCTM], 1998). A diagram is a particularly effective problem representation because it exploits spatial layout in a meaningful way, enabling complex processes and structures to be represented holistically (Winn, 1987). For some students, generating a diagram is the first step towards a successful solution (van Essen & Hamaker, 1990). However, students can also be misled by self-generated diagrams in the solution process (Antonietti & Angelini, 1991). Inadequate diagrammatic representations of problems may limit children's problem solving capabilities (Klahr, 1978), hence, it is important to investigate factors that influence problem representation (Goldman, 1986) and address these in an instructional program. Although there is a vast literature base on students' difficulties in some areas of mathematics (e.g., counting), the literature on primary students' difficulties in generating diagrams is scant. Hence, the purpose of this paper is to explore the various difficulties students' experience in the generation of diagrams.

Knowledge Acquisition through Diagrams

Representing written problem information on a diagram is initially a translation process that involves the decoding of linguistic information and the encoding of visual information. During this process, there is the potential for knowledge acquisition (Karmiloff-Smith, 1990) through the reorganisation of information (Weinstein & Mayer, 1986) and subsequent inference-making (Lindsay, 1995). For example, knowledge about family relationships that cannot be easily inferred from a description can be established on a family tree. Sternberg (1990) proposed that there are three knowledge acquisition components, namely, selective encoding, selective combination and selective comparison. Selective encoding relates to the relevance of the information that is represented. Some students' representations are unhelpful for problem solving because relevant problem information is not included (Dufoir-Janvier, Bednarz, & Belanger, 1987). For example, students may represent the surface (irrelevant) features of a problem rather than the structural (relevant) features (Dufoir-Janvier et al., 1987). Selective combination refers to how new information is integrated as a discrete entity. The diagram is an effective problem representation because problem information is represented by location on a plane, and hence, a large number of perceptual inferences about problem information is possible (Larkin & Simon, 1987). Selective comparison focuses on the relationship between new knowledge and prior knowledge. This component highlights the importance of background knowledge about general purpose diagrams that have applicability in problem solving (e.g., networks).

General Purpose Diagrams

General purpose diagrams assume an important role in mathematics because they provide representational frameworks that are applicable to a range of problem structures. These diagrams are networks, matrices, and hierarchies, and a range of diagrams that exhibit part-whole characteristics. Novick, Hurley, and Francis (1999) developed a theoretical framework for networks, matrices, and hierarchies, which described the conditions of applicability and distinguishing properties for these spatially-oriented diagrams. For example, a network is a path-like representation (e.g., a train line map), whereas a hierarchy is a tree-like representation (e.g., family tree). Part-whole diagrams were not included in Novick et al.'s framework because they have no unique external form. As effective instruction is informed by pedagogical content knowledge about students' difficulties and errors (Carpenter, Fennema, & Franke, 1996), it is important to explore students' difficulties and errors with each of the general purpose diagrams.

Design and Methods

This paper discusses the similarity between the results of structurally dissimilar problems from two separate studies.

Study 1 was an explanatory case study (Yin, 1994) in which the effect of instruction on children's use of diagrams in novel problem solving was investigated. The participants were 12 ten- to eleven-year-old Year 5 students. These students were presented with sets of five isomorphic novel problems during 30-minute interviews conducted before and after a series of lessons on the use of diagrams in problem solving. The interviews were video-taped and subsequently transcribed for analysis. One key finding of this study was the recognition of the variety of difficulties and errors students experienced in generating general purpose diagrams (Diezmann, 1999). As difficulties and errors in diagram generation constitute obstacles to students' problem solving performance, they were explored further in Study 2.

Study 2 focused on identifying the range of difficulties that students experience in generating general purpose diagrams for novel problems. This research was undertaken using an inductive theory-building framework, which requires description and explanation (Krathwohl, 1993). Data were collected until a "saturation" point was reached in which new observations did not provide further insight into the phenomenon. The participants in this study were a class of 25 ten- to eleven-year-old Year 5 students. Data comprised observations of individual children engaged in novel problem solving and the diagrams they generated during interview sessions. Over four videoed interview sessions, the students were presented with a total of eight tasks. Two tasks that could be represented using the same general purpose diagram (e.g., networks) were presented at each interview, which lasted approximately 20 minutes.

The findings of Study 1 and Study 2, respectively, revealed that difficulties in diagram generation can be due to (1) a lack of understanding of the problem structure, and (2) a lack of understanding of specific general purpose diagrams. During the analysis of the Study 2 data, it was noted that while there was limited correspondence between the difficulties students experienced on the two network tasks. However, there was an unanticipated similarity between students' difficulties on one of the pre-instruction network tasks in Study 1 and one of the network tasks in Study 2. As this correspondence can neither be explained by similarity in problem structures nor attributed solely to the general purpose diagram used, the findings from these network tasks were investigated further.

Results

The two network tasks that revealed similar diagram generation difficulties and errors are shown on Figure 1. Although the problems are not isomorphic, there is some obvious similarity in the features of the problems (e.g., height measurement and movement). Henceforth, these types of tasks are referred to as feature-similar tasks. The analysis of students' performance on these tasks revealed a range of diagram generation difficulties and errors. Three categories of difficulty emerged from students' performance on these tasks: (1) a lack of measurement sense, (2) a lack of spatial sense, and (3) a lack of number sense. Due to space limitations, the reporting of these difficulties is limited to one set of examples from each category.

Study 1 - The Koala: A sleepy koala wants to climb to the top of a gum tree that is 10 metres high. Each day the koala climbs up 5 metres, but each night, while asleep, slides back 4 metres. At this rate, how many days will it take the koala to reach the top?	**Study 2 - Bouncing Ball:** Sylvia dropped a tennis ball from a balcony 8 metres above the footpath. Each time the ball bounced it travelled half as high as on the previous bounce. Sylvia's brother caught the ball when it bounced exactly 1 metre from the footpath. How many times did the ball bounce?

Figure 1. The Koala and bouncing ball tasks.

A Lack of Measurement Sense

Although, measurement is integral to daily life and commonly encountered in applied contexts, some students' diagrams revealed the inadequacy of their measurement sense. One of the most pervasive errors on these feature-similar tasks was the incorrect measurement of "ground level". On both tasks, many children incorrectly labelled or referred to the ground height as one metre. This error is evident in Damien's diagram in which he incorrectly identified the first "mark" as one metre, instead of the second mark (see Figure 2). Damien's initial error was compounded because the ten marks on his tree only represent a nine metre tree rather than a ten metre tree as specified in The Koala task (see Figure 1). Jane's diagram also showed a lack of understanding of ground level. In contrast to Damien, Jane correctly indicated that ground height was zero metres. However her diagram suggests that she lacked an understanding of the location at which a ball rebounds. Her depiction of a bouncing ball shows the ball rebounding from one and two metres above the ground. Thus, Jane's representation is not consistent with the behaviour of a ball rebounding from ground level nor is it internally consistent because Jane has it rebounding from two different heights. In this task, Jane realised that if the ball was caught at one metre then ground level could not be one metre. However Jane may not have recognised her error if the ball had been caught at two metres. Both Damien and Jane's difficulties can be interpreted as a lack of measurement sense though either failure to identify ground level as zero metres, or the limited everyday knowledge that a ball rebounds from ground level and rebounds from the same height on each successive bounce. A lack of measurement sense was not restricted to difficulties with "ground level", further difficulties and errors were apparent in other students' diagrams.

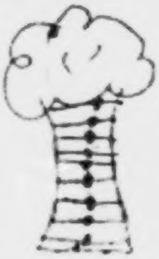
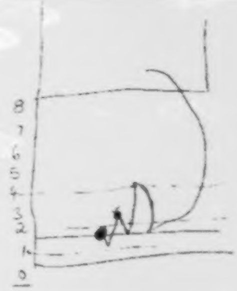
The Koala	The Bouncing Ball
	
Damien: He starts here. [Pointing to a **line at the base of the tree**] Researcher: He starts on that mark. What mark is that? What number? Damien: The first one. Researcher: The first what? Damien: The first part of the tree. Researcher: Okay. If you told me in metres how many metres would it be? Damien: **One metre.**	Jane: He caught it when it was ... [referring to the problem]...one metre. Researcher: One metre. So where is one metre? Jane: Away from...well, **there** is the ground [indicating the "1" on the diagram]. That's one metre away from the ground. No! **Zero's the ground..oh!**

Figure 2. A lack of measurement sense.

A Lack of Spatial Sense

Spatial sense is critical in the representation of information on diagrams (Battista, 1999). However students have a tendency to lack precision in their representation of position on a diagram. For example, although Helen understood the forward and backward motion of the koala, she did not keep track of the koala's precise location on the diagram (see Figure 3, Helen). Consequently, when Helen had completed the koala's movements she was unsure of its exact finishing position. Casey also had difficulty with spatial representation. While Casey accurately represented the pathway of the bouncing ball, she was unsure about what was represented by her lowest horizontal line (see Figure 3, Casey). Casey's uppermost line depicted the balcony and her series of shorter horizontal lines represented the metres from ground level to the balcony. Her inability to identify the lowest horizontal line as representing ground level is a particular concern given the difficulties that students experienced with "ground level" (see Figure 2). As diagrams capitalise on the use of graphics to structure and represent information spatially, the student who produces the diagram should understand each graphic component of a self-generated diagram. Thus, despite the differences in Helen's and Casey's difficulties, their diagrams both revealed a lack of spatial sense.

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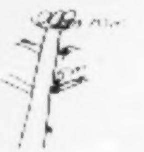
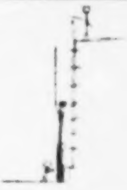
The Koala	The Bouncing Ball
	
Helen: He had to he climbed up another five [metres] and then he slept again and the second day when he climbed up —I mean the third day he climbed up to the top. Researcher: How do you know he climbed up to the top are you sure or could he have been just a bit lower? Helen: **He might have been a bit lower.**	Researcher: What's this line at the bottom? Casey: It's just **the finish of the thing**. It's just the —I don't know. I just —we just do it in class.

Figure 3. A lack of spatial sense.

A Lack of Number Sense

Students' diagrams also demonstrated a variety of numerically-based errors. The two errors that are shown in Figure 4 relate to confusion over concepts that are opposites. Kate's and Ellen 's errors respectively were in confusing "upper and lower", and "halving and doubling". While Kate recognised that 10 metres was a height constraint in The Koala problem, she failed to realise that 10 metres was the *upper* height constraint of the tree (the point the koala climbed to) rather than the *lower* height constraint (the point to which the koala slid back). This is considered to be a numerical error rather than a measurement error because Kate's measurements of the koala's movement were accurate. Ellen's error was in confusing halving with doubling. The Bouncing Ball problem states that on each bounce the ball travels "half as high as on the previous bounce" (see Figure 1), however Ellen doubled numbers commencing with "eight" when she should have been halving numbers. Ellen made two further numerical errors. First, she made a calculation error in doubling 64 to reach 127. Second, once Ellen had reached 127 she stopped doubling for no apparent reason. There is no correspondence between her answer of 127 and the finishing height of the ball of one metre. Both Kate's and Ellen's errors highlight the importance of sound number sense and the interrelationship among number, measurement, visualisation and language in problem representation.

In summary, the findings of this cross-study comparison revealed that a lack of understanding of specific mathematical features of problems results in similar difficulties being detected in the diagrams of feature-similar problems. Although each of the errors made by the students in the examples presented was to some extent unique, these difficulties are unified by a lack of sense-making about mathematics in measurement, space or number as the students attempted to portray particular problem information on a diagram. Given, the findings of cross-study regularities of difficulties and errors for network tasks, a search for difficulties and errors in feature-similar tasks for the other three general purpose diagrams will be followed up.

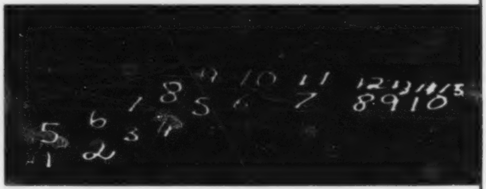
The Koala	The Bouncing Ball
	
Kate: And then he would climb up to number eight and down to four. And then he'd climb back up to number nine and slide back down to five and then he'd climb up to ten and slide back to six and then he'd climb up to **eleven.** I'd have to make the tree again. Eleven and slides back down to seven and then up to **twelve** and back to eight and then climb up there [13] and he'd slide back down to …	Ellen: Well, each time I'm – I'm sort of bouncing it, **I'm trying to halve the amount** that it's going down at …. Ellen: [Draws closer and closer segments on the diagram and writes numbers 1, 2, 4, 8, 16, 32 and 64] 127 [answer] Researcher: So you're saying the ball bounced 127 times before he caught it. Ellen: Mmm. Researcher: Can you tell me how you worked that out? Ellen: Well, you have the one and then it bounces two times and then it bounces four times and you see **it's just sort of doubling** each time.

Figure 4. A lack of number sense.

Conclusion

Students' difficulties and errors in generating accurate and effective diagrams are generally associated with students' lack of expertise in diagrammatic representation (e.g., Dreyfus & Eisenberg, 1990). However the results of this cross-study comparison suggests that effective diagrammatic representation also depends on a sound mathematical knowledge base, which includes sense-making in mathematical situations.

The findings of this cross-study comparison have four key implications for problem solving instruction and future research on diagrams. First, knowing students' errors and difficulties in generating diagrams is an important component of effective instruction in diagram generation. Second, instruction should provide opportunities for the explication of graphic components of the diagram and the relationships depicted by them. Third, students' diagrams provide an insight into the strengths and weaknesses of their mathematical knowledge. Fourth, though diagrams can support the conceptualisation of a problem, they cannot substitute for a lack of basic mathematical knowledge. Thus, diagrams should be considered as both representations that stimulate reflection on the problem structure and reflections of students' mathematical knowledge.

The goal of mathematics education is to produce numerate citizens for the 21st century who have access to mathematics, who are able to reason analytically, and who can make informed decisions (NCTM, 1998). Hence, students' lack of sense-making in mathematics

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with representations, such as diagrams, is a particular cause for concern in an increasingly "data-drenched" and technological society (Steen, 1997):

> As information becomes even more quantitative and as society relies increasingly on computers and the data they produce, an innumerate citizen of today is as vulnerable as the illiterate peasant in Gutenberg's time. (p. xv)

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Recognizing and Realizing "Best Practice" in Initial Mathematics Teacher Education and Classroom Teaching

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This research paper describes a longitudinal study which tracked a cohort of seven student teachers through the secondary mathematics method component of a one year, full-time, preservice university-based teacher education course and then as beginning teachers into their first year of teaching in the Western Cape region of South Africa. Qualitative data in the form of interviews, students' reflective journals and other students' texts as well as lecture and classroom observation were collected over a two-year period. The study found that in constituting their teaching repertoires, beginning secondary mathematics teachers recontextualized certain discrete tasks introduced on the mathematics method course, as well as a professional argot, a way of discussing the teaching and learning of mathematics. There appeared to be a disjuncture between what student teachers were taught on the preservice course, and the way in which they practiced in mathematics classrooms. There also appeared to be a disjuncture between what teachers said about their classroom practice, and the way in which they actually taught. An explanation for this seeming hiatus emerges from a sociological model developed from the work of Basil Bernstein and Paul Dowling and turns centrally on the extent to which student teachers are given access to the recognition and realization rules of "best" practice.

Introduction

Why is it so often the case that teachers do not implement in their classrooms, or at least do not seem to implement, the practices they acquire on teacher education courses? This has been variously attributed to educational biography (for example Lortie's (1975) 'apprenticeship of observation'), school setting (see for example Zeichner and Tabachnik (1981)), differential engagement by students with their teacher education courses (see for example Lacey (1977)) or a failure to change student teachers' and teachers' belief systems (Cooney 1985, see also Ensor, 1998). Less commonly, the seeming disjuncture is related to the structuring of the preservice teacher education course (see for example, Borko et al (1992) and Eisenhart et al (1993).

This issue is an important one, since the entire rationale for teacher education rests upon its ability to prepare students as classroom teachers. In the South African context most particularly, where the educational system is in so many respects dysfunctional, the manner in which teacher education might be used to change teachers' classroom practice has particular saliency. The professional development of teachers, through both preservice and inservice teacher education programs, is regarded as a central pivot for educational transformation.

Any teacher education course, whether preservice or inservice, elaborates what I would call a *privileged repertoire*. Such a repertoire is privileged because it involves a particular selection and combination of mathematics for the production of pedagogic tasks, a particular selection of pedagogic resources to facilitate this and the arrangement of these tasks into sequences as lessons. The privileged repertoire also includes features of classroom arrangement, the regulation of teacher-pupil communication and the deployment of appropriate forms of assessment. The task of inservice and preservice providers is to make available to teachers this privileged repertoire, this particular embodiment of "best practice". How this is configured clearly varies according to the particular teacher education program on offer.

In some cases, it may be the intention of teacher educators simply to make available to student teachers and/or teachers a varied collection of resources from which they can select at will. Hatton (1989, 1997) refers to this as *bricolage*. Such resources might include exemplary mathematical tasks, particular ways of organizing classroom communication and so forth. Or, teacher educators might want to make available to teachers not only this "smorgasbord" of resources, but also the principles for selecting one set of resources over others, producing new and different resources consistent with their particular, privileged view of teaching, as well as the principles for evaluating classroom practice. The question must then be asked, how are these principles made available to students (or in the case of inservice education, qualified teachers) and to what extent do teachers draw on these practices in developing their teaching repertoires?

The Study

These questions provided the focus of a two-year longitudinal study (see Ensor, 1999a, 1999b, 1999c) that tracked a cohort of seven students through the mathematics method component of a preservice Higher Diploma in Education (HDE) program and then as beginning secondary mathematics teachers into schools. The aim of the study was to provide a theoretical account of the recontextualizing of pedagogic practices by beginning teachers from this mathematics method course. By '*pedagogic practice*' I mean any practice, whether utterance or action, which is implicated in the transmission of a pedagogic discourse, such as for example school mathematics, and which forms part of a teaching repertoire. The privileged repertoire of teacher education is constituted by pedagogic practices, as are the repertoires of beginning teachers. Recontextualizing is a notion drawn from Bernstein (1977, 1990, 1996) and Dowling (1996, 1998), and points to the transformation of discourses as they are disembedded from one social context and inserted into others.

The study was stimulated by a broad theoretical interest in the articulation between sites of practice (such as signaled in the work, for example, of Rogoff and Lave (1984), Carraher et al (1985), Lave (1988), Noss and Hoyles (1996), Walkerdine (1988) and others). It was concerned with how privileged forms of knowledge and practice about teaching were made available to student teachers and the extent to which these were used by beginning teachers in classrooms. Drawing on the work of Basil Bernstein (1977, 1990, 1996) and Paul Dowling (1996, 1998), a model was developed to provide an analysis of a mathematics preservice teacher education course, of secondary mathematics classroom teaching and the recontextualizing of practices between them.

A variety of data was collected for the purposes of the study. In the first year, I took field notes of sessions of the mathematics method course and interviewed the two teacher educators responsible for it. I also collected a range of student teacher texts, such as reflective journals, examinations, a curriculum project and conducted interviews with students. In the second year of the study, I interviewed each of the group of seven teachers four times over the year, and in conjunction with the third set of interviews, video recorded a number of lessons with each teacher on a specific day. I also interviewed the head of the mathematics department in each school, or, where there was no head, a senior mathematics teacher.

On the mathematics method course, students were exposed *inter alia* to a range of exemplary mathematics tasks, research in mathematics education as well as an approach to mathematics teaching that favored visualization and intuition as a gateway into formal, conceptual mathematics. "Relational" understanding was privileged over "instrumental" understanding. Students were given access to a range of resources: teaching resources such as geoboards and geostrips, the use of history in the teaching of mathematics, ideas for organizing classrooms as well as exposure to issues such as racism and sexism. This privileged approach to teaching was made available through a range of exemplary mathematical tasks, and through explicit discussion. All of the teaching took place in the university context, however, and student teachers did not watch their teacher educators teach in classrooms, nor, whilst on teaching practice, did they gain the opportunity to put their own practices up for evaluation by mathematics specialists. In terms of the way in which the teacher education course as a whole was structured, students were supervised on teaching practice by lecturers who were not necessarily mathematics education specialists.

Over the period of the method course, students were given access to a *professional argot*, a way of talking about mathematics teaching which privileged, for example, "visualization", "verbalization", "relational over instructional understanding" and learners "discovering things for themselves". A professional argot is a species of speech genre (Bakhtin, 1986), a register (Halliday, 1978) of terms and modes of argument used by members of a profession when either engaged in it or discussing it. What aspects of the argot are foregrounded or backgrounded and the level of specificity of the language used, depend on the evoking context. Different features are likely to be foregrounded when discussing with a colleague than with a layperson. A professional teaching argot comprises part of the privileged repertoire, and provides the student teacher with access to a vocabulary and modes of argument to describe "best practice". What is significant about the particular preservice course of my study is that this argot was elaborated independently of reference to actual classrooms, and thus embodied potential ambiguity about what practices would, for example, constitute "relational" and "instrumental" understanding.

Use of such an argot can be illustrated in the following extract from an interview with a student teacher, Thabo Monyoko, towards the end of the mathematics method course:

> Since the beginning of the academic year I think my whole perception of what mathematics is and importantly how it is supposed to be taught has undergone a complete revolution (Saul changed Paul on the road to Damascus). Previously the impression I had of maths was that it is a theoretical subject having nothing to do with some practical, experiential activities. In this respect I think through the whole lot of activities we did during our lessons, I have developed a different view of how maths can/should be taught, which is indeed a marked departure from traditional method of teaching, the "mug and jug" method.

In his teaching practice journal and in an interview at the end of the HDE year, Thabo spoke positively again about the mathematics method course, which he described as having effected a "complete revolution" in his thinking about teaching. He said he had been exposed to a new approach in terms of which "pupils come to discover some of the things on their own [...]. They actually see how some of the things they do in mathematics is practical and some of the things they discover on their own." He no longer "monopolized classroom activities", "standing in front like a priest", simply giving the formula "raw from the book" so that "people have got to ram it into their heads [..] In the past I would simply give the formula from the book and give them an exercise and they apply the formula, that's all". On his second teaching practice he said he tried "to implement some of, you know, the hands on approaches [..] I remember I implemented some of these self-discovery approaches by pupils, I mean they were very fantastic, they were very interesting to the pupils and I think my lessons went pretty well, you know."

These ideas were re-iterated by Thabo when I interviewed him as a beginning teacher the following year. He indicated that the mathematics method course had "turned me around" in that he was now more "responsive to students' needs" and interacted with them more instead of "teaching from the front". For him, this meant walking around the class and "finding out what pupils were having difficulties with." This he related directly to lessons which I observed him teach. The following is an extract from the beginning of such a lesson with Grade 12 students.

> T: Let's say that the sixth term of a geometric sequence is 3125 and the fourth term is 125. Now find the eighth term. [as he speaks, he reads from a textbook and writes on the board:]
>
> 6th = 3125
>
> 4th = 125 Find 8th
>
> [He repeats again, given that the 6th term is 3125 and the 4th term is 125, find the 8th term]
>
> Now because you don't know the value of a, now remember in a geometric sequence the general term is
>
> $T_k = ar^{k-1}$ [writes this on the board and speaks as he writes]
>
> Now in order to find any term of a geometric sequence you must first find the value of a and the value of r. When you have a and r you can then find out the term. Now given that the sixth term is 3125 we can write
>
> $T_6 = ar^5 = 3125$
>
> Now let's call this our equation 1 [he writes 1 and circles it after the equation given above]
>
> Now the fourth term is 125. Therefore
>
> $T_4 = ar^3$ which is equal to [he turns his head to the class and cocks his head in expectation of a response. Someone says 125] 125 . OK we'll call this equation 2

Now we have two simultaneous equations with two unknowns. We don't know the value of a and we don't know the value of r. We have two variables which are unknown so we must find the value of a and r. Now in solving simultaneous equations you must first do away with one variable. In this case here we want to do away with a so we are left only with r. Once we are left only with r we can be able to find a. Now in finding the value of r we divide our equation number 1 by our equation number 2. We divide the big r by the small r. [He goes on to explain that "you can divide equation number 2 by equation number 1" but this gives rise to a negative exponent which makes calculation more complicated so they should "take the big r and divide by the small r"] (Thabo: extract from transcript of recorded lesson 2)

Thabo continued his explanation from the board, turning at one point to ask a student to check a calculation. When he completed the solution to the problem, he turned to the class, and for the first time and ten minutes into the lesson, he asked students by name if they had any problems: "Mr. Nzo, Nyamende, Zola?" He then proceeded to solve another problem on the board. Again, students were required to listen and take notes. Occasionally he posed a question, normally to ask students to calculate for him. After his explanation, he chose a question from the textbook for students to try on their own and walked around the class, discussing with the students in Xhosa and English.

In discussing the lesson afterwards, Thabo pointed to his use of questions, and his circulation around the class towards the end of the lesson, as evidence of the practices he had acquired from the mathematics method course. For him there was no disjuncture between what he said about his practice and the way in which he actually taught. Yet, what Thabo said, both as a student and as a teacher, *appears* to be at variance with what he did in the classroom, an apparent variation which was evident across the interviews and classroom practices of all seven beginning teachers. All seven teachers used *discrete tasks* (exemplary mathematical tasks and pedagogic resources made available to them on the course) and a *professional argot* of varying range (a way of describing forms of classroom practice). Teachers deployed this argot, descriptions such as "verbalization", "visualization" and "self discovery", in ways consistent with their *own* practice. Thabo, for example, drew on the professional argot ("not teaching from the front", and facilitating "student interaction") to describe his classroom practice, which was in many ways at variance with the approach developed on the course. From the *viewpoint of the privileged repertoire*, it would seem that Thabo is saying one thing and doing another. Yet from Thabo's vantage point, this is not necessarily the case. He has acquired a professional argot and turned it to his own purposes. The ambiguity associated with the transmission of the privileged repertoire made this possible, an issue which I will return to below.

All seven beginning teachers recontextualized from their HDE method course, discrete tasks and a professional argot. However, in interviews with teachers and in observation of their lessons, I found that they were not able to demonstrate access to the principles (or "rules") of selection, production or evaluation of the privileged repertoire. They said they could not produce tasks like those introduced on their mathematics method course, that is tasks which embedded its particular, privileged view of mathematics teaching, nor could they evaluate their practice in they way that the teacher educators might do. Putting this differently, I would suggest that these teachers had gained some access to *recognition rules* (they could describe aspects of "best practice" via the professional argot) but not *realization rules* (they were unable to produce tasks themselves which were consistent with the principles underpinning the privileged repertoire).

Recognition and Realization Rules

Bernstein (1990) distinguishes between recognition and realization rules in the following way:

> Recognition rules create the means of distinguishing between and so *recognizing* the speciality that constitutes a context, and realization rules regulate the creation and production of specialized relationships internal to that context. (Bernstein (1990), p. 15, emphasis in original)

Bernstein elaborates these rules as part of his code theory. The present analysis is differently motivated methodologically but the distinction Bernstein makes can be related to the present project. Here we are interested in the extent to which student teachers are given access to both recognition and realization rules: how a privileged repertoire is to be recognized and how it is to be realized in practice. In the sense in which I am using these terms here, access to recognition and realization rules enables appropriate use of a professional argot as well as its realization in practice. Access to recognition rules alone provides students with the ability to differentiate "best" from "poor" practice and a professional argot that enables acquirers to describe "best practice" discursively. However, since this argot was not demonstrated or illustrated in the site of application, it is potentially ambiguous with regard to the classroom practices it indexes.

The ambiguity referred to above stems from the particular structuring of teacher education as a form of knowledge. Teacher education can be regarded as a hybrid of explicit, discursive practices and implicit, tacit practices. Discursive practices exhibit what Dowling (1998) refers to as high *discursive saturation* (DS+), which means that they are relatively context independent and can be realized to a high degree in language. Tacit practices are those which exhibit low discursive saturation (DS-), which are more craft, context-dependent activities and which cannot be grasped fully in language. Mathematics teacher education can be thought of as making available a form of "best classroom practice", a privileged pedagogic repertoire, which comprises both explicit DS+ and tacit DS- elements. This means that the principles of selection, production and evaluation (which comprise what I call, following Dowling, an *esoteric domain*) can be made available explicitly, through language, but not exclusively so. To become a teacher, one needs to watch teachers teach, teach oneself and open one's efforts up for evaluation. Just as the crucial aspect for the transmission of discursive (DS+) practices is that the generative principles are made explicit in language, the crucial aspect for the transmission of tacit (DS-) practices is that they are transmitted and *acquired in the site of practice*, through demonstration and correction. It is not sufficient to speak to student teachers or teachers *about* "best practice"; one has to show them what this means, in actual classrooms, and allow them to put their own practice up for evaluation. This provides the basis for acquiring recognition and realization rules.

In the case of my study, students were exposed to the privileged repertoire, described in part above, exclusively in a university setting. As I have said, because of the way the HDE as a whole was structured, students did not observe mathematics teacher educators teach in classrooms, nor teach in classrooms themselves and have their performance evaluated by mathematics teacher educators. This means that while students were able to gain some access to discursive aspects of the privileged repertoire in the university setting, the more tacit, implicit aspects, those which required elaboration and exemplification in classrooms, were not made available. This invested the professional argot with considerable ambiguity, restricting access not only to realization but also recognition rules. This allowed teachers subsequently to use it to describe practices that were in many ways quite different to those privileged on the preservice method course.

Conclusion

I have suggested that teacher education and classroom teaching are hybrid activities, incorporating both explicit, discursive aspects as well as tacit, non-discursive aspects. The implication of this for pedagogy is that teacher education courses need to incorporate explicit discussion of privileged forms of teaching, as well as provide opportunities for these to be "tried out" in classrooms.

In my study, which for reasons of space I cannot discuss fully here, I suggested that school settings produced constraints on teachers in terms of how they developed their teaching repertoires, how they taught, and the extent to which they drew on their preservice course in doing this. School settings, however, did not seem to constrain teachers in a simple or obvious way, and did not appear to be decisive in shaping the recontextualizing of pedagogic practices for the teachers of my sample. A further factor which antecedent literature suggested might be important in shaping the development of teaching repertoires, that of educational biography, was also not decisive. What seemed overwhelmingly to affect recontextualizing was access to recognition and realization rules. Access to recognition and realization rules expands the potential of tasks and approaches that can be drawn on and provides the possibility for the production of new tasks and pedagogic choices. Lack of access restricts the recontextualizing potential to tasks actually encountered by students on the initial teacher education course and a professional argot which teachers deployed selectively, and not always appropriately, to describe their own practice.

I began this paper with a question about why teachers appear to practice differently in their classrooms from the ways in which they speak about them, and from the ways privileged on the preservice and inservice courses they attend. I have attempted to offer a possible reason for this in the particular structuring of teacher education discourse and its modes of transmission. Insofar as teacher education occurs exclusively in a site removed from that of classroom practice, it is not possible to make available fully the principles, the recognition and realization rules, that generate any particular privileged repertoire. Instead, the latter will stand as a reservoir of resources for potential, selective recruitment by teachers in forming their individual teaching repertoires. Insofar as they turn a professional argot to describe their own practices, they might well seem to be doing other than what they say, and what teacher educators advocate.

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Students' Visualisations of Three-dimensional Shapes

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This paper reports a cross-sectional pilot study of 30 students from Grades 1, 3 and 5, that investigated students' visualisations and representations of three-dimensional shapes. The study describes how students focused on critical and non-critical aspects of three-dimensional shapes and whether any differences exist between students' visual images, verbal descriptions and drawn representations. An interview-based assessment showed that although the quality of some students' visualisations may improve with grade level, non-mathematical qualities feature strongly in most students' visualisations.

In recent years, an increased emphasis on spatial concepts in the primary school mathematics curriculum has aimed at improving students' spatial development and understanding of geometric ideas (Board of Studies NSW, 1999). This change has reflected widespread concern over secondary school students' lack of understanding of key geometric concepts as well as acknowledgement of the intuitive spatial abilities of young children (Owens, 1994). However, there has been insufficient research into the teaching and learning of geometry and a perceived lack of a theoretical framework to underpin this (Pegg & Davey, 1998). More research is needed to improve teachers' knowledge and understanding of students' visualisation abilities particularly because visualisation is essential for both spatial concept development and spatial problem solving.

Research on Students' Visualisation Abilities

Several researchers have focused on the ways students use mental imagery, or visualisation, in dealing with three-dimensional objects (Battista & Clements, 1991; 1996; Fennema & Tartre, 1985; Owens, 1994, 1999; Pegg, 1997; Presmeg, 1986). Owens (1994) showed that visual imagery was important in young students' noticing features of shapes and in deciding how shapes could be used. This helped students to form geometric concepts.

Fennema and Tartre (1985) found that students who differed in spatial visualisation skills did not differ in their ability to find correct problem solutions, but they concluded that an emphasis on spatial visualisation skills will improve mathematics learning (Fennema & Tartre, 1985, p. 203). Presmeg (1986) discovered that while visual imagery did assist many students in solving problems, visualisers could experience some disadvantages, such as one-case concreteness of image or diagram, inflexible thinking, vagueness with concepts or difficulty in communicating concepts.

Battista and Clements (1991) studied students' responses when using LOGO to draw geometric figures and noted that spatial visualisation may be a more important factor in geometric problem solving for students who are at the visual level (in terms of van Hiele's hierarchy) than at the higher levels of geometric thinking. However, they pointed out that "visual imagery, when properly developed, can make a substantial contribution at all levels of geometric thinking" (Battista & Clements, 1991, p. 20). In a later study, Battista and Clements (1996) found that some students had considerable difficulty in dealing with 3D cube arrays due to their inability to visualise and enumerate the cubes in such arrays. They indicated the importance of further research into the nature of visualisation, particularly visualisation of three-dimensional shapes.

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Theories on the Development of Spatial Concepts

Theories about how students develop spatial concepts highlight the importance of the visualisation process in dealing with both two-dimensional and three-dimensional shapes. Visualisation of objects is an important mathematical process which is linked to growth in mathematical development. Davis, Tall, and Thomas (1997) outline some of the conceptual development theories that deal with the transition from *process to object* in mathematical thinking (Davis, 1984; Dubinsky, 1991; Gray & Tall, 1994; Greeno, 1983; Piaget & Inhelder, 1971). Davis, Tall, and Thomas (1997) assert that although the various *stage* theories about spatial development do not necessarily correspond, a commonality can be observed.

> Each student passes through a development of growing sophistication from some kind of process/procedure, usually performed step-by-step, and ending with an object that can be manipulated as an entity in its own right (Davis, Tall, & Thomas, 1997, p. 134).

Recent work by Gray, Pitta, and Tall (1997) has shown that students with poor visual skills may focus on non-mathematical aspects of shapes and this may inhibit effective learning of geometric ideas. Their related research on elementary number development suggested that children interpret information from the world as objects with various properties. Thus the particular *object*, of which a student chooses to create an image, affects the quality of the image formed. The present study investigates Gray, Pitta, and Tall's assertion by examining the quality of students' visualisations, and by describing which aspects students focus on when they form images of three-dimensional shapes. It is critical to ascertain whether young students focus on mathematical properties such as the six congruent faces of a cube.

Battista (1994) supports the notion of investigating mathematical properties of objects through mental models. He refers to Kosslyn's assertion that "one of the most striking things about objects in images is how they mimic properties of real objects" (Kosslyn, 1983, quoted in Battista, 1994, p. 91).

The van Hiele (1986) theory comprising five differentiated levels of geometric thinking distinguished how students operated on shapes and other geometric configurations as visual wholes. However, the students did not explicitly focus on geometric properties. Pegg and Davey (1989) concluded that a great number of students between Year 3 and Year 7 describe a geometrical figure in terms of one property, namely, the number/equality of sides. They also found that "the ability to draw correct diagrams stems from images that students possess and often these images do not reflect student understandings in terms of the properties of a given figure" (Pegg & Davey, 1989, p. 25).

The early development of geometric concepts is based on the images students create, thus it is critical to examine the way these images are formed and used. It appears that in order to better understand and assist young students with the development of spatial concepts, there is a need for further investigation of students' understanding of the properties of three-dimensional shapes. If it can be ascertained what students choose to focus on when analysing shapes, then teaching programs can be developed to improve students' attention to critical aspects.

This study aims to address three key research questions:

1. How well do students visualise three-dimensional objects?
2. In their visualisations, do students focus on critical or non-critical aspects of three-dimensional objects? Are these aspects mathematical properties?

3. What are the differences, if any, between students' visual images, verbal descriptions and drawn representations of three-dimensional objects?

Method

A cross-sectional descriptive study was considered appropriate so that students' visual imagery could be examined as closely as possible.

Table 1
Assessment Tasks

Task 1: Visualise a three-dimensional shape

I want you to think about a cereal box, for example, a cornflakes or rice bubbles box. Tell me all you can about the shape of this box.

Task 2: Identify similar shapes

Can you think of any other things or shapes in the real world that are the same shape as this cereal box shape? Why are they the same?

Task 3: Name the mathematical shape visualised

Do you know the mathematical name of this cereal box shape?

Task 4: Draw the visualised shape

Can you draw this cereal box shape for me? Can you explain your drawing to me?

Task 5: Describe a (held) shape

I've got a real cereal box here. You can pick it up and turn it around if you want to. Now can you describe the shape of this box to me? (If the description was quite different from the original visualisation, the investigator said "You said 4 sides before, and now you have told me there are 6 sides. Why did you say 4 sides before? How was the picture in your mind different from the real box?")

Task 6: Identify shapes needed to make up into three-dimensional shape

Here are some cardboard shapes. If you wanted to stick some of these shapes together to make one cereal box, which shapes would you need? Hand them to me.

Task 7: Identify net of a shape

This is the shape of a cornflakes' box flattened out. Now if you cut out these shapes (paper with nets A- E was shown) and folded them up along the dotted lines, which ones could you make into a small cornflakes box shape?

Task 8 : Describe a blank (held)shape

Now look at this box (child was shown a muesli bar box which had been folded inside out so that all faces appear blank to avoid distraction. The student was allowed to handle the box for a few seconds. Then it was taken from view). Now describe the shape of the box.

An interview-based assessment of students' understanding and visualisations of three-dimensional shapes included eight tasks (see Table 1) adapted from instruments used in prior studies (Battista & Clements, 1996; Shaughnessy, 1999, in correspondence). The tasks also reflected sample activities from the NSW Mathematics K-6 syllabus (NSW Department of Education, 1989).

A sample of 30 students, ten from Year 1, nine from Year 3 students and eleven from Year 5 was drawn from a NSW Department of Education and Training school in a lower socio-economic area of Sydney. Interviews were administered on a one-to-one basis by the chief investigator. During the interview the students were given the opportunity to represent their visualisations in a variety of ways before and after they had observed and manipulated actual shapes. Responses were recorded on an audiotape and students' drawings and explanations were retained for later analysis. Solution methods were coded for correct, incorrect, or non-response before being analysed for key mathematical aspects. A coding scheme was devised and later revised after analysis had been complete. Coding of responses was supervised and re-coded by an independent coder. Classification of responses reflected 80% agreement between coders for the purpose of initial pilot analysis.

Discussion of Results

Interview transcripts were analysed and responses classified according to (i) student performance, (ii) the mathematical or non-mathematical aspects of the responses and (iii) differences between drawn, visualised and verbal descriptions.

Table 2

Percentage of Students' Responses by Category and Year Level for Tasks 1-8

	Year 1 n = 12	Year 3 n = 11	Year 5 n = 11
TASK 1: visualise three-dimensional shape			
described shape using non-math props only	17%	0%	0%
described shape using non-math and math props	83%	80%	73%
described shape using math props only	0%	20%	27%
unable to name any maths props correctly	92%	60%	9%
name one prop correctly	8%	30%	27%
name two props correctly	0%	10%	55%
name three props correctly	0%	0%	9%
made incorrect estimate of either faces, corners or edges	50%	70%	36%
TASK 2: identify other things with the same shape			
unable to name anything with similar shape	33%	30%	18%
named one other thing with the same shape	58%	40%	45%
named more than one thing with the same shape	8%	30%	36%
TASK 3: name the mathematical shape visualised			
gave correct math name for shape	0%	20%	36%
TASK 4: draw visualised shape			
drew shape as 2D drawing	80%	30%	9%
drew shape as poor 3D drawing	20%	45%	36%
drew shape as a good 3D drawing	0%	25%	27%
TASK 5: describe a held shape			
described shape using non-math props only	17%	0%	18%
described shape using non-math and math props	83%	90%	82%
described shape using math props only	0%	10%	0%
unable to name any maths props correctly	75%	10%	9%
name one prop correctly	25%	70%	55%
name two props correctly	0%	20%	27%
name three props correctly	0%	0%	9%
made incorrect estimate of either faces, corners or edges	8%	50%	45%
TASK 6: identify shapes needed to make up into 3-D shape			
chose 6 correct shapes	0%	30%	36%
chose 6 shapes but incorrect ones	8%	20%	36%
chose 4 shapes only	33%	30%	18%
chose other incorrect combination of shapes	58%	20%	9%
TASK 7: identify net of shape			
identify correct nets	0%	10%	0%

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TASK 8: describe "blank" held shape			
described shape using non-math props only	8%	0%	0%
described shape using non-math and math props	92%	100%	91%
described shape using math props only	0%	0%	9%
unable to name any math props correctly	83%	30%	9%
name one prop correctly	17%	50%	36%
name two props correctly	0%	20%	45%
name three props correctly	0%	0%	9%
made incorrect estimate of either faces, corners or edges	75%	70%	67%

The results of the analysis are shown in Table 2 above. In the discussion following, the research questions will be addressed in turn. Excerpts are used to elucidate some distinctions between students' responses.

Overall, students found difficulty in visualising three-dimensional objects with an accurate awareness of their mathematical properties. Table 2 shows that for Task 1 most children could visualise a three-dimensional object, but could not describe the three-dimensional aspects of it. Results from Task 2 showed that a number of students were unable to match their visualisation of a cereal box shape to any similar shape in the real world (33% in Year 1, 30% in Year 3 and 18% in Year 5). Some of the responses given by students were related to a two-dimensional rather than a three-dimensional image eg "a door", "a piece of paper".

Results for Task 3 indicate that most students were unable to give the visualised box shape its correct mathematical name and even those students who did name the shape correctly in Task 3, sometimes misnamed it in later tasks. It seems that, considerable confusion existed at all grades about the correct meaning of the terms *side, face, corner, edge, rectangle,* and *square* as well as *rectangular prism.*

Task 6 revealed that most students had difficulty selecting the cardboard shapes that would make up into a cereal box shape. Table 2 shows that only 36% of Year 5 and 30% of Year 3 students completed the task accurately. The large number of students (a total of 91% for Year 1, 50% for Year 3 and 27% for Year 5), who choose either four shapes only or any other incorrect combination of shapes (i.e., not six shapes) reinforces results of Task 1 where students were unable to describe the number of faces of the cereal box shape. Although Task 6 was completed after the students had handled a cereal box in Task 5, it required a good deal of visualisation ability to imagine the cardboard shapes, which were laid flat on the table, built up and stuck together as a single three-dimensional shape. It appears that the visualisation ability necessary to complete this task had not yet developed for the majority of students.

Task 7, in which students were asked to examine five possible nets of a cereal box shape and identify the correct nets, was poorly answered by all students. This task required more developed visualisation ability than for Task 6 as the students had to imagine all the faces folded up without the opportunity to handle the individual faces.

2. In their visualisations, do students focus on critical or non-critical aspects of three-dimensional objects? Are these aspects mathematical properties?

The results showed that non-mathematical aspects featured strongly in students' responses across grade levels. Overall, a majority of students used a combination of non-mathematical and mathematical properties. Table 2 shows that for Task 1, 17% of Year 1 students described the shape using non-mathematical properties only. These students talked about the colour, the illustrations and the texture of the cereal box shape, but did not use mathematical terms such as *faces, vertices* or *edges.* For example: Mark (Year 1): "It has pictures on it, it's fat and sort of hard and it's got red and white writing on it".

A further 83% of Year 1 students, 80% of Year 3 and 73% of Year 5 students described the shape using a combination of non-mathematical and mathematical properties. However, 20% of Year 3 and 27% of Year 5 were able to describe the shape using mathematical terms only, even though at times the mathematical descriptions were not always correct. For example: Toa (Year 5): "It's a rectangle and it's got 6 faces and 4 sides."

Similar results appear for Task 5, when the students were asked to describe the shape of a cereal box they were given to hold. Again most students used a combination of non-mathematical and mathematical properties in their descriptions (83% in Year 1, 90% in Year 3, and 82% in Year 5). The attention to non-mathematical properties in Tasks 1 and 5 continued despite the researcher's prompt, "Is there anything else you can tell me about the shape of the box" (with emphasis on the word *shape)*. Similarly in Task 8, despite the removal of stimuli, students continued to mention the colour, design and texture of the shape as well as some mathematical properties. For example: Amber (Year 1): "It's brown and it feels rough and it has sticky tape on it and it has four corners and it's a square".

Responses to Tasks 1, 5, and 8 which described the shape using mathematical terms only were low (0% Year 1, 20% Year 3 and 27% Year 5 respectively). In their visualisation for Task 1, most Year 1 students (92%) were unable to name correctly any mathematical properties of the shape. With the box held in their hands, 75% of these Year 1 students were still unable to name correctly any mathematical properties of the shape. Does this imply that their visualisation of the shape was incorrect, or does it reflect their unfamiliarity with correct mathematical terms and properties? Similarly, in Year 3, 60% were unable to name any mathematical properties in describing their initial visualisation. However, with the shape held in their hands for Task 5, only 10% were still unable to name any properties. It seems likely that by Year 3 these students had had greater exposure to teaching about the mathematical properties of rectangular prisms and this teaching was reflected in their answers when the shape was held. The fact that these properties are not mentioned in their descriptions of their visualisations seems to indicate that their visualisations of the box shape may indeed be flawed.

By Year 5, only 9% of students were unable to name any mathematical properties both in their description of their visualisation of the box (Task 1) and their description of the held shape (Task 5). However, it cannot be assumed that visualisation ability has improved for all students. This may be the case for some, although 36% of these Year 5 students were still making an incorrect estimate of the number of either faces, or vertices, or edges in their description of their visualisation.

3. What are the differences, if any, between students' visual images, verbal descriptions and drawn representations of three-dimensional objects?

In general, it seemed that there was no relationship between students' visualisations, their verbal descriptions and their ability to draw a three-dimensional shape. Indeed there may be considerable differences between students' abilities on these three aspects. Results for Task 4 showed that students' ability to produce a drawing that was a good representation of a three-dimensional shape improved markedly with grade level. Most of the Year 1 drawings consisted of a simple two-dimensional shape, often including illustrations or an attempt at words. By Year 3, 65% of students were producing a drawing that indicated the shape had a three-dimensional quality, although most of them were still including illustrations or words. By Year 5, only 9% of students, compared with 80% of Year 1 students were still drawing the cereal box they had visualised as a simple two-dimensional rectangle or square. Most drawings of Year 5 students indicated that the shape had a three-dimensional quality. Whether it was rated a poor or good three-dimensional drawing reflected only the students' individual drawing skills. However the ability to draw a good three-dimensional figure of a cereal box shape did not necessarily mean that the student had an accurate visualisation of that same cereal box shape. For example, Amanda (Year 5) was able to draw a fairly good representation of a cereal box shape indicating its three-dimensional nature. Yet in her description of her visualisation, Amanda said: "It's a rectangle at the front and back and on the side it has strip things. The top opens and it has a plastic packet with cereal inside and there's writing on it." Amanda's language, while containing mathematical terms, is far from accurate. Jeremy (Year 5), on the other hand, drew a simple two-dimensional representation, but his visualisation was more accurate: "It's like a 3D shape. Sometimes it has a bowl of cornflakes on it and writing and other stuff. It's got 6 faces and 8 corners." It would appear therefore that the accuracy of drawing a three-dimensional shape which a student has just visualised does not necessarily reflect the student's visualisation ability.

Implications

This study has highlighted the difficulties that many young students may experience in visualising three-dimensional shapes. Gray, Pitta, and Tall (1997) indicated the need to examine the quality of the object which dominates the child's visualisation. This study supports their findings that non-mathematical or non-critical qualities appear to dominate, or at least feature strongly, in students' visualisations of objects at the expense of mathematical properties. It also appears that the quality of some students' visualisations may improve with grade level, but that students may remain focused on non-mathematical or non-critical aspects of shapes. The study found that students drawings of the visualised shapes also may, or may not, reflect the description of the visualisation. In addition, students' descriptions of their visualisations of three-dimensional shapes do not necessarily match with their description of the same shape after handling it. This relates to Owen's (1999) notion that verbal expressions are associated with visual imagery and help to define it. Owens found that as students group and regroup shapes, they refine their images of these shapes.

It may be advantageous to analyse students' visualisations of shapes according to qualitative differences in their attention to mathematical and non-mathematical properties. Levels could be assigned which may reflect the van Hiele (1986) levels of cognitive development, but this may produce a stage-like model that detracts from the rich and complex development of the individual. Southwell (1997) also discusses the use of the van Hiele levels of geometric development either to establish a curriculum or to ascertain the level of understanding at which students are operating. Southwell suggests that both aspects may be possible, but when using van Hiele levels as an assessment instrument, difficulties may arise when a student operates at different levels for different aspects of spatial thinking. Pegg and Davey (1998) highlight the importance of "identifying individual paths of development: to seek out variability" (p. 133) rather than using van Hiele's unidimensional descriptive model of learning. The present study indicates that students' visualisation ability is fundamentally formed at an early level where imagery plays a critical role. Investigating students' use of imagery might then be considered an essential element of any overall assessment of spatial development. The study also confirms Battista and Clements' (1996) suggestion of the importance of further research into the nature of visualisation, particularly the visualisation of three-dimensional shapes.

Owens (1999) developed a framework which showed how visualisation strategies are central to the development of spatial thinking. Further research needs to use a framework as a basis for designing a teaching program aimed at improving students' visualisation ability of three-dimensional shapes. This research could assist in establishing how practical activities could improve visualisation skills. Such activities might involve handling and describing three-dimensional shapes, comparing and contrasting the properties of such shapes, folding and unfolding nets to make up three-dimensional shapes and reproducing shapes from memory. Further study is warranted to examine the relationship between improving students' use of correct names and properties of three-dimensional shapes, and their visualisation ability. Classroom studies are needed to ascertain whether improved teaching and assessment might lead to a much closer match between students' visual images, verbal descriptions and drawn representations of three-dimensional images.

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Gender, Early Encouragement, and Survival in Mathematics Studies

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Tertiary mathematics students reported on the encouragement they perceived that parents and teachers gave them to continue in their mathematics studies. Women had perceived stronger encouragement than men, and correlations between early encouragement and university achievement were stronger for women. Interviews confirmed the importance of encouragement for the women, and indicated that their confidence could be shaken by stereotyped views about gender. Men's interviews indicated that, for them, the greatest social danger to their studies was the anti-intellectual peer group.

Introduction

For some considerable time, there have been widespread efforts to encourage girls not to opt out of participation in mathematics studies, from the secondary school level onwards. There is still, however, clear evidence from Europe (for example the report edited by Alting, Brand, Groenendal, Hermanussen, van Vonderen, & Weyers, 1992), the United States (Bae & Smith, 1997) and Australia (Malone, de Laeter, & Dekkers, 1993) that gender imbalance in enrolment is still very strong. At university level, continuing gender imbalance in enrolment is obvious. Past investigations of survival factors among women doing very high levels of university mathematics have often found early encouragement important (Maines, Wallace, & Hardesty, 1983). The more recent wide ranging review of research made by Bae and Smith (1997) found that encouragement at home and at school still relates strongly to enrolment in mathematics courses. More qualitative work such as that of McSheffrey (1992) indicates similar patterns of influence among students who are not necessarily highly selected. One might therefore assume that early encouragement is a survival factor for groups of women at all university levels. That is, it seems reasonable to hypothesise that a high proportion of female university mathematics students would have received stronger encouragement at home and at school than their male counterparts did, precisely because mathematics is a field more traditional for males. Students' perceptions of parents' and teachers' attitudes on the stereotyping of mathematics as a male domain could also be relevant.

The problem about investigating such effects is that it is difficult to sort out whether higher enrolment in mathematics studies, and higher encouragement on the way, are not both purely the result of a student's existing record of good performance. One can, however, make a valid comparison between female and male university students using statistical control for past performance as reflected in entrance qualifications. Similarly, assessment of the importance of the perceived attitudes of parents and teachers would have to depend on finding links between students' achievement and these attitudes, which, although not establishing causality, do provide a *prima facie* case, especially if they go beyond school achievement. Ideally, evidence for influences going beyond school achievement would require links with university achievement to be still perceptible after control for school achievement. This is a stringent requirement, particularly for mathematics teachers' influence, so closely bound to school results. It follows that links that exist before control should also be given some attention, as well as overall patterns of relationships between different achievement and attitudinal variables. Multiple approaches, including qualitative work, have the potential to be a rewarding supplement to quantitative work. The current project therefore uses questionnaires about home and school experience, intended for quantitative analyses, but includes also a set of interviews with a small number of the students from the survey sample, to investigate possible those aspects of students' experience that were most salient to them, and to allow open ended answers to reveal factors not foreseen in the questionnaire design.

Method

Questionnaire Section

The questionnaire sample consisted of 203 women and 523 men in first year mathematics classes at a large Australian university. Some of the calculations are based on slightly smaller numbers, because of missing scores, but numbers are always very close to being maximal. Students were selected as a half-sample of all tutorial classes at each possible time, to ensure a representative sample, because distribution into tutorial classes at a given time is random, but distribution of students into time slots reflects enrolment in different degree courses, and hence reflects selection and motivational factors.

Questionnaire data were collected in two successive years, in class time, using five-point scales adapted from the Fennema-Sherman Attitudes to Mathematics Scales (Fennema & Sherman, 1976). High scores indicate favourable attitudes. The scales covered perceived encouragement from parents and school teachers, and perceptions of their attitudes on the stereotyping issue. Achievement scores came from the New South Wales Higher School Certificate (HSC) results and university examination marks. The HSC scores were separated into scores obtained in mathematics, and scores obtained in other subjects (HSC non-mathematics scores), converted into percentages. University examination scores were used to construct a common score over three different levels of first year mathematics. The marks were pieced together as a single distribution, using performance on common examination material, together with university rules for transfers between subject levels, and conversion of scores for quota selection, to connect the three levels.

Comparisons between women and men were made using analyses of covariance, with HSC mathematics scores as a covariate. Analyses were originally factorial, with year of collection as the second factor, as a control check. Since almost no significant results involving the second factor emerged, results involving gender comparisons only have been extracted for presentation here. Correlations, partial correlations controlled for HSC mathematics, and block canonical correlations between vectors of attitude and achievement scores were calculated on samples maximal with respect to having all scores.

Interview Section

A sample of 22 women and 25 men took part in the interviews. The sample was selected randomly, subject to approximate coverage of degree types and inclusion of high and low achievers. Individual semi-structured interviews were conducted by one of the research team. Material was tape recorded, with the students' consent. Questions were about home and school experience in connection with the students' mathematics studies.

Results

Questionnaire Section

The comparative results of the students' perceptions of their parents' are in Table 1. All were significantly in favour of women, beyond control for HSC mathematics, indicating a helpful environment for women with respect to both encouragement and perceptions of stereotyping.

Table 1

Perceived Parents' Attitudes

	Means		
	Women	Men	Significance
Perceived encouragement: mother	3.87	3.70	0.002
Perceived encouragement: father	3.95	3.80	0.008
Perceived stereotyping: mother	4.19	3.67	0.001
Perceived stereotyping: father	3.85	3.45	.001

Correlation coefficients, and partial correlation coefficients controlled for school achievement, between students' mathematics achievement and students' perceptions of their parents' attitudes, are in Table 2. Those for encouragement are all significantly positive, but the only one to survive control is mother's encouragement of women. The stereotyping scales have zero correlations for women, while those for men tend to be weak, but negatively significant.

Table 2

Parents' Attitudes: Correlations and Partial Correlations with Achievement

	Women			Men		
	HSC Maths	Year1 University Maths		HSC Maths	Year 1 University Maths	
		Initial	Controlled		Initial	Controlled
Encouragement: Mother	0.39*	0.29**	0.13*	0.19**	0.09	-0.03
Encouragement: Father	0.17**	0.17**	0.12	0.25**	0.13	-0.05
Stereotyping: Mother	0.18*	0.09	-0.02	-0.15**	-0.17**	0.12
Stereotyping: Father	0.15*	0.12	0.04	-0.08*	-0.11**	-0.10*

Note. * p < 0 .05; ** p < 0.01.

Canonical correlations with achievement are in Table 3. They give no surprising information, confirming the greater importance of the same sex parent, and indicating that the main achievement link is with school mathematics. The men's relatively high link between mother's perceived acceptance of stereotyping and achievement is also consonant with the pattern of single correlations. But the main advantage of the pairing of correlated vectors is in the larger correlation coefficients arising from the combination.

Table 3

Students' Perceptions of Parents' Attitudes: Canonical Correlations with Achievement

Variables	Women	Men
HSC non-maths score	0.55	0.10
HSC maths score	-0.86	-1.01
Year 1 university maths	-0.32	-0.06
Encouragement: mother	-1.13	-0.35
Encouragement: father	0.30	-0.63
Stereotyping: mother	0.11	0.64
Stereotyping: father	-0.17	-0.10
Canonical correlation	0.44**	0.32**

Note. ** $p < 0.01$.

Results of comparisons of students' perceptions of their school teachers' attitudes are in Table 4. Here too women perceived their teachers' attitudes as more favourable than the men did, and the difference went beyond control for HSC mathematics achievement. Scores for men are reasonably high in absolute, so that one need not regard them as perceiving any absolute lack of encouragement or high levels of stereotyping.

Table 4

Students' Perceptions of their School teachers' Attitudes

	Means		
	Women	Men	Significance
Perceived encouragement	4.06	3.96	0.064
Perceived stereotyping	4.29	3.75	0.001

Correlations and partial correlations with school teachers' attitudes are in Table 5. Positive correlations and zero partial correlations with school teachers' encouragement are roughly what one might expect, given that the school teachers' influence on university mathematics achievement is so closely bound to their influence on school achievement. Perceived rejection of stereotyping has a positively significant link with school achievement for women, and a weak negatively significant controlled link with university achievement for men.

Table 5

Students' Perceptions of School Teachers' Attitudes: Correlations and Partial Correlations with Achievement

	Women			Men		
	HSC Maths	Year1 University Maths		HSC Maths	Year 1 University Maths	
		Initial	Controlled		Initial	Controlled
Encouragement	0.40**	0.21**	-0.01	0.34**	0.26**	0.01
Stereotyping	0.22**	0.04	-0.14	-0.01	-0.07	-0.12**

Note. **$p < 0.01$.

Canonical correlations involving school teachers' attitudes are in Table 6. First pairings for both men and women tend to confirm the information obtainable from single correlations. The second pairing for women, though of lesser importance, indicates that some women who do well in university mathematics tend to be uncertain about their school teachers' rejection of stereotyping. The direction of the next largest coefficient, for HSC non mathematics scores links them with higher stereotyping, as is also the case in the men's second pairing, which does not involve mathematics at all. It is possible that these links reflect characteristics of the secondary schools attended by students, which the interview material clarifies below.

Table 6

Students' Perceptions of Parents' Attitudes: Canonical Correlations with Achievement

Variables	Coefficients			
	Women		Men	
	1	2	1	2
HSC non-maths score	-0.19	0.84	0.18	1.01
HSC maths score	1.11	0.40	0.83	0.06
Year 1 university maths	-0.09	-1.02	0.08	0.06
Encouragement	0.92	-0.62	1.00	-0.02
Stereotyping	0.17	1.09	-0.07	1.00
Canonical correlation	0.38**	0.19*	0.39**	0.15*

Note. *$p < 0.05$; **$p < 0.01$.

Interviews

Women and men alike had assumed for some time while still at school that they would be going on to tertiary education (77% women, 76% men). Women more than men had chosen university mathematics for interest and expectation of doing well (54%, 19%), whereas men were more career oriented (27%, 64%). But one should bear in mind that more men do degrees oriented to a specific occupation, in which mathematics is compulsory, so that it might seem inconsistent to them to say they had chosen mathematics.

Encouragement from parents about doing mathematics was reported more often by women than by men (54%, 32%). In some cases, parents' encouragement of women had been crucial in the overcoming of prejudice. Several female engineering students had had special encouragement from their fathers, including discussion of experiences of gender based stereotyping.

> "I went to a Catholic school, and they were sort of against girls doing science and maths ... but Dad was all for it." (Marie S, Chemical Engineering)
> "Dad was overjoyed. ... My father is always helping me." (Teresa M, Chemical Engineering)

Encouragement from school mathematics teachers was also reported much more frequently by women than by men (82%, 40%). Some women thought they had been well treated because they had female teachers, or because they went to a single sex school, but the sample included all types of school. Discouragement of any student by mathematics teachers was very rare, but the men tended to say they had been neither encouraged nor discouraged. About a quarter of the women had encountered gender based stereotyping of subjects by science teachers, the worst case being a man who stated he did not want any girls in his physics class (the victim, who was a high achiever in mathematics, failed Physics 1 at university). Thus, in this sample, school science was the clearer barrier.

The egalitarian, anti-intellectual peer group was mentioned as a hazard to their own progress more often by men (32% women, 54% men), but more women than men described the protective value of a studious friendship group (55%, 32%). Both sexes (27%, 32%) showed awareness of acceptable rituals to overcome prejudice against high achievers (mainly attribution of success to hard work, and enthusiastic social participation), but women (32%) also mentioned being thought strange because they were interested in mathematics, not a specific issue for the men. One should add that, unlike the graduate students interviewed by Maines et al. (1983), those who mentioned stereotyping tended to show hurt rather than boredom with the issue.

Discussion

Results of comparisons all go beyond differences predictable from school achievement, indicating that the gender factor is more likely to be a real discriminator. Encouragement scores may imply upward selection for the university group or greater encouragement of females in general, but either way they indicate that encouragement is associated with women's survival into university mathematics. Perceptions of rejection of stereotyping are a different type of factor, because the issue is not symmetrical for males and females, and the difference probably indicates that it is not a dead issue, while the high absolute scores are compatible with it not having strong implications for women.

The links between encouragement and achievement give some supporting evidence for the argument that the women are favoured. It is, however, clear from the controlled links that their going beyond school achievement is uncertain, because the only significant controlled link with university achievement is for mothers' encouragement of females. First pairings of canonical variables give no different information about dominant patterns. Combining encouragement and stereotyping, therefore, one may assume the women are a favoured group. This may mean that, in general, they are more vulnerable, but also indicates directions for intervention. Second pairings of canonical variables are likely to reflect consonance between school types, subject choices and overall high achievement. The argument is that many of the most successful secondary schools are selective or private, and often single sex. They may therefore specialise more in subjects more traditional for each sex, so that students whose scores are high in subjects other than mathematics might tend to perceive teachers' attitudes as overall more traditional. The relatively small size of the canonical correlation coefficient, compared with those for the first pairs of canonical variables, means that it seem reasonable to concentrate on the first pairs. In the interview material, it seems very clear that women perceived more explicit encouragement from parents and mathematics teachers than men. Given that it has been more frequent to find more favourable treatment of males (from earlier observational studies, such as that of Becker, 1981, through to the work reviewed by Bae & Smith, 1997), or well-meant underestimation that has deleterious results (Walden & Walkerdine, 1986) or no differences in perceived encouragement (as in the now classic studies by Fennema and Sherman ,1977, 1978; and Armstrong, 1979), one may conclude that the present group of women was to some extent favoured. The finding that science teachers are more of an obstacle needs following up. It is also clear that the peer group is seen as a hazard, involving the stereotyping issue for women. Nevertheless, the more acute perception of anti-intellectualism by men indicates that each sex has special difficulties, and it is hard to say one is in a more favourable position than the other with respect to peer group influences.

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Factors Affecting Mathematics Achievement in Primary and Secondary schools: Results from TIMSS

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A previous paper has shown that in the Third International Mathematics and Science Study (TIMSS) teacher effects contribute little to student achievement in mathematics, but that classroom differences are important. The present study used data from TIMSS to examine classroom and school differences in maths achievement at secondary school level to find out which classroom and school variables affect achievement. It found that the pooling of student resources associated with grouping has a large effect on mathematics achievement at primary level, while at secondary level policies regarding pupil management such as grouping according to ability have the greatest effect on mathematics achievement.

Introduction

The Third International Mathematics and Science Study (TIMSS) provides a rich data set for investigating the effects of student, classroom, and school-level variables on student achievement in mathematics. A previous paper (Lamb & Fullarton, 2000) has shown that while student background variables are important for achievement in mathematics, classroom and school variables also contribute substantially.

Schools don't Matter ... or do they?

The early literature on school effectiveness placed an emphasis on the ability and social backgrounds of students in identifying the factors that shape academic performance, and suggested that schools had little direct effect on student achievement. Coleman, Campbell, Hobson, Portland, Mood, Weinfeld & York (1966), for example, in a major study of US schools seemed to cast doubt on the possibility of improving school achievement through reforms to schools. They found that differences in school achievement reflected variations in family background, and the family backgrounds of student peers, concluding that "schools bring little influence to bear on a child's achievement that is independent of his background and general social context" (Coleman et al., 1966, p.325). A later analysis of the same data set by Jencks and his colleagues reached the same conclusion, "our research suggests ... that the character of a school's output depends largely on a single input, namely the characteristics of the entering children. Everything else—the school budget, its policies, the characteristics of the teachers—is either secondary or completely irrelevant" (Jencks, Smith, Acland, Bane, Cohen, Gintis, Heyns, & Michelson, 1972, p. 256). However, the methodology employed in this early work did not take account of the hierarchical nature of the data, and was not able to separate out school, student and classroom factors. The importance of recognising this structure was noted by Raudenbush & Willms (1991):

> An irony in the history of quantitative studies of schooling has been the failure of researchers' analytic models to reflect adequately the social organisation of life in classrooms and schools. The experiences that children share within school settings and the effects of these experiences on their development might be seen as the basic material of educational research; yet until recently, few studies have explicitly taken into account of the effects of particular classrooms and schools in which students and teachers share membership. (p. xi).

More recent school effectiveness research has used multi-level modelling techniques to account for the clustering effects of different types of data. The results of such studies show, according to the meta-analysis of school effectiveness research undertaken by Bosker &

Witziers (1996), that school effects account for approximately eight to ten per cent of the variation in student achievement, and that the effects are greater for mathematics than for language. It has also been found that classrooms as well as schools are important and that teacher and classroom variables account for more variance than school variables (Scheerens, 1993; Scheerens, Vermeulen, & Pelgrum, 1989). But several studies also show that there are substantial variations between schools (Mortimore, Sammons, Stoll, Lewis, & Ecob, 1988; Nuttall, Goldstein, Prosser, & Rasbash, 1989; Smith & Tomlinson, 1989; Lamb, 1997) .

Other studies have shown that contextual variables such as student body composition and organisational policies play an important role in mathematics achievement. Teacher background attributes such as gender, number of years teaching and educational qualifications have been shown to be important factors in student achievement (Anderson, Ryan, & Shapiro, 1989; Larkin & Keeves, 1984), as have a variety of school effects such as school size (Lee & Smith, 1997) and mean student social composition.

These studies suggest that classrooms and schools matter, as well as student background. A range of studies has examined different effects, however few have been able to utilise the range of contextual variables available in the TIMSS data set. This paper uses the TIMSS data to investigate the interrelationships among different factors at the student, classroom and school levels.

Variables Explored

The main aim of this analysis of the TIMSS data was to explore the relationships between student achievement in mathematics and factors at the student, classroom and school levels for both the primary and secondary school populations. Table 1 provides details of the variables that were used in the analysis.

Student Background Variables

The sex of each student was recorded, as well as the number of people living in the student's household and the number of books in the home. This latter measure is an indication of the cultural background of the student. A variable representing socioeconomic status (SES) was computed as a weighted composite comprising the higher of mother's or father's occupational status, the higher of mother's and father's level of education (secondary students only), and the number of possessions in the home[1]. Ethnicity was measured as a weighted composite variable based on student's birthplace, the birthplaces of their parents and the primary language spoken at home.

Student Mediating Variables

Students completed a word knowledge test (Thorndike, 1973) as a measure of their prior verbal ability. This test presents the student with a number of word pairs and requires the student to decide whether the words in each pair are (near) synonyms or (near) antonyms. The test was used in both the First and Second International Science Studies, and the importance of language skills has been shown to be important for achievement in mathematics. A composite variable was derived to represent the student's enjoyment of mathematics. This variable consisted of positive responses to five attitude prompts: 'I usually do well in mathematics', 'I like mathematics', 'I enjoy learning mathematics', 'Mathematics is boring', and 'Mathematics is an easy subject'. A further variable was computed for secondary students only to represent student's perceptions of the importance of mathematics. This variable was comprised of responses to the items 'Mathematics is important to everyone's life', 'I would like a job involving mathematics', 'I need to do well in mathematics to get the job I want', 'I need to do well in mathematics to please my parent(s)', 'I need to do well in mathematics to get into the university/post-school course I prefer', and 'I need to do well in mathematics to please myself'. These items were not given to primary students

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[1] Number of possessions was measured by asking students which of the following possessions were in their homes: calculator, computer, dictionary, separate desk/table for study, personal bookshelves and books, separate wardrobe, dishwasher, cd or video player in their own room.

Classroom and Grade-level Variables

A number of classroom variables were collected or derived for this analysis. This included the student's year level and the stream or level of the class if setting was a practice used in the school to organise mathematics classes. Mean word knowledge scores and socioeconomic status were derived at the class level. Teacher background attributes—gender, age, number of years teaching and educational qualifications—were also controlled for. Scales representing teacher attitudes and beliefs about mathematics and teaching were developed using factor analysis. These scales are described in greater detail in a forthcoming TIMSS publication, and are summarised in Table 1. There were 6 subscales derived for primary teachers and five for secondary teachers.

Table 1
Student, Classroom and School Variables

	Population 1	Population 2
STUDENT LEVEL		
Student background variables:		
Sex	Student's gender	Student's gender
Books	Number of books in student's home	Number of books in student's home
Family size	Number of people living in student's home	Number of people living in student's home
Ethnicity	A composite of student's birthplace, birthplace of parents and language spoken at home	A composite of student's birthplace, birthplace of parents and language spoken at home
Socioeconomic status	A composite variable representing family wealth	A composite variable representing family wealth
Student mediating variables:		
Word knowledge	Verbal ability as measured by the word knowledge test	Verbal ability as measured by the word knowledge test
Attitude to mathematics	A composite variable reflecting the student's attitude towards mathematics.	A composite variable reflecting the student's attitude towards mathematics.
Importance of maths		A composite variable reflecting the perceived importance of mathematics to the student.
CLASSROOM LEVEL		
Grade-level and classroom composition variables:		
Year level	Grade or year level of class	Grade or year level of class
Mean word knowledge	Average score on Word Knowledge test for the class	Average score on Word Knowledge test for the class
Mean SES	Average SES for the class	Average SES for the class
High band		Whether student is in one of the two the top streams in mathematics
No band		Whether classroom is non-set or non-streamed
Classroom teacher variables:		
Age	Teacher's age	
Gender	Teacher's gender	Teacher's gender
Education. qualifications	Teacher's qualifications	Teacher's qualifications
Years teaching		Number of years teaching

	Population 1	Population 2
Classroom teacher variables:		
Factor 1	Problem-solving approach to teaching	Problem-solving approach to teaching
Factor 2	Discipline oriented approach to teaching	Discipline oriented approach to teaching
Factor 3	Process oriented approach to teaching	Process oriented approach to teaching
Factor 4	Eclectic approach to teaching	Eclectic approach to teaching
Factor 5	Algorithmic approach to teaching	Teacher satisfaction with job
Factor 6	Teacher satisfaction with job	
SCHOOL LEVEL		
School variables		
School size		Number of students in the school
Policy to stream		Whether the school has an explicit streaming policy
Mean SES	Average SES for the school	Average SES for the school

School Level Variables

Mean socioeconomic status scores were derived for each school to provide a control for the social composition of the school. For secondary schools only, a measure of the school size was used ranging from schools of less than 250 students through to schools of more than 1250 students. Explicit school policy to stream in mathematics was also included in the analysis.

Method

Hierarchical linear modelling (HLM) was used to look at the interrelationships between factors at the student, classroom and school levels. This procedure allows modelling of outcomes at several levels (e.g. student level, classroom level, school level), partitioning separately the variance and effects at each level while controlling for the variance across levels. To examine the effects of different variables on mathematics achievement, several models were tested each adding successively a new group or layer of variables. The first model included the group of student background variables comprising sex, socioeconomic status, family size, ethnicity and number of books in the home. The second model added a set of mediating variables to the set of student background factors. The mediating variables included results on a standardised word-knowledge test, and attitudes towards maths. The third model contained grade- or year-level and the set of classroom composition variables relating to mean word-knowledge score, and mean socioeconomic status (SES). The next model added the set of teacher variables including the sex of the teacher, age, qualifications, years of teaching, and scores on the scales related to teachers' attitudes and practices in mathematics teaching. The final model added the school-level variables.

Results

Table 2 presents the results for Population 1 (primary school students) and Table 3 presents the results for Population 2 (secondary school students). In the model which included student background variables, shown in the second column of Table 2 and the second column of Table 3, it can be seen that all of the variables, other than ethnicity, had a significant effect on achievement in mathematics for both primary and secondary students. Gender had a significant negative effect on mathematics achievement. That is, girls achievement levels were still not equal to that of boys, at either primary or secondary level. Also, consistent with previous studies, students from a higher SES background, those with more family cultural resources (as measured by books at home), and those from smaller families tended to have higher achievement levels in mathematics.

The student-level mediating variables—word knowledge, attitudes towards maths, perceived importance of mathematics—had strong independent effects. They were influential predictors of maths achievement. But they not only had independent effects, they also transmitted or relayed the effects of the different student background variables. This is evident from the marked drop in the sizes of the estimates for SES, books in the home and family size when the mediating variables were included in the model. It should be noted that the values for the variable *perceived importance of mathematics* for secondary students are bracketed in Table 3. The model did not include this variable because of collinearity problems with the variable, *positive attitudes towards mathematics*. When both were included, the effect was to make the estimate for importance negative. The bracketed estimates for the importance of maths measure were derived running the same model but excluding the measure for attitudes towards maths.

In summary, gender, number of books in the home, and socioeconomic status had both a direct effect on achievement and a transmitted effect through their influence on word knowledge and attitudes towards mathematics. These findings reinforce earlier studies showing that student background has an effect, both directly and indirectly, on student achievement in mathematics.

Our previous paper has shown that, as well as student-level factors, classrooms and schools also matter (Lamb & Fullarton, 2000). The next stages of the modelling investigated the effects of classroom variables on achievement. As classroom organisation is different at the primary and secondary school levels, these will be discussed separately.

Primary Students: Male Teachers Make a Difference

With the primary school population, several variables at the classroom level exerted strong effects. Year-level had a large influence on achievement. Achievement in Grade 5 classes was higher than achievement in grade 4 classes and higher again than grade 3 classes. In addition to year-level, the social composition of classes (as measured by mean SES) had a strong positive effect on achievement, as did average word knowledge. All else equal, the higher the mean SES of the class the higher the achievement. This also applied to the mean ability of the class as measured by word knowledge.

The addition of the next group of classroom-level factors—teacher attributes and teacher beliefs about mathematics and about teaching mathematics—had little impact. The only factor that was significant was teacher's sex. Neither years of teaching experience nor educational qualifications had a significant effect on mathematics achievement. However, a student with a male teacher would, according to the results, be advantaged in mathematics and the effect is significant. There are several possible explanations for this finding. One is, as suggested by previous research, that female primary teachers, more than male primary school teachers, lack confidence in their ability to teach mathematics. Another possible explanation is related to the types of male teachers who enter primary teaching. The data show that only 27 per cent of the primary teaching population was male, indicating that perhaps the males that do go into primary teaching are a more selected group of mathematics teachers or perhaps have more confidence in their ability than females. None of the teacher attitudes or beliefs had any effect on achievement, nor did school level socioeconomic status.

Table 2

HLM Estimates of Mathematics Achievement: Schools and Students, Population 1, TIMSS

	Level 1 model – student background variables	Level 1 model – student mediating variables	Level 2 model –classroom composition variables	Level 2 model – classroom teacher variables	Level 3 model –school variables
Intercept	530.5***	529.5***	530.1***	529.8***	529.7***
Student-level variables					
Background variables					
Female	-4.0**	-7.3***	-7.0***	-6.9***	-6.9***
Books in the home	8.8***	3.9***	3.9***	3.9***	3.9***
SES	12.1***	7.2***	6.7***	6.7***	6.7***
Ethnicity	0.33	0.1	0.1	0.1	0.1
Family size	-4.7***	-2.5***	-2.5***	-2.5***	-2.5
Mediating variables					
Word knowledge		41.1***	39.4***	39.4***	39.4***
Positive attitudes towards maths		5.0***	5.2***	5.2***	5.2***
Classroom-level variables					
Grade-level and classroom composition					
Year-level			28.8***	28.8***	29.3***
Mean word-knowledge			3.7***	3.6***	3.6***
Mean SES			13.1***	14.0***	11.7**
Teacher attributes					
Age of the teacher				1.0	1.0
Sex of the teacher				-10.0***	-10.0***
Educational qualifications				0.4	0.4
Attitudes towards maths					
Problem-solving approach to teaching				-0.4	-0.4
Discipline oriented approach to teaching				-0.4	-0.4
Process oriented approach to teaching				-0.2	-0.1
Eclectic approach to teaching				-0.3	-0.3
Algorithmic approach to teaching				0.3	0.4
Teacher satisfaction with job				0.1	0.2
School-level Variables					
Mean SES					3.5

Note. *$p < 0.10$; **$p < 0.05$; ***$p < 0.001$.

Secondary Students: Setting Makes the Difference

Table 3 shows that for the secondary school population setting or streaming had an impact on mathematics achievement. Compared to lower band or lower stream classes, there was a strong positive effect for classes in the top band in schools with setting or streaming policies, and a more modest effect for classes in schools with no such policies. Setting and streaming clearly benefited those students in the higher band classes, but led to significantly poorer achievement in lower band classes. The achievement in classes in the lower bands or streams was moderately, though significantly, lower than classes that were not streamed or set.

Year-level had a significant effect on achievement. Achievement in Year 9 classes was higher than for Year 8 and Year 7 classes. Classroom social composition (mean SES) also had a strong independent effect on student achievement in mathematics, as did classroom ability composition. These results show that the higher the mean SES composition of classes, the higher the achievement. Also, the higher the average verbal ability of classes, the higher the achievement. Thus, there are benefits for students to be in classes where they rub shoulders with other higher SES and higher ability students.

Table 3

HLM Estimates of Mathematics Achievement: Schools and Students, Population 2, TIMSS

	Level 1 model – student background variables	Level 1 model – student mediating variables	Level 2 model –classroom composition variables	Level 2 model – classroom teacher variables	Level 3 model –school variables
Intercept	523.0***	524.5***	525.5***	525.5***	525.5***
Student-level variables					
Background variables					
Female	-6.4***	-4.6***	-4.6***	-4.5***	-4.5***
Books in the home	10.1***	6.9***	7.0***	7.0***	7.0***
SES	7.0***	5.1***	4.6***	4.6***	4.6***
Ethnicity	-0.8	-0.9	-0.9	-0.9	-0.9
Family size	-1.9***	-1.0*	-0.9	-0.9	-0.9
Mediating variables					
Word knowledge		26.8***	25.2***	25.2***	25.2***
Positive attitudes towards maths		9.0***	9.1***	9.1***	9.1***
Perceived importance of maths		[1.7***]	[1.7***]	[1.7***]	[1.7***]
Classroom-level variables					
Grade-level and classroom composition					
Year-level			18.3***	18.2***	18.2***
Mean word-knowledge			7.0***	6.8***	6.8***
Mean SES			11.2***	11.2***	11.2***
High band			28.5***	28.5***	28.5***
No band			5.3*	5.3*	5.3*
Teacher attributes					
Sex of the teacher				3.5	3.2
Educational qualifications				-0.4	-0.3
Years in teaching				0.3**	0.3**
Attitudes towards maths					
Problem-solving approach to teaching				-0.2	-0.2
Discipline oriented approach to teaching				-0.2	-0.3
Process oriented approach to teaching				0.4	0.5
Eclectic approach to teaching				-0.2	-0.1
Teacher satisfaction with job				0.3	0.2
School -level Variables					
School size					1.4
Policy to stream in maths					-1.8
SES					-2.1

Note. *p < 0.10; **p < 0.05; ***p < 0.001.

Teacher Attributes

The next step in the analysis was to add the teacher attribute variables to the achievement models. Sex of the teacher and educational qualifications had no significant effect on student achievement. Years of teaching had a small but significant positive effect, suggesting that the more experienced teachers achieved better results. None of the teacher attitudes to mathematics were found to have any direct effect on student achievement.

School Variables

Neither school size nor mean school SES were found to have any direct or indirect effect on mathematics achievement, controlling for all other variables.

Discussion

What can we learn from the TIMSS data? One thing we learn is that differences between classes and schools matter. Early studies examining patterns of student achievement in mathematics had concluded that schools have little impact above and beyond student intake factors. The results from TIMSS show, consistent with current research on school effectiveness, that not only do schools make a difference, but classrooms as well. There are strong classroom effects and modest school effects on maths achievement. These effects are linked to particular classroom and school-level factors.

For primary schools, grade-level was a major factor. Classroom-composition variables also made a difference. The pooling of pupil resources that is associated with the grouping of students—reflected by mean SES and average verbal ability—influences mathematics achievement. Achievement was highest in those classes and schools with higher concentrations of students from middle class families and students with higher verbal ability. Therefore, the effects of residential segregation more broadly and school-level pupil management policies more locally shape the contexts within which differences in maths learning and achievement develop. Other factors also can play a role. Classes with male primary teachers have higher levels of achievement, a finding that needs further analysis to identify whether this might be related to differences in the scholastic backgrounds of male and female primary teachers or to differences in confidence in the teaching of maths or differences in maths-related teaching practices.

For secondary schools, class and school effects were also influential. Policies regarding pupil management are critical. Schools which formally group students according to maths achievement or ability promote differences in mathematics achievement. The benefits of this practice are large for students who enter higher band or top stream classes. They receive substantial gains in achievement. The cost is for those students in the lower band or stream classes. They had significantly lower levels of achievement compared to their top streamed peers and also their unstreamed peers. It suggests that the different learning environments created through selective pupil grouping work to inhibit student progress in the bottom streams and accelerate it for those in the top streams. Teachers matter less in this context. Their beliefs about mathematics and teaching mathematics, and feelings of self-worth as a teacher, had no significant effect on achievement, after controlling for other factors. The one exception was teaching experience, as measured by years of teaching. This factor did have a significant effect, suggesting that more experienced teachers promote higher achievement.

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Manipulator or Magician: Is there a Free Lunch?

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We report on research into outcomes that emerge when MAPLE software forms a central component of both the lecture presentation and laboratory activity in a first-year undergraduate mathematics course. We have noted an increase in both the number and type of questions asked by students compared with conventionally taught classes, and have identified factors associated with a symbolic manipulator environment that appear to link with task demand and student success. While enabling capable students to move faster and further, our evidence suggests that the use of such software does not compensate for, replace, nor render irrelevant, flaws in mathematical background.

Undergraduate mathematics courses in Australia have for some time been integrating symbolic algebra software into their teaching programs (Pemberton, 1997). This local reflection of an international trend, has received impetus from national reports such as Mathematical Sciences-Adding to Australia (NBEET, 1996). This report noted that the mathematical sciences are becoming increasingly laboratory based, with significant implications for how they will be taught. It recommended that mathematics departments re-design courses to make best use of this increased computer power which is heavily centred around the impact of symbolic manipulator packages, such as DERIVE, MATHEMATICA, and MAPLE.

Computer-Based Undergraduate Programs

The form of computer-based instruction varies widely. Olsen (1999) describes one of the most extensive examples of automated instruction. She describes how politicians visiting Virginia Tech's Mathematics Emporium, a 58 000 square-foot (1.5-acre) computer classroom:

> see a model of institutional productivity; a vision of the future in which machines handle many kinds of undergraduate teaching duties-and universities pay fewer professors to lecture…On weekdays from 9 am to midnight dozens of graduate-student and undergraduate helpers can be observed strolling along the hexagonal pods on which the emporium's computers sit. The helpers are trying to spot the students who are stuck on math problems and need help. (p. 31)

This program is openly driven by economic rationalism. At the other extreme Shneiderman, Borkowski, Alavi, and Norman (1998) describe a model, in which electronic classroom infrastructure is extensive and expensive, containing full computer and multi-media facilities as well as designer courseware. Courses are scheduled into the electronic classrooms on a competitive basis, and successful applicants must guarantee that the resources will be used as designed. It is required that full use be made of the interactive, collaborative, multi-media environment. Between such extremes lie a variety of models of instruction, whose users are concerned in varying degrees about factory production on the one hand, and student understanding on the other. Of those valuing the latter Alavi (1994) directly imported constructivist principles into computer-based learning by emphasising that learning is best accomplished by acquiring, generating, analysing, manipulating and structuring information. However Templer, Klug, and Gould, (1998) raised problems in this arena, that were perceived to arise as a direct result of a symbolic manipulator (MATHEMATICA) environment. They noted that having mastered the rudiments, the majority of students "began to hurtle through the work, hell bent on finishing everything in the shortest possible time". The following comment (or a close relative), was noted as occurring frequently: "I just don't understand what I'm learning here. I mean all I have to do is ask the machine to solve the problem and it's done. What have I learned?" Kent and Stevenson (1998) on a similar theme saw both positives and negatives, questioning first whether computer-controlled mathematical procedures can be learned effectively without appreciation of their place in mathematics. Their evidence and observation suggested that unless some kind of breakdown in the functionality of some concept or procedure (say integration) was provoked, the student would not focus on the essential aspects of that concept or procedure. However they noted that the demands for formal precision that a programming environment places on its user, serves both to expose any fragility in understanding, and to support the building and conjecturing required of the re (construction) of concepts by learners. Templer et.al., also saw the symbolic manipulator environment of MATHEMATICA helpful in this respect, to the extent that its language was sufficiently close to that of mathematics for the two to be treated in tandem. Similar claims could be made on behalf of MAPLE and DERIVE. We note similarities to observations from earlier work, where in noting gains in some performance measures, reference was also made to dangers of reifying a "black box syndrome" (Park, 1993).

The program that is our focus of interest, is a mainstream course located between the extremes referred to above. It represents a model that sits within present university structures and resources. Issues associated with its implementation connect with those raised in Kent and Stevenson (1998), and Templer, Klug, and Gould, (1998). Like the latter we are concerned with the links between computer-controlled processes and their mathematical underpinnings, noting the similarities and differences between the respective symbolism. With the former we share an interest in the range of questions raised by students as they work with the software, as well as in their performance. Many examples we have noted reinforce that the three-way interaction between students, technology, and mathematics, creates interactions that a focus on examination performance alone is unable to address.

The Study Context

The first-year undergraduate mathematics subject, forming the context of this study provides for a population of 650 students studying within Science and Engineering degree courses. The teaching comprises a lecture series complemented by weekly workshops, in which approximately 40 students are timetabled into a laboratory containing networked computers equipped with MAPLE software. The lecture room is fitted with computer display facilities so MAPLE processing is an integral and continuing part of the lecture presentation. To support their workshop activity students are provided with a teaching manual (Pemberton, 1997), continually updated to contain explanations of all MAPLE commands used in the course, together with many illustrative examples. During workshops two tutors and frequently the lecturer also, are available to assist the students working on tasks structured through the provision of weekly worksheets. The students can consult with the lecturer during limited additional office hours, and unscheduled additional access to the laboratory is available for approximately 5 hours per week. The course is also available on the Web a medium that is attracting increasing custom. Solutions to the weekly worksheets are provided subsequently. The formal assessment in the subject is constrained by departmental protocol to a pen and paper test at the end of semester (90%) supplemented with a MAPLE based assignment (10%). Consequently to succeed students must transfer their learning and expertise substantially from a software supported environment to written format. This means that they must be able to develop understanding through the medium with which they work, while simultaneously achieving independence from it - involving the ability to learn and maintain procedures that a MAPLE environment does not enforce. The educational implications of this characteristic need pursuing in their own right, but additionally attention is focused on the relationship between the mathematical demands of tasks, and their representation in a MAPLE learningscape. With this in mind the following questions emerge as requiring attention, noting that they remain central questions whatever the relationship between symbolic manipulators and assessment.

1. What is the range of student generated questions that emerge when learning of mathematical content interacts with a symbolic manipulator environment?
2. Can structural properties associated with the MAPLE environment be identified that link task demand and student success?

Data Sources

The data for addressing these questions come from two sources. Tutors assigned to the MAPLE workshops were provided with diaries in which they entered, on a weekly basis, examples indicative of the range of questions raised by students in the course of their workshop activity. The second source of data was a test given 7 weeks after the program started. This test was a voluntary exercise, and comprised a series of questions to be addressed with the assistance of MAPLE in its laboratory context. It was intended to provide formative feedback to the students on their performance, and ranged from simple school level manipulations to new material introduced in the tertiary program. Three sample questions are included in the appendix, together with their MAPLE solutions. As an additional incentive the test was directly relevant to preparing for the formal assessment at the end of semester, for the procedures required are ones that the students need to be proficient with, irrespective of software support. Additionally several questions contained an explanatory component, where the students were required to interpret the meaning of graphical output. Two sets of data were obtained from the tests, which were analysed and marked by two of the course tutors using criteria designed by the researchers. One of these involved the recording of correct and incorrect solutions, except that for this purpose the quality or indeed presence of a final interpretation of graphical output was not taken into account. This meant that the correct/incorrect dichotomy was on the basis of MAPLE operations only. The second set of data was obtained from an analysis of the errors that led to incomplete or incorrect answers. These are elaborated below. It was necessary to restrict the questions included in the analysis because poor results on later questions may be attributable to cursory attention imposed by time constraints. On the basis of a review of the 250 (approx.) scripts submitted, it appeared we could assume that the first 16 questions had been attempted seriously by the group as a whole. For technical reasons two of these were deemed unsuitable for inclusion, so that responses to 14 questions formed the final data set.

Results

Error Classification

A combined total of approximately 180 questions indicative of the range of concerns displayed by students when working in a MAPLE environment was assembled. These have been classified into categories in decreasing order of prevalence, as shown below in Table 1, together with the respective percentages. Clearly an element of informed judgment is present in selecting categories and assigning questions to them. The number of questions per category varies from a maximum of 30 (15.6%) to a minimum of 10 (5.2%).

Table 1
Student Question Types

Question Category	Percentage
1. How to get result or use particular operation (HGRO)	15.6 %
2. Identify problem caused by a typo (TYPO)	15. 6%
3. Resolve syntax error (SYNTAX)	15.1 %
4. General request for help (STUCK)	11.5 %
5. Interpret a technical term (ITECH)	8.9 %
6. Answer a procedural query (PROC)	8.3 %
7. Problem assigning variables (VAR)	7.3 %
8. Problem specifying function (SPEC)	6.8 %
9. Query result (QRES)	5.7 %
10. Query meaning of symbol (QSYM)	5.2 %

The error analysis from the test results generated a range of individual flaws (over 600 in total), which could be coarsely grouped into four main categories as shown in Table 2. Again these are judgment based with an element of subjectivity-they are essentially errors of commission. Errors of omission, as evidenced for example by failure to invoke appropriate commands, could not be so readily quantified. Ignoring the last category, of an omitted verbal explanation, which has no necessary connection with MAPLE facility, we may examine the others in terms of their role in this mathematical context. As far as operations with MAPLE are concerned we may identity two categories (at least) of facility. One of these is the requirement of accurate syntactical representation of common elementary operations, such as are represented in the first row in Table 2. The other is the more sophisticated and demanding selection and specification of functions to achieve identified mathematical ends. Clearly there is interaction between mathematical understanding and function specification, for if the former is flawed the wrong selection may be made or functions combined inappropriately. Alternatively if the mathematics is correct, the desired outcome can be defeated by mistakes in the technical detail of function specification. Considering jointly rows 2 and 3 in Table 2 covers this interdependence.

Table 2
Test Error Classification

Category Title	Percentage
1. Syntax related errors [*, ( ), ^ etc.]	31.1 %
2. Function choice and specification errors	17.3%
3. Errors in mathematics	17.3 %
4. Comment omitted or inadequate	34.3 %

Table 3
Regression

Regression statistics	
Multiple R	0.8710
R Square	0.7586
Adjusted R Square	0.7148
Standard Error	0.1419
Observations	14

Table 4
Analysis of Variance

	Df	SS	MS	F	Sig F
Regression	2	0.6957	0.3479	17.29	0.0004***
Residual	11	0.2213	0.0201		
Total	13	0.9171			

Table 5
Other Statistics

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.0947	0.0961	11.383	2E-07
SYNTAX	-0.0482	0.0168	-2.874	0.015*
FUNCTION	-0.0396	0.0122	-3.246	0.008**

Discussion

In reviewing the outcomes of our study we can relate to the comment of Ramsden (1997) that "the impact upon educational practice of powerful software…has been less profound than optimists hoped or pessimists feared". Almost all reports contain statements tempering enthusiasm with caution, or disappointment with optimism. A continuing challenge is articulated by Olsen (1999) following a description of the most extensive budget driven, automated, attempt at mass produced learning that we have so far identified.

> Instructional software issues are unlikely to be resolved quickly... If we want the software to help at all... it's got to understand how students might misconceive what is presented to them--and to figure that out from the student's response. And right now, only people do that well. (p. 35)

With respect to our first question the patterns evident in Table 1 reinforce that when students interact with mathematics through technology questions are generated rapidly, and their scope is vastly increased. We can identify at least four types of inquiry from the responses: Those that are simply procedural (what to do next); those that are mathematical in the traditional sense; those that are software related (syntax and symbols); and those generated by the interaction of mathematics with software (function choice and specification, interpretation of output). The intensity and scope of student questioning has ballooned in comparison with that in traditional practice classes, with software the major contributor through properties of fast processing and scope for formatting and specification errors, together with student initiative in exploring. Secondly, in examining the analysis relevant to our second question, we observe that while achieving more rapid and efficient closure to algorithmic procedures the use of MAPLE has not reduced the need for the mathematical attributes of understanding and attention to detail. We note this in the significant impact of the variables SYNTAX and FUNCTION on success rate. SYNTAX errors penalise those who lack sufficient care in expressing their work symbolically, while the demands imposed by FUNCTION are proportional to the principles and sophistication of the associated mathematics. On the other hand, for those students who possess conceptual understanding and due regard for precision, the MAPLE environment has provided a means to progress rapidly and successfully at a greater rate than could otherwise be achieved. So our conclusion to this point is that there is no free lunch (indeed laboratory tutors are lucky to get lunch at all). The propensity of students to alter their approach to reduce the learning potential available to them (Templer, Klug, & Gould, 1998) is apparent. It is hoped that as student performance is mapped more carefully, and lessons learned from their responses to both mathematical tasks and in teaching situations, new insights for teaching-learning options will be identified. New properties that emerge from the mutual interaction of students, mathematics, and technology can support new approaches extending beyond the models that thus far appear to have motivated many of the proponents of automated learning. Goals of doing faster and more cheaply that which was done formerly with blackboard, chalk, and paper are limited indeed.

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Appendix: Sample Questions

(Questions in italics: MAPLE commands in bold: MAPLE output in ordinary type)

Q2. Factorize $x^3 - 6x^2 + 11x - 6$

Maple Solution

```
➢ factor(x^3-6*x^2+11*x-6);
```

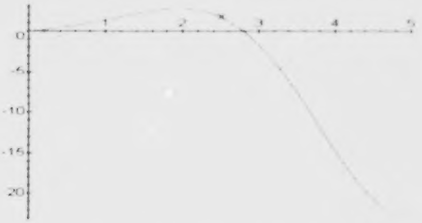
Q8. Find where the graph of $x^2 \sin x + x \cos x$ *for* $0 \le x \le 5$ *is :*

(a) above the x – axis (b) below the x – axis (c) cuts the x – axis.

```
(x - 1) (x - 2) (x - 3)
```

Maple Solution

```
> plot(x^2*sin(x)+x*cos(x),x=0..5);
```



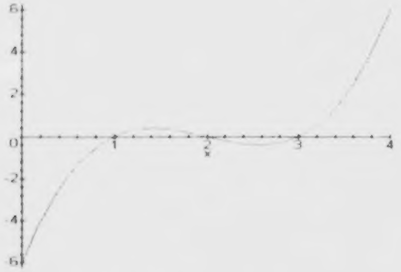
```
> x1:=fsolve(x^2*sin(x)+x*cos(x),x=2..3);
   x1 :=2.798386046
```

****

Q14. Plot the graph of f(x) = (x-1)(x-2)(x-3) and use this to find the physical area under the graph from x= 1 to x= 3.

Maple Solution

```
> y:=(x-1)*(x-2)*(x-3);
➢ plot(y,x=0..4);
```



```
➢ int(y,x=1..2)-int(y,x=2..3);
➢ 1/2
```

Using Growth Point Profiles to Identify Year 1 Students who are at Risk of not Learning School Mathematics Successfully

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It is important for teachers to identify children who are at risk of not learning mathematics successfully as early as possible in their schooling. This enables school communities to put in place strategies to assist these children before they lose confidence. This paper explores assessment interview data for 1667 Victorian Year 1 primary school students with a view to developing profiles that may be used to identify children who are at risk. These profiles are based on growth points within the Early Numeracy Research Project assessment framework.

An important challenge for school communities is providing mathematics programs and teaching that ensure that all students learn mathematics successfully. Partly, this challenge is about recognising and assisting the children who are at risk of not learning school mathematics successfully as early as possible, before they lose confidence. At present, there is considerable debate about how teachers may identify children in Prep to Year 2 who are at risk. Currently, teachers in Victoria compare children's mathematical progress to the broad standards stated in the Curriculum and Standards Framework (CSF, Board of Studies, 2000) and thus identify the students who are not meeting these standards. However, teachers have no finer measure to determine who are the students at greatest risk and who may benefit from specialised additional assistance in mathematics.

To enhance the mathematical learning of children in the early years of schooling, the Victorian Department of Education, Employment and Training (DEET) has launched a three-year (1999-2001) research project: the *Early Numeracy Research Project* (ENRP, Clarke, 1999). The aim of the ENRP is to identify those teacher, coordinator and school community characteristics that enhance numeracy learning in the early years of schooling (P-2). The project has as its basis the eight design elements used in the Early Literacy Research Project (Hill & Crevola, 1997). One of these elements is intervention and special assistance. The inclusion of this design element recognises the fact that for some children, learning mathematics successfully requires specialised assistance and support beyond that provided by the regular classroom program.

This paper presents a preliminary analysis of ENRP assessment interview data for 1667 Victorian Year 1 primary school students with a view to identifying students who are at risk of not learning school mathematics successfully, and developing profiles of students who may benefit from additional assistance in mathematics.

Identifying Students Who May Benefit from Additional Assistance

Two programs operating in Australia which provide specialised assistance for Year 1 children who are at risk of not learning mathematics successfully are *Mathematics Recovery* (Wright, Cowper, Stafford, Stanger, & Stewart, 1994), and *Mathematics Intervention* (Merrifield & Pearn, 1999). These programs use the Early Arithmetic Stages (Steffe, von Glasersfeld, Richards, & Cobb, 1983) to identify participants for the programs. For example, in a study of *Mathematics Recovery* participants and their counterparts (Wright et al., 1994), students were initially assessed as prenumerical if they had not attained Stage 3 of the Stages of Early Arithmetic Learning. That is, these students were not yet able to count-on to solve additive and missing addend tasks involving screened collections. In addition to the Early Arithmetic Stages, *Mathematics Recovery* assessment also considers children's facility with the forward and backward number word sequences, ability to identify numerals, and ability to subitise (Wright et al, 1994). Wright (1994) has described the stages that children attain in each of these areas. These stages have been used as the basis of the *Count Me In Too* program initiated by the NSW Department of Education (Stewart, Wright, & Gould, 1998), which, in addition to these areas, also discusses stages in the development of children's multiplicative thinking.

In order to identify the Year 1 students at risk of not learning mathematics successfully, *Mathematics Intervention* uses a clinical interview (Pearn, Merrifield, Mihalic, & Hunting, 1994) which is designed to allow young students to talk about their mathematical strategies. Merrifield & Pearn (1999) consider this to be a more effective method of obtaining information about children's own mathematical constructs and knowledge than traditional paper and pencil testing. The interview includes tasks that determine the facility of children's verbal counting skills, knowledge of the number word sequence, and tasks that would help ascertain their Early Arithmetic Stage. Merrifield & Pearn (1999) suggest a further stage that precedes the rest during which students recite the verbal number word sequence, either successfully or unsuccessfully, but do not seem to realise the purpose for counting. Children who participate in *Mathematics Intervention* are those who display difficulties with most tasks and are at Stage 0 or Stage 1 and use procedural strategies such as count-all. That is, they are children who have not yet reached the count-on stage. This strategy for identifying students at risk of not learning mathematics successfully aligns closely with that used in *Mathematics Recovery*.

It is important to note that the initial assessment for both the *Mathematics Recovery* Program and the *Mathematics Intervention* Program is undertaken by specialist teachers who have participated in extensive professional development which assists them to analyse assessment information and determine the Early Arithmetic Stages children have attained. In general, classroom teachers do not have knowledge of the Early Arithmetic Stages that may identify children who are at risk of not learning mathematics successfully. Two recent professional development programs which have provided classroom teachers with the opportunity to analyse their students' mathematical learning using the Early Arithmetic Stages are the NSW *Count Me In Too* program (Stewart et al, 1998), and the Melbourne Catholic Education Office program *Supporting Children's Early Numeracy Learning* (Gervasoni, 1998).

Using the ENRP Assessment Framework to Identify Students at Risk of not Learning Mathematics Successfully

The key to developing profiles of students at risk of not learning mathematics successfully is collecting rich, reliable assessment data that provides an insight about children's mathematical development. *Mathematics Recovery* Teachers and *Mathematics Intervention* Teachers use clinical interviews to gain assessment information and understand children's mathematical constructs and thinking. The ENRP has also developed an extensive 30-40 minute interview which provides the opportunity for classroom teachers to learn about their students' thinking and determine the mathematical growth points students have reached in nine mathematical domains: counting, place value, addition and subtraction strategies, multiplication and division strategies, measuring time, measuring length, measuring mass, classification of shape, and visualisation.

As part of the ENRP, project teachers use an interview to assess their students' mathematical learning in both March and November each year. Experienced teachers who have been trained by the ENRP research team interview students in the reference schools. Following the interviews, the ENRP research team collects the assessment information obtained from the interviews for analysis, and determines the growth points each child has reached in each of the mathematics domains assessed. This process provides rich data for over 5000 students. With such a large sample, it is possible to achieve a picture of the mathematical learning of young children. Two analyses of this data are important in developing profiles of Year 1 children who are at risk of not learning mathematics successfully: first, a calculation of the number and percentage of Year 1 students in March who have not reached each of the *on-the-way* growth points for each of the number domains; second, the frequency of each of the number domain profiles for Year 1 students in March who have not yet reached all of the *on-the-way* growth points.

In order to achieve this, 1999 ENRP assessment interview data were analysed for 1667 Year 1 students. These students were between six and seven years of age and beginning their second year of primary school education. They each attended one of 54 Victorian Government Schools that began the ENRP in February 1999. This group comprises 27 project ('trial') schools and 27 control ('reference') schools from the nine DEET regions.

On-the-way Growth Points

It is anticipated that the ENRP Assessment Framework of growth points (Clarke, 1999) may assist teachers to identify the children in their class who are at risk of not learning mathematics successfully, and who may benefit from additional assistance beyond that possible within the regular classroom mathematics program.

In using the growth points to monitor children's learning, it is suggested that there is a growth point for each domain beyond which children may be considered to be learning successfully (the *on-the-way* growth point). Growth points below this may be indicative of students who are at risk and who may benefit from additional assistance or careful classroom monitoring. For example, the growth points for the Counting domain of the ENRP Framework are as follows:

0. Not apparent.
   Unable to state the sequence of number names to 20.
1. Rote counting
   Rote counts to at least 20, but is unable to reliably count a collection of that size.
2. Counting collections
   Confidently counts a collection of around 20 objects.
3. Counting by 1s (forward/backward, including variable starting points; before/after)
   Counts forwards and backwards from various starting points between 1 and 100; knows numbers before and after a given number.
4. Counting from 0 by 2s, 5s, and 10s
   Can count from 0 by 2s, 5s, and 10s to a given target.
5. Counting from x (where $x > 0$) by 2s, 5s, and 10s
   Given a non-zero starting point, can count by 2s, 5s, and 10s to a given target.
6. Extending and applying counting skills
   Can count from a non-zero starting point by any single digit number, and can apply counting skills in practical tasks.

It is conjectured that for Year 1 students at the beginning of the year, the *on-the-way* growth point is growth point 2: counting collections of about 20 items. Not reaching this growth point provides one piece of evidence to suggest that a child be at risk of not learning mathematics successfully.

Profiles of Children who may be at Risk

In developing profiles that may identify children who may benefit from additional assistance beyond that provided by the regular classroom program, I have decided to focus on the four number domains of the ENRP framework. Although children's learning in the measurement domains is most important, an understanding of number underpins much of this learning. Also, a preliminary analysis of the ENRP data indicated that very few children had not reached the *on-the-way* growth points in the measurement domains. Therefore, it seemed that profiles based on the number domains would provide the best initial indication that a child was at risk.

The first step in developing profiles of children at risk of not learning mathematics successfully was analysing the ENRP data in order to determine how many children had not reached the *on-the-way growth* points for the number domains in the ENRP framework. Tables 1-4 describe the number of students who have attained the *on-the-way* growth point for the number domains in March, and the number of students who have reached each growth point below this. The percentage of the cohort attaining each of these growth points is shown also. The heavy lines in Tables 1-4 indicate the growth points beyond which children may be considered *on-the-way* to learning mathematics successfully. The shading indicates growth points that suggest children may be at some level of risk.

Counting

It would be hoped that all Year 1 students in March could count a collection of about 20 objects. The data indicates that in March, 88% of Year 1 students can at least count a collection of about 20 objects. Indeed, 23% of Year 1 students can at least count forwards and backwards by one from a given number. The 5% of students who know some number names but cannot yet rote count may require additional assistance. A further 7% of Year 1 students who can rote count but not yet count a collection of about 20 objects may also benefit from targeted assistance.

Place Value

In March, 89% of Year 1 children can read, write, interpret and order one digit numbers, and 16% of children at least can read, write, interpret and order two digit numbers. The 11% of students who in March cannot yet read, write, interpret and order one digit numbers should be carefully monitored by the classroom teacher.

Addition and Subtraction Strategies

Programs such as *Mathematics Recovery* and *Mathematics Intervention* suggest that a key to recognising Year 1 children who are at risk of not learning mathematics successfully is Stage 3 of the Early Arithmetic Stages: count on. This is equivalent to the second growth point in the ENRP addition and subtraction strategies domain. Table 3 shows that 58% of Year 1 students in March had not yet reached this growth point, and that 25% of students had not reached the *on-the-way* growth point of count-all. If these children have not yet reached the *on-the-way* growth points for the other number domains, particularly the counting and place value domains, they are at greater risk.

Multiplication and Division Strategies. More children had not yet reached the *on-the-way* growth point for this domain than for any of the other domains. In March, only 64% of Year 1 students had reached at least the first growth point. This may suggest that young children have fewer opportunities to construct ideas about multiplication and division, or that multiplicative thinking develops later than some other number constructs.

Table 1
The Number of Year 1 Children in 1999 who reached each of the Counting Growth Points on the ENRP Learning and Assessment Framework Below the On-the-way Growth Points.

Counting Year One March 1999 (n=1667)		
Growth Points	Count	%
0. Number Names	**89**	5
1. Rote Counting	**124**	7
2. Counting collections & beyond	1454	88

Table 2
The Number of Year 1 Children in 1999 who reached each of the Place Value Growth Points on the ENRP Learning and Assessment Framework Below the On-the-way Growth Points

Place Value Year One March 1999 (n=1658)		
Growth Points	Count	%
0. Some number names	**180**	**11**
1. One digit numbers	1207	73
2. Two digit numbers & beyond	271	16

Table 3
The Number of Year 1 Children in 1999 who reached each of the Addition & Subtraction Strategies Growth Points on the ENRP Learning and Assessment Framework Below the On-the-way Growth Points.

Addition & Subtraction Strategies Year One March 1999 (n=1658)		
Growth Points	Count	%
0. Not Yet	**420**	**25**
1. Count all	543	33
2. Count on & beyond	695	42

Table 4
The Number of Year 1 Children in 1999 who reached each of the Multiplication & Division Strategies Growth Points on the ENRP Learning and Assessment Framework Below the On-the-way Growth Points.

Multiplication & Division Strategies Year 1 March 1999 (n=1658)		
Growth Points	Count	%
0. Not Yet	**597**	**36**
1. Count all	448	27
2. Modelling when groups are perceived & beyond	613	37

March Growth Point Profiles for the Number Domains

One way to identify the children who are at risk of not learning mathematics successfully may be to consider the growth point profiles of the students who have not reached the *on-the-way* growth point in at least one number domain. This gives a more holistic picture of children's number constructs and understandings than considering each number domain in isolation. Of particular interest is determining whether there are any common profiles that are illustrative of children who need additional assistance in mathematics. These may provide a powerful tool for identifying students who are at risk of not learning mathematics successfully.

The March four-digit profiles for the 729 (44% of cohort) children who have not yet reached the *on-the-way* growth points for at least one of the four number domains are shown in Table 5. The first digit in the string relates to the counting growth point, followed by growth points for place value, addition and subtraction strategies, and multiplication and division strategies respectively.

The shading in the Table 5 highlights the profiles of children who have not yet reached the count-on stage which is used by *Mathematics Recovery* and *Mathematics Intervention* as the basis for identifying children at risk. Reference to the count-on growth point in the ENRP framework identifies the majority of children at risk, but misses three groups of children who can count-on: those who cannot yet count a collection of about 20 objects, those who cannot yet read, write, order and interpret one digit numbers, and those who cannot yet count all to solve multiplicative problems. The first two of these groups certainly need to be considered for additional assistance.

There are some 60 profiles listed in Table 5. This shows that there is great diversity in children's understanding of number, and emphasises the difficulties classroom teachers face in meeting the individual needs of children at risk, let alone the needs of all other

Table 5
The ENRP MARCH 1999 Number Domain Growth Point Profiles for YEAR 1 Students who have not yet reached the On-the-way Growth Points.

Growth Point Profile	Total Domains for which children not yet *on-the-way*				*Total for Profile (n = 729)*	*Growth Point Profile*	Total Domains for which children not yet *on-the-way*				*Total for Profile (n = 729)*	*Growth Point Profile*	Total Domains for which children not yet *on-the-way*				*Total for Profile (n = 729)*
	1	2	3	4			1	2	3	4			1	2	3	4	
0000				*24*	24	**1120**		3			3	**2300**		1			1
0001			9		9	**1121**	8				8	**2420**	1				1
0002			2		2	**1122**	5				5	**3000**			1		1
0020			1		1	**1130**		2			2	**3012**	1				1
0100			*24*		24	**1132**	2				2	**3020**		1			1
0101		11			11	**1201**		1			1	**3100**		15			15
0102		3			3	**1202**		1			1	**3110**	3				3
0110		1			1	**2000**			*43*		43	**3120**	*20*				20
0111	2					**2010**		7			7	**3130**	7				7
0120		2			2	**2011**	*32*				32	**3200**		3			3
0121	3				3	**2012**	8				8	**3220**	6				6
0122	1				1	**2020**		7			7	**4022**	1				1
1000				*24*	24	**2021**	6				6	**4100**		5			5
1001			5		5	**2022**	2				2	**4120**	5				5
1011		1			1	**2030**		3			3	**4130**	1				1
1021		1			1	**2100**		*189*			189	**4200**		2			2
1022		1			1	**2110**	*33*				33	**4220**	5				5
1100			*37*		37	**2120**	*89*				89	**4230**	2				2
1101		7			7	**2130**	7				7	**4300**		1			1
1102		8			8	**2200**		4			4	**4330**	1				1
1110		5			5	**2210**	1				1	**5220**	1				1
1111	9				9	**2220**	9				9	**5230**	1				1
1112	2				2							**Total**	274	285	122	48	729

Note. Shading identifies those students who have not yet reached the count-on stage used by *Mathematics Recovery* and *Mathematics Intervention* as the basis for identifying children at risk. Profiles which describe 20 or more students are italicised.

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students. However, about 500 of the 729 Year 1 children who have not yet reached the *on-the-way* growth points on the ENRP framework are described by only 9 profiles. Brief descriptions of these profiles follow.

Profiles 0000 and 1000

These children have not yet reached the *on-the-way* growth point in any of the four number domains. They are not yet able to count a collection of about 20 items, or read, write, interpret and order one digit numbers, or count all as a strategy to solve simple addition or multiplication problems.

Profiles 0100 and 1100

These children are able to read, write, interpret and order one digit numbers and are therefore *on-the-way* in place value. However, they have not reached the *on-the-way* growth point in the remaining three number domains. They are not yet able to count a collection of about 20 items, or count all as a strategy to solve simple addition or multiplication problems.

Profile 2000

These children have not yet reached the *on-the-way* growth point in three number domains, with the exception of counting. They are able to count a collection of about 20 items, but are not yet able to read, write, interpret and order one digit numbers, nor count all as a strategy to solve simple addition or multiplication problems.

Profile 2100

This profile described about one quarter of the children at risk. These children have not yet reached the *on-the-way* growth point in two number domains: addition and subtraction, and multiplication and division domains. They are able to count a collection of about 20 items, and read, write, interpret and order one digit numbers. They are not yet able to count all in addition and subtraction, or multiplication and division contexts.

Profiles 2110, 2120 and 3120

These children have not yet reached the *on-the-way* growth point in one number domain: multiplication and division strategies. They are able to count a collection of about 20 items and read, write, interpret and order one digit numbers, and count all to solve simple addition problems. Some students can count-on to solve simple addition problems. They are not yet able to use the count all as a multiplicative strategy.

Levels of Risk

It is possible to use the ENRP profiles to describe children's level of risk of not learning mathematics successfully. This method, however, does not take into account important factors such as children's health, motivation, interest, parental support, culture, and language background etc. Such factors need to be considered also when deciding who may benefit from additional assistance in mathematics. Moreover, the ENRP profile is based on a single interview and needs to be considered in light of other assessment information collected by the classroom teacher. The levels of risk described below may be useful in identifying children who may benefit from additional assistance in mathematics.

The highest level of risk relates to profiles describing children who cannot yet count a collection of about 20 items, interpret one digit numbers, nor count-on. The lowest level of risk relates to those children who do not yet use a count-all strategy in multiplicative situations, but who are *on-the-way* in all other domains.

Level 5	children who cannot yet count a collection of about 20 items, nor count-on.
Level 4	children who can count a collection of about 20 items but not yet read, write, interpret one-digit numerals, nor count-on.
Level 3	children who can at least count a collection of about 20 items, but not yet read, write, interpret two digit numerals, nor count-on.
Level 2	all other children who cannot yet count-on.
Level 1	all other students who cannot yet count all in multiplicative situations.

Conclusion

It is possible to use the ENRP assessment framework growth points to develop profiles of children who may be at risk of not learning mathematics successfully. Further, these profiles may be used to suggest the level of risk children may be facing, and to assist teachers decide who in their class may benefit from specialised assistance additional to the regular classroom program. Additional analysis of ENRP data is recommended so that profiles of Prep and Year 2 students who may also be at risk of not learning mathematics successfully may be developed also. It is important to identify these children as early as possible in their schooling, so that they can be assisted before they lose confidence.[1]

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Rural Students and Graphics Calculators in Examinations: Some Preliminary Findings

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With the introduction of graphics calculators to tertiary entrance examinations in mathematics in Western Australia, a question that arose is whether rural students were placed at a disadvantage when compared with those in the city[1]. A preliminary analysis of examination data suggests that rather than disadvantage, there was some evidence of an improvement in the achievement of rural students when compared with those in the city.

Background

The use of graphics calculators was allowed in the Tertiary Entrance Examinations (TEE) in Western Australia for the first time in 1998, in each of the three mathematics subjects. The decision to allow the use of graphics calculators was based on the rationale that, given the technology exists, its use should be allowed, and that developments with scientific calculators had made it difficult to police their use in examinations (personal communication with Curriculum Council officer, November 1999). Some four years' notice was given for their introduction to allow time for schools to make decisions pertaining to curricula and the selection of a suitable calculator. This time-line afforded teachers the opportunity to familiarise themselves with the uses to which the calculators might be put and to make the necessary adjustments to their teaching procedures and materials.

There are three mathematics subjects examined at the TEE level in Western Australia. Applicable Mathematics and Calculus are the two subjects considered accessible to the top 30% of the age cohort, while Discrete Mathematics is deemed appropriate for the next 30%. Each year, approximately 5000 students sit for Applicable Mathematics, 2000 for Calculus and 6000 for Discrete Mathematics; the smaller numbers who study Calculus reflecting its status as a pre-requisite for a limited number of courses at tertiary level.

Theoretical Framework

There is a paucity of research articles that have been published regarding the effectiveness of graphics calculators as a tool for instruction and learning in mathematics. One area in which research has occurred concerns their use in pre-calculus and calculus classes (e.g., Slavit, 1996; Quesada & Maxwell, 1994; Drijvers & Doorman, 1996). Another that has attracted some attention concerns sex differences associated with their use (e.g., Ruthven, 1990; Vazquez, 1991; Boers & Jones, 1991, 1992a, 1993; Cassity, 1997). While some studies involving the use of graphics calculators in examinations were identified (e.g., Boers & Jones 1992a, 1992b, 1992c, 1993), no studies that examined their impact with rural students were apparent.

Studies that have investigated the mathematics achievement of rural students compared with those in the city include that of Young (1994). In her analysis of Monitoring Standards in Education data in Western Australia, Young found students in Years 3, 7 and 10 in metropolitan schools scored significantly higher in mathematics than students in rural and remote locations. However, Greenberg and Teixeira (1995), in their analysis of National Assessment of Educational Progress achievement scores in the USA, found that non-metropolitan 17 year-old students scored only slightly lower than metropolitan students in mathematics. Meanwhile, an analysis of three Common Assessment Tasks conducted as part of the Victorian Certificate of Education (Cox, 1997) revealed that rural students performed worse than their metropolitan counterparts. In none of these three studies was the use of graphics calculators the focus.

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[1] City in this case refers to students who attended schools located in the Perth metropolitan area.

It has been suggested that students in rural schools may be at a disadvantage when compared with their city counterparts, in terms of both human and material resources. Sunal (1991), in a study of rural science teaching in a south central Appalachian state (USA), found that higher science achievement correlated positively with an adequate number of certified (i.e. qualified) science teachers; with higher teacher ratings of effectiveness of in-service programs and greater access to university courses; and to schools rated as having more laboratories, equipment and general resources. In addition, Hall and Barker (1995) reported that their comparison of curricular offerings in mathematics in a Chicago suburban high school, a rural high school district, and a small rural unit, revealed the curricular disadvantages of small rural high schools.

It is possible that students in rural high schools in Western Australia may suffer from some of the disadvantages reported above. Their teachers may be less qualified and experienced than those in city high schools, have less access to professional development (such as that provided prior to the introduction of graphics calculators) and, in the smallest of these schools, have less opportunity to confer with colleagues. In addition, rural schools may not be as well resourced in materials and equipment as city schools. Each of these factors has the potential to impact on the achievement of rural school students.

Penglase and Arnold (1996), in their meta-analysis of research on the graphics calculator in mathematics education, concluded that the use of graphics calculators in examinations was an area of research that was particularly under-represented. In this study, the achievement of rural students in the TEE provided the focus. It sought to determine whether there was any evidence to suggest that rural students were disadvantaged when compared with those in the city, following the introduction of graphics calculators.

Methodology

Data pertaining to student achievement were obtained from the Curriculum Council of Western Australia, the body that administers tertiary entrance examinations. The data provided took the form of the total number of students who sat for each of the TEE mathematics examinations in 1997 and 1998, together with students' mean scores for each whole question (in the case of Applicable Mathematics and Calculus) or part question (Discrete Mathematics). These data were further categorised according to the location of the school (rural or city). Although in each examination students were asked to attempt all questions on the paper, the Curriculum Council excluded non-attempts when calculating the mean scores.

A second set of data was provided by panels of teachers. For each of the three mathematics courses, a panel of five experienced teachers who had taught that course in 1998 were invited to make assessments of the 1997 and 1998 examination papers. With respect to each part question, they were asked to indicate whether the use of a graphics calculator would have been appropriate. Inter-rater reliability coefficients were calculated for each mathematics paper assessed using the formula

$$r = \frac{(MS_b - MS_w)}{((MS_b - (n-1)MS_w))}$$

where MS_b is mean squares between; MS_w is mean squares within, and n is the number of cases (Stevens, 1990).

Data obtained from Teacher Panel members were then aggregated so that each part question on each of the six examination papers was categorised as being appropriate or inappropriate for graphics calculator use.

For each of the six examination papers, the numbers of questions where the mean score of rural students exceeded those from the city, and vice versa, were aggregated, as were the marks allocated to those questions. The same procedure was adopted for questions identified by the teacher panel as being appropriate for graphics calculator use. In addition, questions for which the difference in mean score between rural and city students exceeded five percentage points were identified on each paper.

Results

Table 1 indicates the numbers of students who sat for each of the mathematics examinations in 1997 and 1998. As may be seen, both the total numbers and also the proportions of city and rural students remained consistent between the two years.

Table 1

Numbers (%) of Candidates for TEE Mathematics Examinations

Subject/Year	Total	City	Rural
Applicable '97	4904	4100 (84)	804 (16)
Calculus '97	1888	1636 (87)	252 (13)
Discrete '97	5818	4928 (85)	890 (15)
Applicable '98	4976	4144 (83)	832 (15)
Calculus '98	1885	1632 (87)	253 (13)
Discrete '98	5781	4891 (85)	890 (15)

Table 2

Measurement of Agreement Coefficient for Teacher Rating of Items' Appropriateness for Graphics Calculator Use

TEE Exam	r
1997 Applicable	0.7953
1998 Applicable	0.8976
1997 Calculus	0.7059
1998 Calculus	0.7339
1997 Discrete	0.7180
1998 Discrete	0.8851

Inter-rater reliability coefficients for the teacher assessments are reported in Table 2. They were consistently high, indicating a strong measure of agreement in the responses of the five members of each teacher panel.

The teacher panel assessments are presented in Table 3. Designated are proportions of the total mark that were allocated to the categories of appropriate or inappropriate for graphics calculator use as a result of the teachers' assessments. Of particular note are the proportions of marks allocated to questions deemed appropriate for graphics calculator use in Applicable Mathematics and Calculus. While for the 1998 Applicable Mathematics paper there is a suggestion that, in devising questions, the examiners moved to encourage their use, the opposite appears true for Calculus.

Table 3

Teacher Assessment of Examination Items (Percentages)

Graphics Calculator Usage	Applicable		Calculus		Discrete	
	1997	1998	1997	1998	1997	1998
Appropriate	23	34	26	16	21	23
Inappropriate	77	66	74	84	79	77

The comparative performance of rural and city students on each of the papers for 1997 and 1998 is found in Table 4. Indicated is the number of questions (part questions in the case of Discrete Mathematics) where the mean score of students of one group exceeded that of the other. These data indicate there was no deterioration in the achievement of rural students compared with city students from 1997 to 1998. In fact, they suggest a slight improvement in the performance of rural students, in both Calculus and Discrete Mathematics.

Table 4

Comparative Performance of Rural and City Students: Numbers of Questions on which Mean Scores of Students of One Group Exceeded those of the Other

TEE Subject	1997		1998	
	Rural	City	Rural	City
Applicable	0 (0)	23 (180)	1 (5)	21 (175)
Calculus	0 (0)	20 (180)	4 (39)	15 (141)
Discrete *	22 (57)	46 (122)	26 (68)	43(112)

Note. Numbers in parentheses are marks out of 180 that were apportioned to those questions. *Mean scores were provided for part questions in most cases.

The data in Table 5 focus on questions that were rated by the teacher panels as being appropriate for graphics calculator use. In the case of Applicable Mathematics and Calculus, they include whole questions for which the teacher panel rated at least two thirds of the question appropriate for graphics calculator use. Again, these data suggest no deterioration in the achievement of rural compared with city students on these particular questions.

Table 5

Comparative Performance of Rural and City Students on Questions Rated Appropriate for Graphics Calculator Use

TEE Subject	1997		1998	
	Rural	City	Rural	City
Applicable *	0 (0)	4 (22)	0 (0)	5 (45)
Calculus *	0 (0)	4 (47)	1 (5)	2 (17)
Discrete **	4 (10)	8 (24)	5 (12)	11 (31)

Note. Numbers in parentheses are marks out of 180 apportioned to those questions. * Whole questions in which at least two-thirds rated appropriate for graphics calculator use. ** Part-questions rated appropriate for graphics calculator use.

Data showing the number of questions on each of the six examination papers where the difference in mean scores between rural and city students exceeded five percentage points are found in Table 6. That there is only a small number of such questions indicates the difference in achievement overall between rural and city students was not great in either year. In comparing 1998 with 1997 data, the most notable change is in Calculus, favouring rural students. In both Applicable Mathematics and Discrete Mathematics, little change was apparent.

Table 6

Comparative Performance of Rural and City Students on Questions Where Means Differed by More Than Five Percentage Points.

	1997		1998	
	Rural	City	Rural	City
Applicable *	0 (0)	11(91)	0 (0)	12 (106)
Calculus *	0 (0)	11 (103)	0(0)	3 (25)
Discrete **	1(2)	10 (24)	0 (0)	4(14)

Note. Numbers in parentheses are marks out of 180 apportioned to those questions. * Whole questions in which at least two-thirds rated appropriate for graphics calculator use. ** Part-questions rated appropriate for graphics calculator use.

A closer examination of the questions for which the means differed by more than five percentage points reveals that three in Applicable Mathematics, two in Discrete Mathematics, but none in Calculus were rated as appropriate for graphics calculator use. The three in Applicable Mathematics involved the solution of an inequality, a normal probability distribution problem, and a probability density function. The two in Discrete Mathematics were part of a problem involving linear programming and part of a problem involving a geometric sequence. There doesn't appear to be a particular reason why city students might score higher on these questions than their rural counterparts.

Discussion

Differences in achievement favouring city over rural students in both 1997 and 1998 are commensurate with those found by Young (1994), Greenberg and Teixeira (1995) and Cox (1997). However, the question to be addressed in this study was whether rural students were placed at a disadvantage with the introduction of graphics calculators into TEE mathematics. From the analysis of the rudimentary data available, it appears not. There was no deterioration detected in the achievement overall of rural students compared with city students, or in questions that were rated by the teacher panels as being appropriate for graphics calculator use. There was even a suggestion among the data that rural students may have improved in some instances. On the basis of this finding, it would appear that none of the factors identified earlier – namely, the relative qualifications and experience of rural teachers, access of rural teachers to professional development and the opportunity to confer with colleagues, and the level of resources in rural schools – has impacted on rural students' facility with graphics calculators.

As the analysis presented here was based on limited data, further research is necessary to confirm whether or not rural students have been placed at a disadvantage. Access to individual student data and to that for the 1999 examinations would allow for more sophisticated analyses to be performed. Furthermore, examination of student scripts, focussing on questions rated appropriate for graphics calculator use and those where the difference in scores between rural and city students was greatest, would allow any differences between rural and city students in their usage of the calculators to be detected.

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Understanding of Functions Among Maldivian Teacher Education Students

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Maldivian teacher education students' understanding of functions and rates of change was investigated. The written responses of 22 students to 10 examination questions were first analysed, and then 5 students were interviewed in depth as they solved 4 of the most difficult questions. It was found that students had a good grasp of basic concepts, but tended to rely on remembered formulae or procedures. They also had difficulty visualising tangents and linking them to rates of change and derivatives. Implications for teaching are explored.

The National Diploma of Secondary Teaching (NDST), a tertiary program which prepares teachers for Maldivian lower secondary schools (Grades 8-10), started at the Institute of Teacher Education (ITE) in February 1997. The two-year program combines academic courses in mathematics, various sciences, and education with field-based teacher education. The mathematics and mathematics education elements have been developed and taught by the first two authors in consultation with the third author.

Students enter the NDST with London A-level passes in at least two subjects. However, experience suggests that most students have achieved these passes largely through rote learning. Regarding this as an inadequate basis for teaching, the designers of the NDST aimed to develop a program which would strengthen students' understanding of elementary concepts as well as teaching more advanced content. For example, the first-semester mathematics course MATH101 includes a 5-week module on Functions and Rates of Change which seeks to put calculus techniques learnt in school on a firm conceptual footing. It was important for us to ask the question: How effective is this module?

Functions and Rates of Change

The apparent tendency of calculus courses to promote rote, manipulative learning has been a subject of concern for more than a decade. We only have space to cite a few related research studies: Poor understanding of the underlying concept of function has been shown among mathematics students (Eisenberg, 1991), teacher education students (Even, 1993) and teachers (Cooney & Wilson, 1993). The concept of rate of change causes particular difficulties (White & Mitchelmore, 1996). Most recently, Santos-Trigo (1998) analysed errors among first-year calculus students and found that several were due to inadequate mastery of elementary concepts such as use of functional notation, evaluation of limits, and a restricted tangent concept.

There have been numerous curriculum responses to what some have called a crisis in calculus teaching (White, 1990). Some authors (Barnes, 1992) have used everyday situations as the basis for developing students' intuitions about functions, following an approach similar to that used by the Dutch *Realistic Mathematics Education* movement (Streefland, 1991). Others have focussed on the relation between functions and graphs, often using computers or graphics calculators to assist and develop visualisation (Keiran, 1993). Still others have emphasised the importance and value of being able to translate fluently between different representations of a function, another area where technology can be particularly helpful (Kaput, 1992).

The MATH101 module on Functions and Rates of Change was developed with the above research and development in mind. The teaching approach adopted is well exemplified in Chapters 1 and 2 of the chosen textbook (Hughes-Hallett et al., 1994): Students sketch graphs of realistic functions, learn to recognise the graphical forms of a basic set of functions (polynomials, exponential and logarithmic functions, etc.), measure rates of change in various ways, and sketch graphs of rates of change. The textbook strongly emphasises what it calls "The Rule of Three: Every topic should be presented geometrically, numerically, and algebraically" (p. vii). We also use Barnes (1992) to provide additional realistic examples of functions and rates of change. Unfortunately, no technological aids are available at the present time.

Despite the efforts which have gone into designing MATH101 (which also includes modules on Vectors & Matrices and Number Theory), examination results have been rather disappointing. After an analysis of the archived examination scripts showed poor performance on the Functions and Rates of Change items (Hassan, Mohamed, & Mitchelmore, 1999), a small number of students was interviewed as they solved related items. The present paper presents an error analysis of the examination scripts and the interview responses. The primary aim of the error analysis is, as suggested by Fong (1995), to identify students' underlying conceptual difficulties and thus to obtain guidelines for improving the teaching of the module.

Script analysis

Method

A total of 27 students have taken the MATH101 examination since the inception of the NDST: 5 in 1997, 10 in 1998, and 12 in 1999. Because 1997 was the first year of the program and involved a small number of students, the 1997 scripts were excluded.

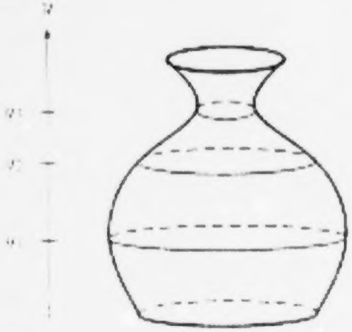
There was a total of ten items on Functions and Rate of Change on the 1998 and 1999 papers. One was rather misleading, and two were completed correctly by almost all students. The other seven are shown in abbreviated form in Figure 1. Items 3 and 4 appeared in identical form in 1998 and 1999. Items 6 and 7 appeared in 1998 and similar items in the 1999 examination. The other three items appeared in one year only.

Results

In the following, a notation such as "(8/12)" indicates that 8 out of 12 students exhibited a particular error at least once.

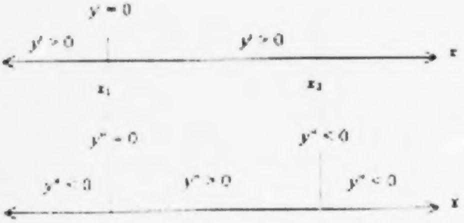
Item 1. Most students (10/12) failed to answer this item correctly. Most of these students had clearly misinterpreted the graph as a graph of displacement against time: They chose a graph which was increasing but concave downward instead of upward, claiming that athletes tend to slow down as they become tired during a race.

1. Which graph models most realistically the relationship between the time it takes to run a race and the length of the race? Explain your choice. If you think none of the graphs is realistic, draw your own version and explain it fully. [Six graphs were given, each showing time as a monotonically increasing or decreasing function of length.]
2. Find a possible formula for the graph below. [The graph of a monotonically decreasing, apparently exponential, function through (-1, 8) and (1, 2) was given.]
3. The depth of water in a tank oscillates sinusoidally once every 6 hours around an average depth of 7 metres. If the smallest depth is 5.5 metres and the greatest depth is 8.5 metres, find a formula for the depth in terms of time, measured in hours. (Sinusoidally is another name for sine curve).
4. Consider the vase drawn below. Assume the vase is filled with water at a constant rate (i.e., constant volume per unit time).



(a) Graph $y = f(t)$, the depth of the water, against time, t. Show on your graph the points at which the concavity changes.
(b) Where does $y = f(t)$ grow fastest? Slowest? Estimate the ratio between these two growth rates.

5. Sketch the graph of $y = f'(x)$ for the function given below. [A continuous graph was given with x-intercepts at 2 and 4, a maximum at $x = 0$, a minimum at $x = 3$, and points of inflection at $x = -2$ and $x = 4$.]
6. Sketch a possible graph of $y = f(x)$, using the given information about the derivatives $y' = f'(x)$ and $y'' = f''(x)$. Assume that the function is defined and continuous for all real x.



7. Identify the local and global maxima and minima of $f(x) = x^{-2/3} + x^{1/3}$ (1.2 ≤ x ≤ 3.5.

Figure 1. Examination items on Functions and Rate of Change module.

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Item 2. All students recognised the given function as possibly exponential but most (6/10) could not, for various reasons, fit an appropriate equation to it. Most of these (5/10) did obtain an equation, but they failed to check that the function so defined passed through both the given points.

Item 3. Most students (16/22) failed to answer this item correctly. Five students omitted the item altogether, and the remainder made a variety of errors (mostly involving the amplitude and the mean height). There was little evidence of students attempting to check their equations—for example, by sketching a graph.

Item 4. All students (22/22) failed to predict the shape of the graph precisely. The main error consisted of assigning the second point of inflection to the height y_2 rather than y_3 — although most students correctly stated that the maximum rate of change is at y_3. All students (22/22) also failed to estimate the ratio of the minimum to the maximum rate of change on Item 4. Nine students omitted this part completely, whereas the remainder appeared to use the diameters of the vase at the heights y_1 and y_3 rather than the cross-sectional areas.

Item 5. Half the students (6/12) made errors responding to this item. All these students drew a graph of $f'(x)$ which had a maximum at $x = 5$ instead of $x = 4$. (On the given graph $f''(x)$ had a minimum at $x = 5$; giving a point of maximum curvature.)

Item 6. This item was difficult in both versions, with almost all students (21/22) making some error. Students had slightly more difficulty interpreting the information about the first derivative (18/22) than the second derivative (15/22).

Item 7. On both versions of this item, most students (19/22) made errors. On the item shown in Figure 1, most students (8/10) had difficulty with the rational powers: Some made errors in the derivative, some were unable to solve the equation to find whether the derivative is zero, and some were unable to calculate the value of the function at the end-points. Several students were unable to reconcile the local minimum they obtained with the values of the function they calculated at the end-points of the given interval. On the other version of Item 7, in which $f(x)$ was a cubic polynomial defined over all real numbers, most students (10/12) calculated the local maximum and minimum correctly and then assumed these values also gave the global maximum and minimum.

Summary

The errors students made on the examination scripts suggested that they were not sufficiently skilled in the following aspects of graphical reasoning:

1. From a description of a physical situation:
   (a) identify two variables, one of which is varying as a function of the other;
   (b) represent the general form of their relationship in a graph.
   (The term "general form" includes whether the function is increasing or decreasing over the domain, whether it reaches a maximum and then decreases, whether the rate of change is constant, and so on.) This skill was essential for Items 1, 3 and 4.

2. Sketching a graph by finding *precisely* where its gradient changes from increasing (becoming steeper if positive or less steep if negative) to decreasing:
    (a) from a description of a physical situation; or
    (b) from information about the first and second derivative.

    This skill was essential for Items 4, 5 and 6.

3. Using a graph as a representation of a function over its entire range of definition, for example:
    (a) to check whether a given equation models the function adequately, or
    (b) to find the range of variation of the function.

This skill was important in Items 2, 3 and 7.

The interviews were accordingly designed to learn more about students' understanding of these aspects of graphical reasoning.

Interviews

Method

It was decided to interview students as they solved four of the most difficult examination items, namely Items 1, 3, 4 and 6. Each item was presented on a separate sheet of A4 paper, on which students also wrote their working. Item 1 was re-worded to require the student to draw a graph rather than to select a graph from several. An additional easy item (finding the equation of a line through two given points) was included as a warm-up to help students learn the technique of "thinking aloud".

Five first-year NDST students, all of whom had taken the MATH101 examination approximately 12 weeks earlier, were interviewed. These 5 students were those judged by their lecturer to be most likely to be able to think aloud. All the students were female.

The students were each interviewed individually for about an hour by the first and third author. Whenever students forgot to talk about what they were doing, the interviewers asked students to describe what they were thinking or trying to do. The interviewers also frequently asked students to explain what their drawings or writings meant and how they had worked them out. When students could not quite complete a task, the interviewers occasionally asked leading questions to help the students reach a more complete understanding of the item.

For technical reasons, the interviews could not be recorded. The data collected for analysis consisted of the students' written work together with the interviewers' notes on (a) their observations of the students' behaviour during the solution process (e.g., long pauses, apparent visualisation movements) and (b) students' answers to the probing questions.

Results

Item 1. All 5 students correctly identified the two variables, explained the concepts of dependent and independent variables, and identified "the time taken to run a race" as depending on "the length of the race". However, none was able to successfully represent this relation graphically. Most students started by drawing a graph of length against time; only 3 attempted to draw a graph of time against length. Two students plotted two pairs of values (10 s for 100 m and 20 s for 200 m), but could not decide whether 200 m would take proportionately longer than 100 m or what this would imply for the shape of the graph. All students interpreted the relation as referring to a single race (i.e., a distance-time graph) and one student said "length means distance" and another said "ah, 'time it takes' means the time as someone runs". But it seemed to the interviewers that the basic problem was that students expected graphs to show a relation between variables which actively change, rather than simply having a range of possible values. As one student asked "Are we talking about *change* or *relationship*?"

Item 3. Although all the students had great difficulty demonstrating the physical motion which this question describes, they all correctly drew a horizontal time axis, a vertical depth axis, and a sine curve with period 6 hr oscillating between 5.5 m and 8.5 m. (The time which some of them required to do this explains why no student drew the graph under examination conditions.) But only 3 of the 5 students were able to correctly calculate or interpret the average depth of 7 m. The same 3 students were also the only ones who were able to calculate the amplitude as the maximum deviation from the average and to construct the correct formula $d = 7 + 1.5\sin(\frac{\pi}{3}t)$. However, all 3 seemed to be working from the memorised formula $d = A \sin Bt$, where A = amplitude and $B = 2\pi \div$ period, together with a vertical translation by 7 m. No student attempted to visualise a transformation from $d = \sin t$ into $d = 1.5\sin(\frac{\pi}{3}t)$ or to find the coefficients A and B by substituting appropriate values of t.

Item 4. All 5 students were able to sketch the general form of the graph, to explain the variation in slope, and (when pressed) to identify the precise heights at which the water level was rising fastest and slowest. However, as in the written examination, several students initially identified a range of depths (usually between y_2 and y_3) where the rate of change was greatest, instead of the single point y_3. Students' explanations suggested that most were thinking in terms of average rates of change rather than instantaneous rates of change. For example, one student marked about 10 intervals on the t-axis and said "In each second, water has to spread out different amounts"; another divided the vase into horizontal layers and said "It takes longer to fill up the wider layers". Also, all students had difficulty drawing tangents at the points of inflection; they would repeatedly draw the tangent at a nearby point, even though they realised the gradient was not as steep there (see Figure 2). Students admitted to believing that a tangent must "touch" the curve, and therefore should not cross it. Students knew the term "point of inflection" and knew that the concavity changes at such a point, but they appeared not to have thought of it in terms of the changing slope of the tangent.

Item 6. All students showed a very weak concept of derivative. Only one student immediately interpreted the first derivative y' as the gradient and used it correctly to sketch the graph. Three students stated that $y' = 0$ meant the graph had a turning point at x_2 and $y' > 0$ meant that the gradient was positive; two managed to resolve this contradiction and sketched a horizontal point of inflection at x_1, but the third could not. The fifth student could not even start to answer the question. Students' concepts of the second derivative were even weaker. Three students said they only knew that you calculate y'' to find whether a turning point is a minimum or a maximum; two of these also remembered that $y'' > 0$ means the gradient is increasing and were then able to deduce that there is a point of inflection at x_1. No student was able to say what y'' meant, or why it was called the *second* derivative.

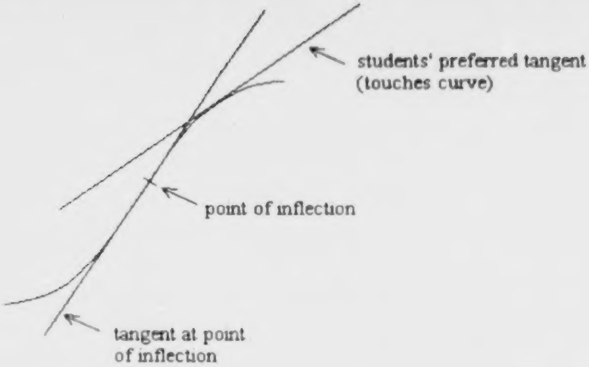

Figure 2. Drawing the tangent at a point of inflection.

Summary

The individual interviews confirm the findings of the script analysis and are generally in line with previous research on students' concepts of function and rate of change. Students knew the names of some important concepts (e.g., function, second derivative, point of inflection) and could identify examples of them, but they were often unable to relate them in the manner required in graphical reasoning. Like Australian students (White & Mitchelmore, 1996), they tended to rely on formulae and rules, and they were severely restricted by their own experience (Goldenberg, 1987). They found it difficult to visualise a tangent moving along a curve, seemed not to distinguish average from instantaneous rate of change, and did not appear to have formed strong links between the concepts of tangent, derivative, gradient, and rate of change (Santos-Trigo, 1998).

Implications

Functions feature widely in the Maldivian school mathematics syllabus, but the emphasis is on algebraic manipulation. In that respect, the approach adopted in MATH101 is quite new and some of the errors we observed may have been due to this novelty. In general, however, our findings suggest that many students have not grasped several concepts which are fundamental to the tertiary study of functions.

We deduce that it is appropriate to include the module on Functions and Rates of Change in MATH101 unit and that no essential changes need to be made. However, there are clearly some aspects which need extra emphasis if it is to be as effective as possible:

- When sketching everyday functions (as in Barnes, 1992), challenge the idea that a graph is not necessarily a static or dynamic picture of actual events.
- Emphasise even more the relation between rate of change, derivative, and gradient. Distinguish more clearly between average and instantaneous rates of change. Do not assume that students have mastered these concepts before commencing tertiary studies.

- Focus more on how the rate of change of a function itself changes. Challenge the idea that a tangent "touches" a curve without crossing it. Obtain technology to "draw" tangents and show how they vary along a curve. Emphasise precision in locating the point on a curve where the concavity changes.
- Emphasise that the second derivative is the rate of change of the first derivative. Sketch graphs of both first and second derivatives of given functions (and vice versa). Derive logical explanations for rote-learnt rules for using the second derivative to classify turning points.
- Hold even more strongly to the "rule of three" by continually reminding students of the arithmetical, algebraic, and graphical representations of mathematical ideas and the links between them.

Our findings have implications for the teaching of mathematics in Maldivian schools. We hope they will also be of interest to Australasian mathematics educators.

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Improving Decimal Understanding: Can Targetted Resources Make a Difference?

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This study investigated whether minimal intervention with new teaching resources could make a difference to children's understanding of decimal numeration. Four teachers were supplied with resources designed specifically to address the common misconceptions children have about decimal numbers, and asked to keep a record of their teaching. Use of the resources was unexpectedly low, but teachers who used them achieved an encouraging improvement in decimal understanding, measured against previous performance of the school over some years, indicating that a small amount of deliberate attention to decimal concepts can make a difference.

The Incidence and Persistence of Difficulties with Decimal Numeration

It is well documented that many students throughout schooling, and indeed many adults, have difficulties with decimals. A central problem (and possible underlying cause of many other difficulties) is that many people lack understanding of decimal numeration. This is a long-standing and international problem. For example, Grossman (1983) reported that, even though over 50% of U.S. students entering tertiary education could add, subtract or multiply decimals, less than 30% could select the smallest decimal from five options.

Recent Australian data from the *Learning Decimals Project* (Steinle & Stacey, 1998) confirm that Australian students have similar difficulties and that misconceptions are widespread and persistent. Using a carefully designed Decimal Comparison Test, they found that the proportion of students able to compare decimals reliably is relatively stable from year 6 (about half) to year 10 (less than two thirds), suggesting that normal teaching is making little difference to the way students think about decimals. The test enables most students' responses to be classified according to the way they are thinking about decimal numeration. Longitudinal data indicates that about half of the students who have a decimal misconception have the same misconception a year later. (Stacey & Steinle, 1999b).

The data above demonstrate that understanding decimal numeration is a complex task. Students have to coordinate their ideas of place value learned in the whole number setting with their understandings of fractions. For example, to compare 0.3 and 0.34, a student who knows that the first is 3 tenths and the second is 34 hundredths has to be able to mentally coordinate the effect of the varying numerators (3 and 34) and the varying denominators (tenths and hundredths). There are other ways of comparing these two numbers, for example by using knowledge of equivalent fractions so that 0.3 can be read as 0.30 and therefore as 30 hundredths, making the comparison easier. Another way is to think of 0.34 as 3 tenths + 4 hundredths. However, all of these methods require coordination of place value knowledge and fraction knowledge.

Some schools and teachers seem to teach decimal numeration well and others not so well. Steinle and Stacey (1998) found that although 52% of grade 6 students were apparent experts, the results by school varied from 0% to 82%. This variability is only partly explained by socioeconomic factors. For this reason, it seems likely that attention to the meaning of decimal numbers of varying length in teaching is a major factor in determining success. It is likely that, given just a small impetus, the understanding of children in this topic could be markedly improved. This study set out to investigate whether this is the case. Could teachers, given minimal support, change their teaching so that most children develop a good idea of decimal numeration over a relatively short time?

Resources for Learning about Decimals

The *Learning Decimals Project* has developed a collection of classroom resources for teaching about decimal numeration, including activities to address common misconceptions. The resources include classroom activities using simple equipment such as cards and the chalkboard, computer games, and a concrete model of decimals which we call Linear Arithmetic Blocks (LAB) (see Figures 1 and 2).

This model was first shown to us by Heather McCarthy, a local teacher, who saw it at an in-service training session given by a person whose name we do not know. LAB is similar to Dienes' Multibase Arithmetic Blocks (MAB) in that it can be used as a hands-on model of decimal numbers. Units, tenths, hundredths and thousandths are represented by hollow tubes of decreasing, and proportionally accurate, length. The unit piece we use is just over one metre long, so that thousandths are just over a millimetre long.

LAB pieces can be arranged in two ways to represent a number, either end-to-end or on what we call an organiser. When placed end-to-end the pieces form a linear representation of decimal numbers, as in Figure 1. This facilitates direct comparison of decimal numbers. The second possibility is to place the pieces on the organiser, as in Figure 2. The organiser consists of upright rods attached to a wooden base, which provide a concrete model for the place value columns. The height of the rods is such that only nine of the relevant pieces can be placed on it, providing an in-built constraint on column overflow. Thus if there are ten or more pieces of any one size, they will not fit on the appropriate rod of the organiser and so ten of them must be exchanged for a single component of the next highest place value.

In this study, we followed the advice of Judah Schwartz who observed that "in the case of education reform, there need to be new curricular artifacts that allow the users to mark the newness of their undertaking" (1994, p. 4). LAB, something that teachers and children had not seen before, served as the marker of this new undertaking.

A number of arguments can be put forward in favour of LAB over MAB as a tool for supporting the learning of decimal concepts.

1. The values of MAB pieces used for decimals are often confused with whole number values that they previously represented. Since LAB is probably new to teachers and children, this confusion is unlikely.
2. LAB is a simpler model than MAB. The underlying representation of the size of number by MAB is by volume (or mass if the density of material used is constant). One number is larger than another if the volume of the pieces assembled to represent it is larger. LAB, however, represents numbers by the length of the pieces assembled, which in view of children's difficulties with volume concepts (Battista & Clements, 1996) may be more developmentally appropriate. MAB material is also complicated by what users may perceive as a switch from the 3 dimensional block, to the 2 dimensional flat to the 1 dimensional long to the 0 dimensional unit.
3. Third, as noted above, no more than 9 pieces of any one denomination fit on a rod of the organiser, forcing trading ten of one unit for one of the next.
4. Fourth, because LAB has structural similarity to the number line, it has important advantages in being able to demonstrate the density property of decimals (the property that between any two decimal numbers, there is another). It also provides an excellent concrete basis for rounding decimals.

5. Two small, unpublished student studies (Archer, 1999; Condon, 1999) reported several advantages of LAB over MAB in practice. Analysis of lesson transcripts revealed significantly more discussion, conjecture and explanation by groups of children using LAB. Some children reported being confused by MAB but there were no such reports for LAB. Children were also more likely to visualise using LAB than MAB when a model was not available. These encouraging results suggest that LAB warrant further investigation.

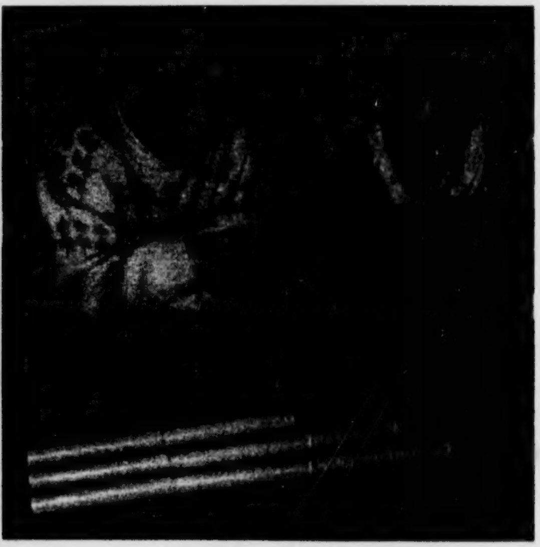

Figure 1 (above). LAB pieces laid linearly to illustrate and compare the numbers 0.2, 0.27 and 0.3

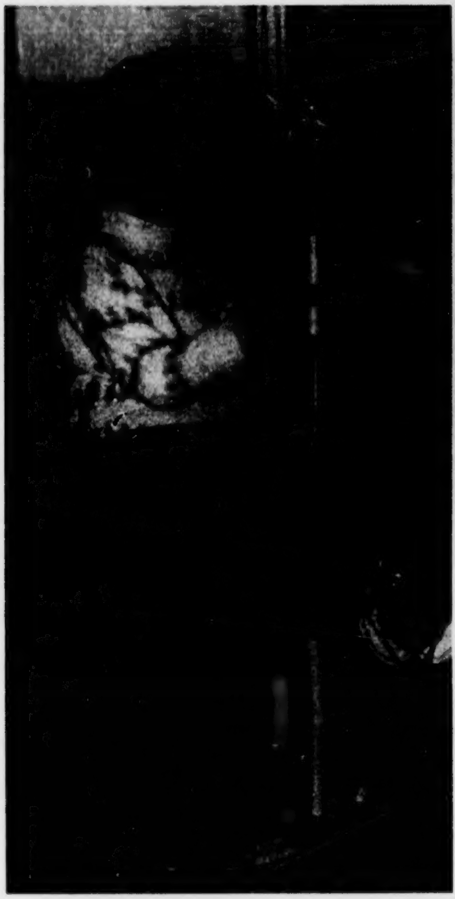

Figure 2 (left). Placing pieces on the organiser to illustrate place value and the base 10 structure of decimals

Method

Six primary schools had participated in the longitudinal study of the *Learning Decimals Project* (Stacey and Steinle, 1999a, 1999b) which had three years of data showing the numbers of students who were apparently expert at understanding decimals and the numbers who held misconceptions of various types about decimal numeration. Two of the primary schools had high percentages of apparent experts and so were not suitable for targetted instruction. After eliminating a distant school, we approached the three remaining schools and invited them to participate. One school, through 'M' their Grade 6 coordinator, willingly agreed to be involved.

Two members of the research team visited the school and spent about one hour with the four grade six teachers (M, S, X and Z) who all agreed to participate. M, S and X taught grade six, and Z taught a combined grade five and six class. Each teacher received two booklets and a set of LAB that was demonstrated. One booklet contained lesson plans for using LAB

(Archer & Condon, 1999) and the other contained general lesson ideas for teaching about decimals (Condon & Archer 1999). (Some of the contents are described below.) Teachers were asked to use as many activities as they wished, but record their experiences of five activities from each booklet on simple feedback sheets supplied by the project team. They were asked to report on how they used LAB, and given space to record their observations of children, as well as general comments. They were also asked to provide a written account of their thoughts on teaching decimals during the time of the intervention. Teachers were asked to send back their responses at the end of the following school term (about three months later).

The teachers offered to test their students prior to the intervention, to obtain more accurate data on their students' misconceptions to supplement the longitudinal data already in the *Learning Decimals* database. Ninety-eight children completed the Decimal Comparison Test, a one-page 5-minute test which presents 30 pairs of decimals and asks students to nominate which of each pair is the larger. (Refer to Steinle & Stacey, 1998 for details). The test results were returned to the teachers, with a description of what the test indicated about each child's thinking. Early in term 4, the school was contacted again and a member of the research team visited the school to finalise the post-testing and meet with teachers again to discuss their participation in the project. Ninety-six children were tested on the second occasion, making 87 children who were tested twice.

Results and Discussion

Although all four teachers had agreed to participate, only M (the Grade 6 coordinator who had volunteered the school for the study) trialled the activities and provided a record of what she did. She commented that other curriculum priorities prevented her and the other teachers from completing the task. Thus her reports were limited to a record of three activities using the LAB model, and four activities from the Lesson Ideas Booklet.

At the meeting, teacher S explained that LAB had been used at least once. Class Y had not used them at all. She was on leave at the time of the first meeting and there was no record of her temporary replacement X, who had attended the training session, having used the materials. Despite repeated requests for information after the end of the study, no record ever became available of use of the activities by Z and it seems reasonable to assume there was no use in this class either. Thus we loosely defined three categories of use: Most use (Class M), Some use (Class S) and No use (Class X/Y and Class Z).

Pre- and Post-test Results

A total of 87 children from four different classes were tested twice and the data below is only for these students. All were grade six classes except for Class Z, which was a combined grade 5 and 6 class. Data collected earlier by the *Learning Decimals Project* over a period of three years (1994-1996) indicated that, for this school, 41% of grade 6 students were apparently expert in decimal notation. In the cohort of 1999, 46 children (53%) tested as apparent experts, a slightly higher proportion than previously.

Table 1 shows the results of the Decimal Comparison Test before and after the intervention for each of the four classes. Improvement occurred for the classes that used the resources, particularly the improvement from 33% to 63% expert for Class M. No improvement was evident for the two teachers for whom there was no record of any intervention. This very clean result indicates that deliberate attention to this topic is likely to make a substantial difference to students' understanding at Grade 6, which will in turn put them in a much better position to understand secondary school mathematics.

Table 1
Number and percentage of experts by class on pre- and post-Decimal Comparison Tests

	Class M		Class S		Class X/Y		Class Z		Total	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Number of students	24	24	23	23	20	20	20	20	87	87
Number of experts	8	15	19	23	10	10	9	9	46	57
Percentage of experts	33%	63%	83%	100%	50%	50%	45%	45%	53%	66%
Percentage gain	30%		17%		0%		0%		13%	

In an attempt to identify any trends in how students' knowledge changed, the data for classes M and S were combined. The Decimal Comparison Test classifies students according to the misconceptions in four major categories: the apparent experts; students who exhibit various of the longer-is-larger misconceptions; students who exhibit various of the shorter-is-larger misconceptions; and unclassified students who do not show any known pattern in their responses. Table 2 reports the changes in classification that occurred over the intervention period.

Table 2
Numbers of students by classification from pre- to post-test for classes M and S (N=47)

Pre-test classification	Post-test classification			
	Apparent experts	Longer-is-larger	Shorter-is-larger	Unclassified
App. experts	27	0	0	0
Longer-is-larger	2	0	0	1
Shorter-is-larger	1	0	0	2
Unclassified	8	3	0	3

All 27 students who were initially classified as apparent experts retested similarly, which suggests that once students reach expertise, they generally retain it. This finding is consistent with other studies using this classification scheme (e.g., Stacey & Steinle, 1999a). Over half of the non-expert students (55%) tested as apparent experts after the intervention period. This is an encouraging result given that Stacey and Steinle (1999a) found that over a period of about six months only about one third of non-expert students in a very large sample moved to expertise. Another encouraging result is that none of the students with misconceptions retained them, either moving to apparent expertise or into the unclassified category. Examination of individual test papers of the students who moved into the unclassified category revealed small improvements in understanding for all three.

Teacher M's use of LAB and the Lesson Plans

As discussed above, Teacher M was the only teacher who provided a written record of what she did with her class and made the most use of the resources. Since she achieved significant gains in performance, we examined more closely what she did.

LAB Activities. The teacher selected the initial three activities. These included familiarisation with the pieces and the organiser; creating decimal numbers with the pieces; placing LAB pieces on the organiser and writing down the number; and comparing decimals of different length. She commented that "the children reacted favourably to the material" and that "most children found this work to be very valuable for their understanding of decimals."

Lesson Ideas. The teacher selected the initial four activities (*Number trails, Decimal Skip Counting, Number between, Stickers game*), details of which can be found in the booklet. *Number Trails* and *Decimal Skip Counting* both involve starting with a particular number and adding or subtracting a constant number, preferably mentally. For example, a beginning class might start at 0.3 and successively add 0.1, producing the sequence 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and then erroneously 0.10. An advanced class might start with 0 and successively add 0.125, obtaining the sequence 0, 0.125, 0.25, 0.375, 0.5 etc. Both of these activities can be used to expose misconceptions, including difficulties with column overflow (eg. the number after 0.9 is 1 and not 0.10). *Number Between* starts with the teacher nominating two endpoints of a segment of the number line and children have to nominate any number between the endpoints. The nominated number and one of the previous endpoints become the endpoints of a new segment. As the activity continues, the number line is divided into smaller and smaller segments. Thus children gain an understanding of the value of numbers from their relative position on the number line and an appreciation of the density of decimal numbers. *Stickers* involves students arranging themselves in order from smallest to largest according to the decimal number they have been given on a sticker. Depending on the numbers chosen, the common misconceptions about the size of decimal numbers can be addressed by these activities, as well as issues such as the function of zeros in different positions in a numeral. The teacher reported that students participated enthusiastically in all activities. She noted the effectiveness of *Number Trails* for addressing column overflow problems, and commented that LAB was useful for helping children to visualise the numbers and understand, for example, why 0.9 plus 0.1 is 1.0 and not 0.10. She also noted that column overflow problems became less evident as children continued to play the game. She reported that children "learnt heaps" from *Number Between,* with children of all abilities participating enthusiastically.

It was evident from M's feedback that the resources were not always used in the way intended, nor to their full potential. This is not intended as a criticism; teachers need to use resources more than once in order to exploit their full potential. However it does point to the need for development, both of the resources (e.g., making instructions more specific; including more detailed explanations) and for teachers themselves (e.g., to increase their understanding of decimals and how to teach the topic effectively). An action-research model that involves teachers trialling and refining the activities and meeting on a regular basis to discuss their progress may be needed, but it would be much more labour intensive than the low intervention model trialled in this study.

Discussion with Teachers

The follow-up meeting and discussion was conducted at lunchtime (the only available time) and due to heavy commitments, only three teachers could attend (M, S and Y). The general impression was that of a school that expected a lot from the teachers, who had been very busy with extra work, such as orientation programs for secondary school.

The group asked to discuss the sorts of problems children have with decimals and the strategies they found effective in overcoming them. The problem first mentioned was column overflow (that 17 tenths, for example, is not 0.17) and M repeated what she wrote on her feedback sheet about the effectiveness of *Decimal Skip Counting* (see above). S mentioned that she did in fact make use of the LAB in this activity, and that it appeared to trigger conceptual change:

> Actually, it was good to show what happens [when there are more than nine of any denomination] because there's nowhere to go so you've got to take it to the left concretely. And I remember doing that at the time because some of them said, "Oh yeah!"

M's description of using the activity *Number Between* in the interval between 0 and 1 is also suggestive of conceptual change. She remembered it powerfully demonstrated that small decimals (i.e., zero point something) could never be less than zero:

> I just remember putting the number line up on the board and showing them where nought was, and one, and then getting them to halve. Remember one of the activities was getting them to write a fraction in between each time, and they were getting smaller and smaller and smaller and they suddenly realised they could never get smaller than nought.

This discussion brought up a related issue; that pressures of competing tasks and responsibilities interfere with good teaching:

> S: You need to talk to them don't you, and say, well, why do you do this?
> M: That was coming out with that maths conference we went to. They said that teachers are too busy giving work and looking at results.
> M: What we need to do is talk to them
> S: Talk to them.
> M: —and get them to think about, and even getting them to write why they got a certain answer.

These pressures may partly explain why the teachers who initially agreed to participate failed to do so. As discussed earlier, the only teacher who made a significant effort to use the materials was the initial contact. She clearly felt an obligation to complete the task. The other teachers, faced with competing commitments and responsibilities, opted out. M reported that, having already taught decimals earlier in the year, they found it difficult to justify spending more time on the topic when they were expected to be covering other areas of the curriculum. An intervention in the period when the teacher plans to teach decimals may have increased participation.

Finally, it emerged in the discussion that teachers did not make use of the pre-test results which the project team supplied to learn about children's difficulties or target children with particular misconceptions. This was the case even though an explanatory sheet was sent along with the pre-test results, and the project team also invited them to ask for clarification. This emphasises the need for professional development to assist teachers identify both students' difficulties and conceptual change when it occurs. It is also consistent with a personal communication from Kathleen Hart, the director of the large CSMS project in Britain around 1980. Very large numbers of children were interviewed and good descriptions of children's thinking were carefully prepared for their teachers. Hart believes that there was no evidence that any teacher ever used any of this information.

Conclusions

This study investigated the impact of resources specifically designed to address decimal misconceptions. The intervention was minimal, in an attempt to model as closely as possible what happens in practice when new resources become available. Use of the resources was unexpectedly low, although this genuinely seemed to be due to other commitments rather than disappointment with the materials themselves. The teachers who used the resources with their children achieved an encouraging improvement in decimal understanding: over half of the non-experts tested as apparent experts after the intervention. Thus children can be given a strong foundation in decimal numeration but it requires deliberate attention from their teachers.[1]

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Assessment Strategies: Using Multimedia to Promote Transfer to Classroom Practice

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This study set out to investigate whether assessment strategies for mathematics classrooms transfer to real world practice when learnt within an authentic context. A multimedia program was designed to incorporate characteristics of 'situated' learning environments and cut to CD-ROM. Preservice teachers used the program in their mathematics method classes, and they and their supervising teachers were interviewed regarding their use of strategies while on professional practice. Results show that all students used a variety of assessment strategies, and according to the beliefs of the students themselves, were influenced in their use of strategies by the multimedia learning environment.

Learning Assessment Strategies

The lack of transfer of pedagogical skills from the theory of teacher education to the practical reality of the classroom has been a source of concern to teacher educators for some time. Several writers have expressed concern that despite the emphasis in teacher education courses on 'reformist' methods of teaching mathematics, teachers frequently revert to methods of teaching derived solely from their own experiences as students (Ball, 1994). Others have noted that preservice teachers' experiences in classrooms during their professional practice have proved inadequate because often students observe teaching 'driven by texts and tests', or are ill equipped to detect the subtle differences between quality and mediocre teaching (Mousley & Sullivan, 1995). Despite the variety of innovative and effective assessment techniques, teachers generally continue to limit their means of assessment to a narrow range of pencil-and-paper methods (Australian Education Council, 1991; National Council of Teachers of Mathematics, 1995).

The present study set out to explore the use of a multimedia program on assessment strategies within a preservice teacher mathematics method unit, and to investigate the extent of transfer of strategies to classroom practice. The learning environment was designed according to critical characteristics of situated learning (described in Herrington & Oliver, 1995; Herrington & Oliver, in press), which focused on the creation of an authentic context, activity and assessment, together with expert performances, multiple perspectives, opportunities for collaboration and reflection, and coaching and scaffolding by the teacher. Transfer was thought to have occurred if firstly, students using the interactive multimedia program on assessment had a good understanding of the types of assessment appropriate in the mathematics classroom and were able to articulate this understanding; and secondly, they employed a variety of the assessment techniques shown in the program, as opposed to the predominant use of pencil-and-paper tests (Cognition and Technology Group at Vanderbilt, 1993).

Design and Development of the Multimedia Program

In order to produce an effective learning environment on the issue of assessment in mathematics, it was necessary to provide preservice teachers with the experience of observing expert teachers using different types of assessment in classrooms. McLellan (1993) points out that a situated context can be represented in *the actual work setting, a highly realistic or 'virtual' surrogate of the actual work environment* or *an anchoring context such as a video or multimedia program* (p. 8).

In order to use the first method, the actual work setting, it would have been necessary to take groups of preservice teachers to a large number of schools and to have them observe expert teachers in their classes, in addition to their professional practice. This scenario, while authentic, may not have provided efficiencies within a teacher education program. As Spiro, Feltovich, Jacobson and Coulson (1991) contend, learning a complex concept from 'erratic exposures to complex instances with long periods of time separating each encounter, as in natural learning from experience, is not very efficient' (p. 30).

The second context recommended by McLellan (1993) as acceptable for situated learning, a 'virtual surrogate' of the actual work environment (such as aircraft simulators), was also ruled out very quickly on the basis of prohibitive costs of development, and lack of resources within the university for use of the finished resource.

The anchoring context seemed the most viable. A video, or series of videos, as an anchoring context was rejected because of the linear format which could not provide ready student access to expert performances and multiple perspectives. Audio-tape and text did not provide the appropriate visual elements to allow peripheral observation of the authentic classroom context, important elements in real-life learning.

One medium that did not have these restrictions was computer-based multimedia. The combination of video clips, sound, text and graphics meant that interactive multimedia was capable of supporting 'the kinds of more intimate, supportive, learning environments called for by the constructivist perspective' (Perkins, 1991, p. 22), and presenting it in an efficient and accessible format. Multimedia would also enable a 'criss-crossing of the conceptual landscape' in such a way that relevant examples can be explored in close-proximity to each other (Spiro et al., 1991, p. 30). In addition, several exemplary published packages within Australia—notably *Investigating Lake Iluka* (1993), *Exploring the Nardoo* (1996), and *Learning about Teaching* (Mousley, Sullivan, & Mousley, 1996)—provided valuable models for the development of successful multimedia learning environments.

Preservice teachers using the program to investigate assessment strategies would need to be able to observe experienced teachers in the field demonstrating a range of strategies and techniques, and to then reflect on the most appropriate strategy to use in a particular situation. Video clips of classroom scenes and interviews appeared to be an appropriate means to provide such opportunities to the students who would use the program. Other important requirements of the program were that the context would need to be situated in a real or simulated classroom, and authentic activities could require students to address the problems of assessment and to select their own alternatives to paper and pencil tests. It was essential to provide multiple perspectives on assessment, and in so doing, focus strongly on the classroom experience. On this basis, the elements included in the final multimedia program were:

- Video clips of teachers using various assessment techniques (see Table 1)
- Video clips of teachers' comments of the strategies
- Video clips of children's comments on the strategies to present their own thoughts
- Interviews with experts in the field to provide theoretical perspectives
- Reflections by third year preservice teachers to provide practical advice
- Text descriptions of each assessment category
- Teacher and children work samples
- An electronic notebook to enable students to copy text and to write their own ideas
- Problems and investigations to enable the students to complete authentic tasks.

Assessment strategies were identified from the literature and are listed below, together with the scenario appearing in the video clips, in Table 1.

Table 1
Assessment Strategies Featured in the Multimedia Program

Assessment Type	Technique	Scenario
Observing	Checklists	Teacher observing students and marking a checklist
	Anecdotal	Student doing problem, teacher writing the record
Questioning	Higher order	Teacher asking how and why questions
	Factual	Teacher asking basic facts questions employing wait-time
	Open-ended	Teacher and students working on good questions
Interviewing	Structured	Teacher doing a Newmann Error Analysis
	Open	Teacher interviewing on understanding of a concept
	Parent	Teacher interviewing parent
Testing	Diagnosis	Teacher using calculator to diagnose
	Performance-based	Students attempting a tangram activity
	Pencil and paper	Teacher discussing student errors and misconceptions on a test
	Multiple choice	Teacher giving instructions for a test
	Problem solving	Teacher giving a problem and developing a rubric with students
	Attitude	Teacher asks: what do mathematicians do?
Reporting	Oral	Student presenting an oral report to the class
	Written	Teacher giving advice on how to do an investigation
	Portfolio	Leafing through student portfolio of work
	Investigation	Marking an investigation
	Modelling	Teacher viewing students modelling projects
Self - Assessment	Journals	Teachers explaining how to write a journal
	Reflective prompts	Teacher directing class to fill in a lesson check
	Self questioning	Teacher going through a self-question checklist
	Peer assessment	Teacher getting students to write their own questions

The Study

The interactive multimedia program on assessment was cut to CD-ROM (Herrington, Sparrow, Herrington, & Oliver, 1997) and introduced to a class of approximately 24 preservice secondary teachers studying mathematics education method. The students were midway through the first semester of the second year of their course. The lecturer was asked to recommend six students who might be considered as 'typical case' (Miles & Huberman, 1994) with an equal representation of gender, to be observed and interviewed for the study. Students were grouped in pairs as they preferred to maximise collaborative interactions.

To begin the lesson, the lecturer held a discussion with the students on the issue of assessment in mathematics. The discussion was prompted with questions such as: What does assessment mean in mathematics? How were you assessed in mathematics when you were at school? Minimal instruction was given in the use of the program itself, except for a brief introduction to the elements of the program and how each could be accessed through the main 'menu', the classroom interface. Students were given an authentic and complex activity to investigate while using the program, simply in the form of two letters. The activity required the group of students to assume the identity of new teachers in a school given responsibility to prepare a report to staff on assessment strategies. The request has been prompted by a letter to the school from a parent whose child becomes very anxious before each test, and who requests whether there are alternative means of assessment that could be used in mathematics. The six students were observed using the interactive multimedia resource over three weeks in their normal class time.

Approximately five weeks after the conclusion of the use of the assessment multimedia package in their mathematics method course, the six students in the main study completed a two week professional practice in six different metropolitan schools. All the students were required to teach mathematics classes in this practice, and it was expected that they would have the opportunity to implement different assessment strategies. In order to assess whether students used a variety of assessment strategies during their mathematics classes on professional practice, both the students and their supervising teachers in the schools were interviewed and the comments were analysed.

Students were given a list of the assessment techniques featured in the interactive multimedia program on assessment (Table 1) and asked whether they had employed any of the strategies listed. The questions mirrored those asked of the supervising teachers to assist structural corroboration of data. Findings are presented below, with names substituted with pseudonyms.

The Findings

A situated view of transfer is not one that suggests that a person can acquire a set of skills that can be lifted and applied in a totally novel situation. The view of transfer adopted by the proponents of situated learning and used here, is that knowledge is more likely to be transferred to novel situations when it is learnt in the context of use and is 'a central or integral part of one's cognitive structure' (Prawat, 1992, p. 375).

It appeared from analysis of the comments made by students that assessment issues had been incorporated into their cognitive structures. They spoke openly and knowledgably about assessment issues after teaching practice. They acknowledged the complexity of the area but were well acquainted with the types of assessment that might be suitable in the mathematics classroom, and they used appropriate language with familiarity and ease. None of the students thought that being unaware of appropriate assessment strategies was a relevant factor in their teaching practice. The students were also aware of the usefulness of assessment in performing more functions than the summative appraisal of students' understanding (for example, as listed by authors such as Burton, 1992; NCTM, 1995; Clarke, 1988) including: to improve the teaching of the child, to inform the teacher, and to make instructional decisions.

Comments by the majority of students indicated that they were aware of many important roles for assessment, and that it could be used, as noted by Jonassen (1991) as 'less of a reinforcement or behaviour control tool and more of a self-analysis and metacognitive tool' (p. 32). For example, Louise distinguished between 'formal' and 'informal' assessment when asked whether assessment was necessary on short professional practice:

> It depends what you mean by assessment. When we used the multimedia, it looked at questioning and monitoring as part of assessment and some people don't think that is assessment. So I think maybe formal assessments like long investigations where a lot of work is done, maybe that's not necessary, but I think the informal like questioning and monitoring is. (Interview with Louise)

Carlo also pointed out that assessment has a critical role in helping him to monitor his own performance as a teacher:

> As a teacher, you need to know where they're at and the objectives you've set yourself. I think it's quite important because you have a feel for how you're doing, especially if you can see that they're learning something. It actually allows you to assess your own teaching. (Interview with Carlo)

The students were able to speak knowledgably and confidently about the issue of assessment. Lave and Wenger (1991) point out that learning the language and stories of a community of practice is necessary for full participation in that practice, and the students' ability to speak both within and about the practice was clearly evident in their discussion.

The second indicator of transfer as described above, was that students employed a variety of the assessment techniques, shown in the multimedia program, in the classes they taught while on professional practice. The findings for each student were analysed and evaluated. These findings are given for one student only, as an example, below.

Example of Students' Use of Assessment Strategies: Rowan

Rowan completed his professional practice in a co-educational, government high school catering for Years 8-12. Under the supervision of Rob, Rowan taught Years 8, 9, 10 and 11 mathematics. Rowan's and Rob's reports of the assessment strategies used during the professional practice are presented in Table 2.

Table 2

Assessment Strategies used by Rowan During Professional Practice

Source	Observing	Questioning	Interviewing	Testing	Reporting	Self-assess
Rowan	Anecdotal records and checklists	Factual questioning	Open interviewing	Two formal tests	None	Reminders as reflective prompts
Rob	None	Factual questioning	Open interviewing	Pencil and paper test of parabola	None	None

While Rowan's supervising teacher was not aware of any observation of students, Rowan himself reported that he tried to use both types of observing presented in the interactive multimedia program on assessment, particularly anecdotal records:

> I was trying to do both of these actually, not fully into the checklists, but observing and ... writing down some little notes about how certain ones were doing ... if you've got those little notes there you can see that they've been having a problem. (Interview with Rowan)

Rowan admitted to having a problem with group questioning, pointing out that he found it difficult not to use rhetorical questions, a point also made by Rob. Nevertheless, he showed that he was willing to use the strategies and try to perfect his techniques:

> Towards the end, in certain classes I was asking the How and Why questions. To actually find out where they're at, I prefer to use individual questions, you know, going around ... then I'll tend to use like all three types of questioning techniques. (Interview with Rowan)

Both the supervising teacher and Rowan concurred that he used open interviewing to help students who were having difficulty understanding mathematical concepts, and that this assistance was provided on Rowan's own initiative. Rowan used two pencil-and-paper tests during his professional practice. The tests were administered in a formal manner, as explained by the supervising teacher:

> [He] stood at the front of the room explaining that it was a test, what they were to have on their desk, pens, pencil and calculator, gave out the test, ensured everyone knew it was 2 pages, right you've got 40 minutes, look at the clock, get to work. (Interview with Rob)

Rowan explained, however, that he used the results as the basis of a discussion on students' understanding of the problem. This procedure was demonstrated in the scenario on pencil-and-paper testing on the interactive multimedia program. Rowan's comment reveals an insight which suggests his use of the tests was to gain a true assessment of students' understanding rather than to obtain a score:

> The teacher wanted me to keep the tests for my own record, but the last day was a sports test and only 10 [students] were there, so I—this was interviewing as well—I went through the test and discussed things and clarified things where they'd got it wrong and to see whether they actually did know it. A lot of the time in a test they get it wrong but they still might understand the concept. That's all part of assessment, not just whether they got it right on the day. (Interview with Rowan)

Rowan did not use any reporting strategies to assess students' understanding during his teaching practice, nor did he use any self-assessment techniques other than encouraging students to assess their own understanding at regular intervals.

Rowan appeared to use a variety of assessment strategies in his classes during professional practice. Like the other students, he was limited in the strategies he could use by the time constraints of the placement. Rowan's comments reveal a far deeper understanding and application of assessment than was evident to his supervising teacher.

The remaining students' uses of assessment strategies are summarised in Table 3. Many of the students expressed concern about the difficulties associated with implementing procedures and techniques in a short space of time on professional practice. All the students in the study, were limited in the types of assessment strategies they could use, but chose varied methods of assessment when they had the discretion to do so.

Discussion

Prior to the commencement of the study, a prediction was made that the students would use a variety of assessment techniques in their mathematics classes during teaching practice, and this was true of all six students. In spite of the fact that one student reported the prediction to be untrue and another was unsure, all the students did use a variety of strategies. Evidence to support this conclusion was provided by the students on their own admission in interviews, and this was generally corroborated by their supervising teachers.

If students had reported using only pencil and paper tests to assess students, only *Testing* strategies would have been used. However, as shown in Tables 2 and 3, all the students used assessment strategies from the *Questioning* and *Interviewing* groups, most used *Observing* and *Testing*, and some students used *Reporting* and *Self-assessment* strategies. Generally, the students used the assessment strategies that had been predetermined for use by the supervising teacher. However, this was supplemented by the use of strategies that were under the students' own control.

Table 3

Assessment Strategies Used by Students during Professional Practice

Source	Observing	Questioning	Interviewing	Testing	Reporting	Self-assess
Evie	Anecdotal records	Mainly higher-order	Open and structured interviewing	Pencil-and-paper problem solving activity (formal requirement)	None	None
Teacher: Carol	Notes on individual students	Mainly factual recall	Open interviewing	As above	None	None
Louise	None	Higher order and factual recall	Open interviewing	Informal pencil-and-paper tests	Students report orally to the class	None
Teacher: Michael	None	Factual recall	Open interviewing	Informal pencil-and-paper tests	As above	None
Carlo	Informal observation	Open-ended questioning	Open interviewing	Mental tests to start Year 8 lessons	Observed modelling	Informal reflective prompts
Teacher: Peter	None	Factual recall questions	Open interviewing	None	None	None
Zoe	Checklists & anecdotal records	Higher order	Open interviewing	None	Oral reports	None
Teacher: James	None	Factual recall	Open interviewing	None	Oral reports	None
David	Anecdotal	Higher order questions	Open interviewing	Pencil and paper test	Students explain their solutions on blackboard	Occasional reflective prompts
Teacher: Frank	Anecdotal	Open ended questions	Open interviewing	Pencil and paper test	As above	None

Analysis of the data shows that all the students could speak knowledgably and confidently about assessment, and all the students used a variety of techniques to assess children's understanding. The students appeared to be influenced very strongly in their use of assessment strategies by the supervising teacher. However, even when assessments had been planned in advance by the supervising teacher, all students used techniques that they were able to use without the contribution or agreement of the supervising teacher. Five of the six students attributed their use of assessment techniques to the interactive multimedia program. Two factors militated against a realistic appraisal of whether the students' use of the assessment program influenced their adoption of a variety of assessment practices in a real-life classroom: the substantial influence of the supervising teacher, and the brevity of a two week professional practice. The choice of the professional practice nearest to the use of the interactive multimedia program meant that any transfer effect could more reliably be attributed to the influence of the situated learning environment on assessment rather than an accumulation of influences and practice in the students' entire course.

While many models of transfer exist, it was difficult to find an appropriate model of transfer to use in the study. While transfer did appear to occur within the parameters given, many questions remain about the types of transfer and whether the effect is long-term, for example: What is an effective model of transfer for situated learning environments? Do students apply assessment strategies presented in the interactive multimedia program when working as fulltime teachers? Is there long-term retention of assessment strategies?

Ideally, the students needed to be appraised in the real world context, possibly in their first or second year as practising teachers, and over a lengthy period of time. (Such a follow-up study is currently under way, where data is being collected from the four students who are teaching, and who are now in their second year.)

Nevertheless, in the current study, the multimedia program on assessment influenced the types of strategies students employed and their thinking about assessment as they taught mathematics and other classes during their professional practice in schools. Further research is needed to verify these tentative findings.

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Preservice Mathematics Teachers on Professional Practice: How can the Internet Help?

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This paper describes the development of a web-based resource designed to support preservice mathematics teachers on school practice. The development of the site resulted in part from a needs analysis conducted with a focus group of students and reported on below. As a result of focus group discussions with students from the target group, the website was developed and refined to provide a comprehensive resource to suit their needs while on professional practice. Elements of the site, together with potential further developments and research, are described.

Introduction

Despite Strommen and Lincoln's (1992) claim that: "The technological changes that have swept through society at large have left the educational system largely unchanged" (para no. 3), almost a decade on, technology has made significant, and one might argue, sustained inroads into higher education. The use of multimedia and the internet have provided educators with powerful tools to create effective and immersive learning environments (Wilson, 1996; Jonassen & Reeves, 1996; Jonassen, 1995). Such technology-based learning environments (for example, Herrington, Sparrow, Herrington, & Oliver, 1997; Mousley, Sullivan, & Mousley, 1996) have been particularly productive in providing pre-service teachers with authentic and realistic contexts within which to explore problems and issues that face teachers today, *before* they move to real classrooms.

The use of a range of technologies in the preservice teachers' learning environment has meant that they have become adept at accessing appropriate technology, and are able to use electronic resources and the internet effectively as an information source. However, the tendency has been to neglect these powerful resources to support preservice teachers once they leave the confines of university and move to schools for professional practice.

The Web as a Support Tool for Preservice Teachers on Practicum

The purpose of this paper is to describe the development of a web-based resource designed to support preservice mathematics teachers on school practice. The idea to develop such a resource arose from a number of concerns of mathematics teacher educators at (deleted) University. As with many teacher education programs, cost reductions have led to a cut back in university staff preventing many preservice teachers from direct involvement with university supervisors on their school practicum. Students are receiving less specialist supervision than in the past to the extent that on block practices, more than one third have a university supervisor outside their chosen specialism. In many cases, school visits have been replaced with phone calls, with the handing over of responsibilities to school staff. Other problems with the professional practice system have been identified in the literature: the disparity between the theory students learn at university and the practical realities of the classroom (Goodlad, 1991); the removal of students' "web of support" while on practice (Roddy, 1999); supervising teachers' inability to relate practical teaching episodes to general pedagogical principles (Kennedy, 1991); the concern that students exposure to a small number of supervising teachers limits the value of the experience (Richardson-Koehler, 1988); and the practical focus on *how* to teach at the expense of having to make decisions about *why* and *what* to teach (Zeichner, 1981).

Many students also find themselves in schools with limited resources for lesson planning, particularly the type that reflect the teaching approaches developed in methods courses, such as concept development through problem solving, and investigations.

With these concerns in mind, and a recognition that the internet could be employed productively for such a purpose (Casey, 1994; Hutchinson & Gardner, 1997), a Committee for University Teaching and Staff Development Grant (CUTSD) was applied for and awarded to develop a web site that would provide:

- Open communications channels, to discuss problems, with peers and lecturing staff;
- A database of prepared lesson plans accessible through a simple search engine;
- Links to other web-based resource materials for use in lessons;
- Video clips of teaching strategies with teacher and student commentary; and
- Answers to frequently asked questions by students on school practice.

Assessing the Need

To establish the effectiveness of the resource, focus group interviews were conducted with eight students, studying mathematics education method, to determine their perceived needs on school practice and their views on how a web resource could assist. These students were required to attend 18 weeks of school practice during their teacher training, varying from continuous blocks of time in a school to distributed practice.

Focus group techniques were used to elicit opinion from the students. The interaction between the students proved to provide data on issues and problems, which were of greater richness and depth than might have been collected by individual interviews. As Morgan (1988) notes one of the strengths of focus group methodology is the intentional and explicit use of the group interaction: "to produce data and insights that would be less accessible without the interaction found in a group" (p. 12).

The purpose of the focus group discussion was to gain from the students their ideas about the type of support that would be most helpful to them as they prepare to teach mathematics lessons, and involve themselves in school life generally. In particular, we wanted to know the types of problems they have, and how the university might be able to assist in these sometimes prolonged periods in school where the students are without their traditional support structures. In the focus-group discussion, students were asked questions related to two principal areas:

1. What are some of the difficulties you face on professional practice? (*Prompt questions*: Can you tell us about some of your professional practice experiences? Have you had any difficulties relating to lesson planning? Teaching the students? Dealing with supervising teachers? Resources? Did you have difficulty contacting lecturers? Have you found educational theory useful to your practice?)
2. Do you have any suggestions on how the university might be able to assist you to overcome these difficulties? (*Prompt questions*: What kind of resources could the university produce that would help you, e.g., booklets, videos etc? How useful would

a web-site devoted to teachers on professional practice in mathematics be? If you were to use such a web-site in your professional practice in school, what would you want it to provide? Content? Communication? Resources?)

The focus group discussion was audio taped and transcribed for analysis. Transcripts were analysed using techniques of qualitative analysis recommended by Miles and Huberman (1994): data reduction, data display, and conclusion drawing and verification. Student comments were grouped to reflect common themes. From these themes and comments, specific recommendations were formulated by the researchers on the type of web-based resource that would best meet the needs of students on professional practice. Table 1 provides a summary of responses for the first area of investigation, together with suggested solutions and their possible manifestation within an online resource.

Table *1*

Summary of Difficulties Encountered on Professional Practice with Possible Solutions

Area of investigation: *Difficulties and problems encountered on professional practice*

Problem	Example student explanation of problem	Possible solution	Possible manifestation in online resource
Problems with mathematical understanding	'Getting across to the kids, how to do things basically, fractions and things like that. Actually explaining to them.... the things I don't understand myself, it's hard to get across'.	Web-based resources that develop mathematical understanding	Links to appropriate websites
Disparity between the way students are encouraged to teach mathematics and the way they learnt mathematics	'I find the hardest thing is that, we were taught a certain way at school, and now we are being taught not to teach how we learnt it'.	Exposure to different teaching approaches	Video clips of teachers using a variety of approaches
Disparity between the way students are encouraged to teach mathematics and the way mathematics is taught in school	'Like not doing multiplication—tables wise—or not doing carrying and that sort of thing. I find it hard because they are still doing it that way in schools, but we are told we should steer away from that. It's really hard to know where to go'.	Exposure to different teaching approaches	Video clips of teachers using a variety of approaches
Lack of resources	'We have to plan the night before. Sometimes they give it to you a couple of nights before, and you have to find resources. You don't have enough time to run about here and there to find these resources'.	Provide resources for mathematics teaching	Lesson plans URLs of appropriate sites
Lack of ideas for lesson plans	'Trying to create some ideas to relate it to what you are trying to teach the children to do, and how to do it. We are told one example, like how to do a shop but not how to set it up and how to make it work'.	Provide students with a range of lesson plans related to strands and year levels	Lesson plans classified according to content, process and year level
Lack of practical strategies for dealing with large classes	'We are told ideas but not really how to go about them, like in a class of 30, you know its such a huge class'.	Access to broader educational issues such as class management	Links to other general educational websites

The student teachers gave several ideas on the type of support the university might provide to make their practice more enriching and productive. These are listed in Table 2, together with the researchers' view of solutions and manifestation in a web-based resource.

Table 2

Summary of Suggestions on how the University might Assist in Overcoming Difficulties

Area of investigation: *How the university might be able to assist in overcoming difficulties*

Suggestions for university assistance	**Example student explanation**	**Possible solution**	**Possible manifestation in online resource**
Practical exemplars	'I would like to be able to see a lot more in practice, like videos in schools, seeing demonstrations of these things, because it's easy enough to read in a book, but to actually see it puts a whole different perspective on things'.	Practical demonstrations of teaching strategies suitable for a variety of class levels	Video clips of teachers in real classrooms using teaching strategies
Resources for lesson ideas	'Have a Resource Centre, may be on the internet to give you some ideas'	Ready access to resources	Lesson plans with downloadable lessons resources Links to lesson plan websites
Support via telephone	'It would be nice to be able to speak to someone instead of an answering machine.'	Ready access to lecturer and other students for mutual support	Email access to lecturer Discussion board with other students
Australian web-based resources	'It would be good to have something Australian ... not dimes and nickels'.	Web-site with variety of cultures represented	Links to sites reflecting cultural and ethnic diversity
Easily accessible lesson support	'When you're on prac, you're just so busy you don't have time to go searching'.	Centrally located resources for just-in-time support	Fully searchable site covering a variety of resources

Generally, the student teachers were enthusiastic about having a variety of lesson planning ideas readily accessible, but they made it clear that they required much more substantial support. While just-in-time support was clearly important, particularly when students had little notice that they were required to give a lesson, more resources on learning mathematics and mathematical pedagogy, that could be accessed in a reflective way, were also required.

The feedback received from the students enabled the basic website to be developed with a more specific focus which would directly address the concerns and suggestions of the target group. The design of the Mathematics Education On the Web (MEOW) site is described in more detail below.

Design of the MEOW Website

The interface of the site resembles a typical (if well-equipped) teacher preparation workspace. It features a desk and other items that act as metaphors for the resources that can be accessed from the site. Figure 1 shows the central interface which acts as a main menu, and it is here that the students choose the type of resource or support they require.

The design of the MEOW website utilises databases to simplify management and provide a flexible, yet efficient means of displaying and representing the content. The database driven approach has been used for a range of reasons:

- management and display of media is separated and hence more easily controlled
- simplifying the design process by requiring fewer HTML source files
- simplifying the maintenance and extension of the website by not requiring any editing of display information, only content;
- cross referencing of lesson plan and teaching/assessment content for flexible navigation and discovery of information.
- future extensibility by the addition of note taking and "shopping cart" of useful MEOW links.

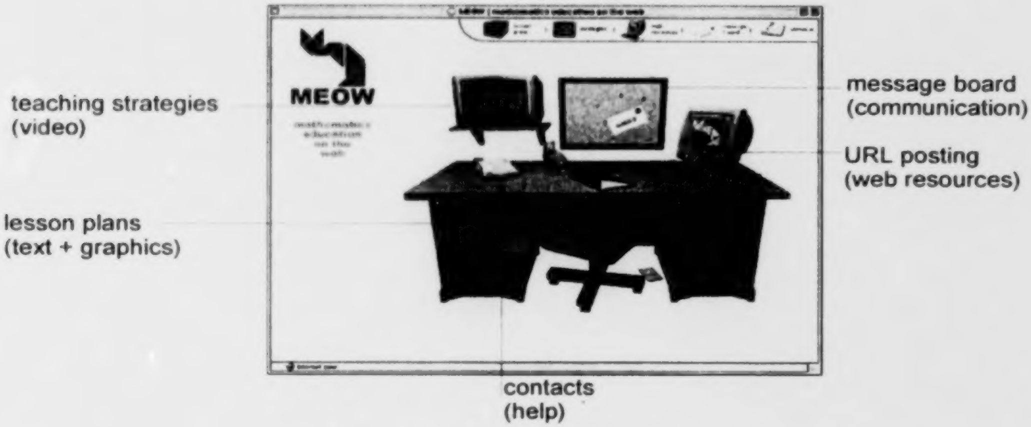

Figure 1. The main MEOW interface.

A student on professional practice can access the following elements:

Contacts: "It would be nice to be able to speak to someone"

Regardless of where the students complete their professional practice, students can have email access to their lecturer via the phone metaphor on the desk on the interface. Clicking on this image gives students access to the lecturer's phone number (with available times), together with a direct email link.

Message Board: "I find it hard sometimes going into prac"

Clicking on the message board gives students access to a threaded discussion board, where messages and all related postings are grouped so that students can follow discussions in logical sequence. If students have problems, they can post messages to the discussion board, where other students and the lecturer can make suggestions and provide support. In such a way, contact with traditional support groups can be re-established.

URL Posting: "You're just so busy you don't have time to go searching"

When students click on the computer on the desk, they have access to a range of relevant websites beyond the MEOW site. Students can access any of these sites by simply clicking on the link. They can also add any interesting sites themselves, together with a short annotation describing its value to other preservice teachers.

Teaching Strategies: "I would like to be able to see a lot more in practice"

Students wishing to access examples of classroom strategies can presently observe video clips of teachers in real classrooms demonstrating teaching and assessment strategies appropriate to mathematics classrooms. These strategies use a database to record a text description of each strategy, its type (assessment or teaching), and details about the video sequences available (Figure 2). Each screen is created dynamically from the teaching strategies database. Students can observe teachers demonstrating a particular strategy, check teachers' and students' perspectives on that strategy and read descriptions, enabling them to explore different classroom strategies reflectively.

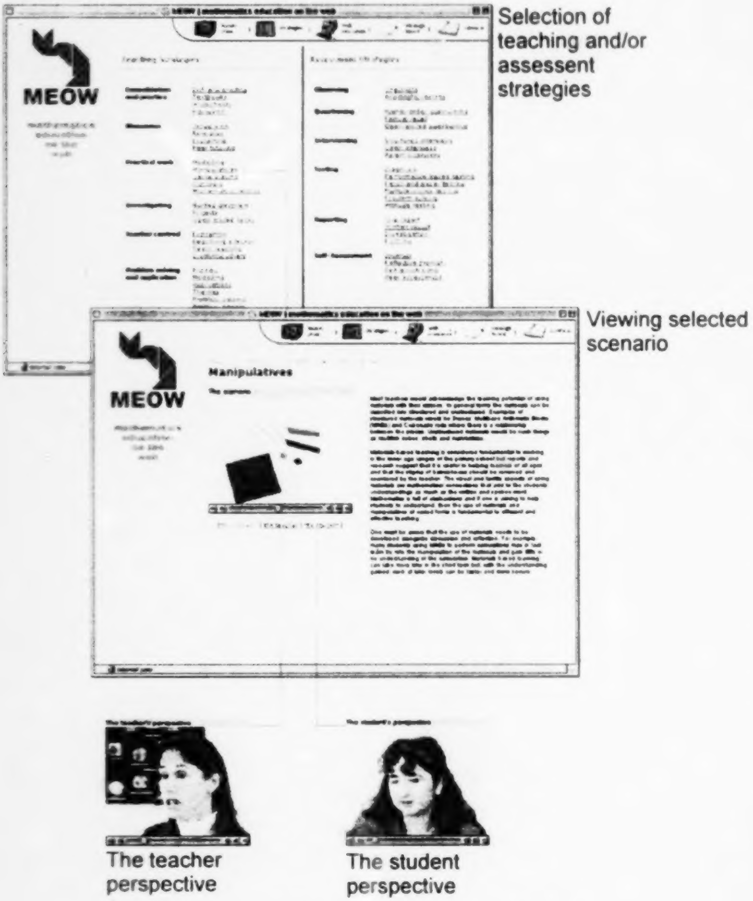
Selection of teaching and/or assesent strategies
MEOW
Viewing selected scenario
MEOW
Manipulatives
The teacher perspective
The student perspective

Figure 2. Teaching strategies in MEOW.

Lesson Plans: "We have to plan the night before"

A student on professional practice, when faced with little notice to prepare a lesson can access a variety of mathematical lesson plans from the drawers of the desk on the interface. The lesson plans database enables students to choose according to a number of different parameters, giving them lesson plans which precisely meet their needs. Lesson plans are categorised according to a number of keys: the *year level* (k-2, 3-5, 6-8 and 9-12), *content* categories (e.g., space, number, measurement) and the *processes* being used (e.g., classifying, visualising). These keys are not only used so that teachers can find plans relevant to their specific requirements, but are used to provide cross references between common content and

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processes. For example, in the Dicey boxes lesson (see Figure 3) the content area is *space* and the processes are *classification*, *prediction* and *visualisation*. A teacher interested in finding other exercises in *classification* can select from a menu of processes and find other plans that also relate to that category.

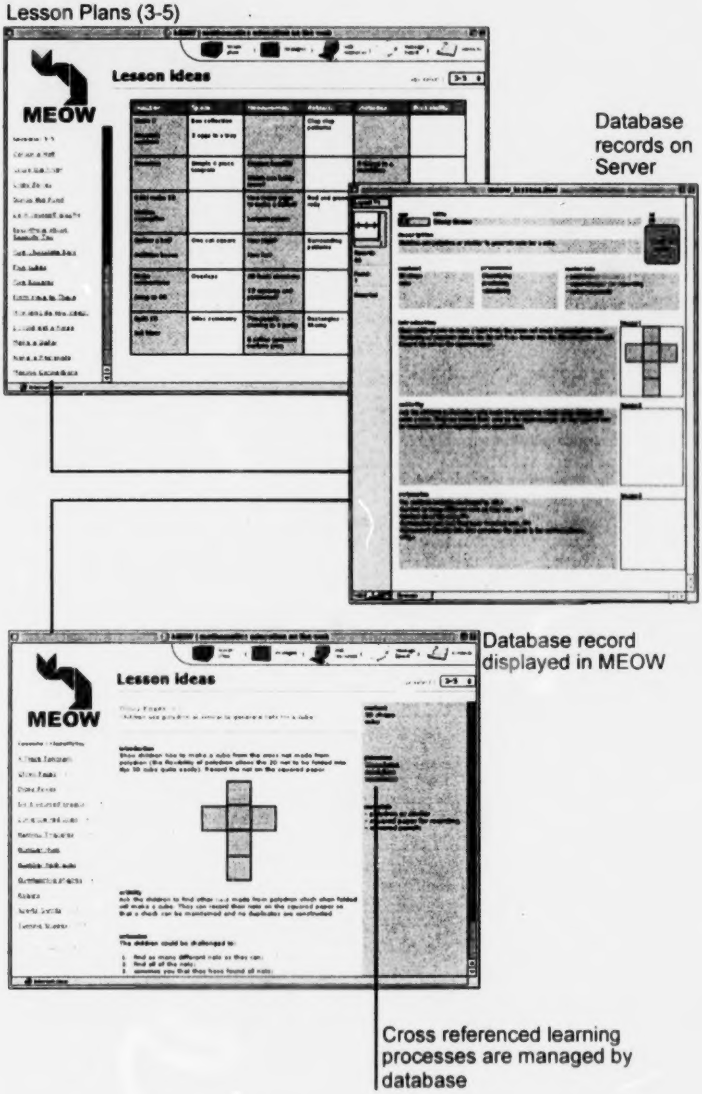
Lesson Plans (3-5)
MEOW
Lesson ideas
Database records on Server
Database record displayed in MEOW
MEOW
Lesson ideas
Cross referenced learning processes are managed by database

Figure 3. Lesson plans in MEOW.

The use of databases means that the content can be extended by simply adding records in the appropriate database—the display of menus and the cross-referencing of content is then automatically updated with no need to update the underlying display (HTML) code. In this way, the resource grows as a vigorous, comprehensive, and current resource.

Conclusion

A web resource of this nature is both dynamic and adaptable. Student teachers will be encouraged to add lesson plans and links to other web resources. A rich collection of advice should grow in response to students' periodic and prevailing concerns. The potential is there

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to extend the functions of the web site and its boundary of support. As a resource developed with Commonwealth funding it is available for use by any University and any mathematics education community that wishes to use its capabilities. However, in building a community of learners, the boundaries need not be national ones. The project team is now working with a team of mathematics educators at the *Institut für Didaktik der Mathematik* at the *Westfälische Wilhelms-Universität Münster* in Germany to extend the capabilities of the site for German student teachers.

While budget cuts will continue to be part of the higher education landscape, the internet can be a useful tool for reducing the isolation and providing preservice teachers on school practice support in both their pedagogical and communication needs. Further research is underway to investigate how students use the resource, and to determine their opinions on ease of use, functionality and development of the site.

Despite recent claims that the internet could be "the ultimate isolating technology that further reduces our participation in communities" (Nie & Erbring, 2000, p. 19), the project described here attempts precisely the opposite. When fully operational, it will provide all the benefits of collaboration, communication and human contact, without the physical constraints imposed by the lonely isolation of teaching practice. In so doing, it will maintain and enhance students' roles as valued members of a community of scholars, able to learn, plan, create, and consult as they practice mathematics pedagogy in a fully supported environment.

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Difficulties Learning Simple Addition Facts: A Persistent Problem

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Research carried out amongst adolescent students with learning difficulties is consequential to the delay/deficit interpretation of performance: Is performance immature and development simply delayed or does a processing deficit underlie performance difficulties so that development is impeded? This study investigated what strategies three students in Year 10 applied to simple addition problems. Performance was analysed on a trial-by-trial basis and strategy variability within problems was examined. After participating in ten years of formal schooling, these students still did not know the basic addition facts (including tie facts, and facts to problems with an addend of two and ten). Their performance based on strategy use was not consistent with a processing deficit interpretation but neither could it be considered simply delayed. Understanding the extent of the difficulty experienced by some students in acquiring basic number facts is imperative to developing appropriate interventions.

If students have difficulty retrieving simple arithmetic facts then they are likely to experience difficulty in other areas of maths learning (Ackerman, Anhalt & Dykman, 1986, Geary, 1994). Inefficiencies in basic fact knowledge have been identified as antecedents of difficulties experienced by students understanding and performing word problems, subtraction and multi-digit addition (Zental, 1990, Cumming & Elkin, 1999).

Students experiencing learning difficulties are not likely to develop a reliance on retrieval to perform simple addition problems in a way that is consonant with their peers (Russell & Ginsburg, 1984; Fleischner & Garnett, 1987; Gray, 1991; Geary, 1990; Goldman, Pellegrino, & Mertz; 1988). These students are referred to in the literature as being: 'learning disabled', 'mathematically disabled', 'not-so-good' and of 'below average ability'.

Developing retrieval strategies to perform simple addition problems involves a complex process of change whereby more efficient procedural strategies supplant less efficient strategies (discovered or constructed by the learner) until eventually answers are retrieved from a network representation of facts (Ashcraft, 1982; Siegler, 1995). Less efficient strategies are referred to as Count All Strategies and include the Long-Sum Strategy and the Sum Strategy (sometimes referred to as the Sum Strategy and Shortcut Sum Strategy respectively). (Using the example 3+5, each strategy is illustrated in Table 1.) More efficient counting strategies are referred to as Count-On Strategies and include the Right Strategy (sometimes referred to as the Max Strategy) and the Min Strategy. Decomposition strategies can also be applied to problems and involve applying a known fact to derive an answer.

Table 1
A Description of Strategies.

Strategy	Description	No. of counts
Longsum	Start at one and count three on one hand (or using any concrete material) then start at one and count five on the other hand, then count each finger up.	16
Sum	Start at one and count to three and then continue to count another five.	8
Right	Start at three and count on five.	5
Min	Start at five and count on three.	3
Decomposition	Eg. double three and count two.	2

Studies based on identifying strategy use by self-report and/or observation have revealed that during certain stages of development, a range of strategies are applied by children to perform simple addition problems: less efficient strategies co-exist with more efficient strategies and continue to compete for selection (Svenson, Hedenborg, & Lingman, 1976; Fuson, 1982; Carpenter & Moser, 1984). For example, Siegler and Jenkins (1989) described one preschool child who applied five different strategies, in mixed order of efficiency, on nine presentations of the problem 5+3. Research into strategy use must take into consideration strategy variability.

Students with learning difficulties do not use retrieval strategies (Geary, Widaman, Little & Cormier, 1987; Geary & Brown, 1991; Russell & Ginsburg, 1984) or decomposition strategies (Gray, 1991) to the same extent as their normally achieving peers. These differences have been interpreted in the literature as being indicative of either a processing delay or a processing deficit (Goldman, Pellegrino, & Mertz, 1988). The delayed view implies that students with learning difficulties do invent for themselves more efficient strategies and that they execute these strategies with increased speed and accuracy over time, but the developmental process is much slower than that documented for 'normal' students. The deficit view implies that students with learning difficulties follow a fundamentally different sequence of skill acquisition for simple addition and require alternative instruction to successfully move-on to develop a reliance on retrieval.

A delay versus deficit interpretation of a learning difficulty remains a topical issue in mathematics (Geary, 1993) as it does in other domains such as reading (Kulak, 1993). An area of enquiry consequential to the delay/deficit debate that has to date been neglected is the simple addition performance of high school students with learning difficulties. This tendency to focus on younger, primary aged children, is common in the learning disabilities research field (Durrant, 1994). Consistent with a delayed view of development, one would expect retrieval to start emerging as the dominant strategy in simple addition for the adolescent student with learning difficulties. Consistent with a deficit view of development, one would expect these students to rely on procedural strategies and other 'unusual' strategies.

This study investigated what strategies were applied to simple addition problems by three adolescent boys (in Year 10) who were experiencing difficulties learning maths. The study was part of a larger study completed in fulfilment of a Ph.D degree (Hopkins, 1999). A single case study methodology was appropriate given the level of detailed, trial by trial analysis required to account for strategy variability (Siegler, 1987; Delaney, Reder, Staszewski, & Ritter, 1998).

Method

The Participants

All three students were in Year 10 at a regular, suburban high school and attended a remedial mathematics class where there was no focus on mastering basic arithmetic facts. Their teacher encouraged them to use a calculator when required and he adhered to a similar but modified curriculum taught in the regular class, which included percentages, indices, compound interest and binomial expansions.

Michael was 16 years old and his performance on the Raven's matrices test (Raven, 1938) was between two and three standard deviations below the mean calculated for his age group (de Lemos, 1989). He was a very enthusiastic student but had difficulty fitting in with his peers at a social level.

Richard was 17 years old and his performance on the Raven's matrices test was between one and two standard deviations below the mean for his age group. He was generally not interested in classroom activities, preferring instead to develop his talent at technical drawing and graffiti art.

Steven was 15 years old and his performance on the Raven's matrices test was also between one and two standard deviations below the mean for his age group. Despite his great difficulty in reading, Steven was a very diligent student who meticulously copied notes from the board and from books.

Procedure

Each student individually came from the classroom to a vacant room within the library to participate in the study. Each school day students performed a practice set of 26 simple addition problems from a problem set of 65 problems until each student had performed the problem set exactly five times (in random order). Due to illness, holidays, and various individual timetable constrains, practice extended over a period of between 16 and 23 school days.

The problem set (displayed in Figure 1) included simple addition problems (problems with addends less than or equal to ten), as well as ten problems with one addend between 10 and 20 to promote the use of decomposition (Siegler & Jenkins, 1989). Only problems with the first addend smaller or equal to the second addend were included to allow for use of the Min Strategy to be distinguished from use of the Right Strategy (Carpenter & Moser, 1984). Problems with an addend of zero were not included.

Problems were displayed on a personal computer in the format m+n. After each trial the student was asked to explain the strategy they had used and they entered their answer into the computer. All performance was videotaped and comments were transcribed. Students were instructed to obtain an answer using whatever strategy they chose and told that speed and accuracy were equally important. The computer was programmed to keep a record of whether the answer was correct or not, and the time taken to perform each problem. (Reaction time results are not reported in this paper). Strategy use was identified by self-report and checked for consistency with observed behavior.

Results

Michael made a total of 34 mistakes, representing an error rate of approximately 10.5%. Richard made 19 errors in total, representing 6.1% of trials (based on a reduced total of 312 trials due to a computer error). Steven made 17 errors representing 5.2% of trials. Only strategy use on correct trials is reported in this paper.

Combining all the correct trials from the five time intervals, the three most common strategies applied were retrieval, decomposition and the Min Strategy. The percentage of trials performed using these three strategies, for each student, is displayed in Table 2.

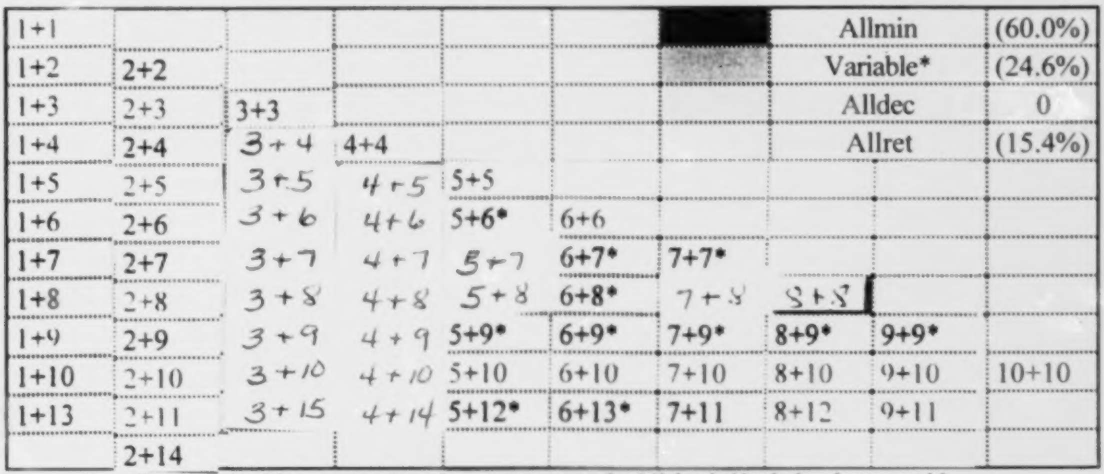
Table 2
Percentage Strategy Use

Students	Min Strategy	Decomposition Strategy	Retrieval
Michael	71.5	0	22.7
Richard	21.2	8.9	69.3
Steven	64.3	1.0	33.8

There was no evidence of unusual strategy use. On one occasion Michael used a Sum Strategy, a Right Strategy and on two occasions, a multiplication strategy. On one occasion Steven reported using a guessing strategy.

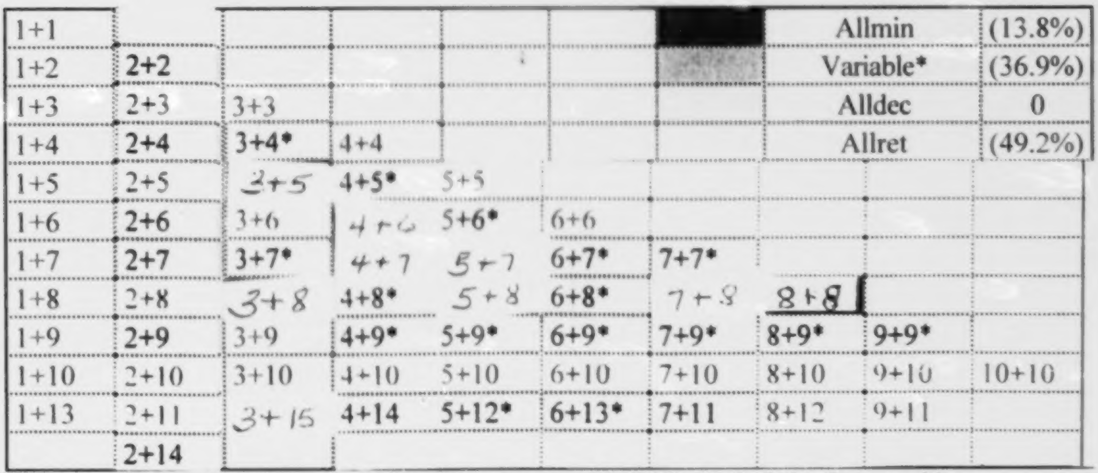
To display constant and variable strategy use for each student, problems were categorised into one of four problem groups depending on what strategies were applied. Constant groups were labelled Allmin, Alldec and Allret problems. If a student correctly retrieved a problem on all five trials then this was classified as an Allret problem. (Classifications were based on performance of correct trials only and therefore may have been based on four trials or less.) Similarly Alldec and Allmin problems refer to problems consistently performed by a student using decomposition or the Min Strategy respectively. The fourth group labelled Variable problems consisted of problems that were performed correctly by a student using more than one strategy. To minimise the possible number of groups for classification purposes, a multiplication and guessing strategy was considered a retrieval strategy and a Sum and Right counting strategy was considered a Min strategy.

Problem groups based on strategy use are displayed in Figures 1, 2 and 3 for Michael, Steven and Richard respectively.



1+1						■	Allmin		(60.0%)
1+2	2+2					▒	Variable*		(24.6%)
1+3	2+3	3+3					Alldec		0
1+4	2+4	3+4	4+4				Allret		(15.4%)
1+5	2+5	3+5	4+5	5+5					
1+6	2+6	3+6	4+6	5+6*	6+6				
1+7	2+7	3+7	4+7	5+7	6+7*	7+7*			
1+8	2+8	3+8	4+8	5+8	6+8*	7+8	8+8		
1+9	2+9	3+9	4+9	5+9*	6+9*	7+9*	8+9*	9+9*	
1+10	2+10	3+10	4+10	5+10	6+10	7+10	8+10	9+10	10+10
1+13	2+11	3+15	4+14	5+12*	6+13*	7+11	8+12	9+11	
	2+14								

Figure 1. Problem groups based on strategy use for Michael. *Includes decomposition.



1+1						■	Allmin		(13.8%)
1+2	2+2					▒	Variable*		(36.9%)
1+3	2+3	3+3					Alldec		0
1+4	2+4	3+4*	4+4				Allret		(49.2%)
1+5	2+5	3+5	4+5*	5+5					
1+6	2+6	3+6	4+6	5+6*	6+6				
1+7	2+7	3+7*	4+7	5+7	6+7*	7+7*			
1+8	2+8	3+8	4+8*	5+8	6+8*	7+8	8+8		
1+9	2+9	3+9	4+9*	5+9*	6+9*	7+9*	8+9*	9+9*	
1+10	2+10	3+10	4+10	5+10	6+10	7+10	8+10	9+10	10+10
1+13	2+11	3+15	4+14	5+12*	6+13*	7+11	8+12	9+11	
	2+14								

Figure 2. Problem groups based on strategy use for Richard. *Includes decomposition.

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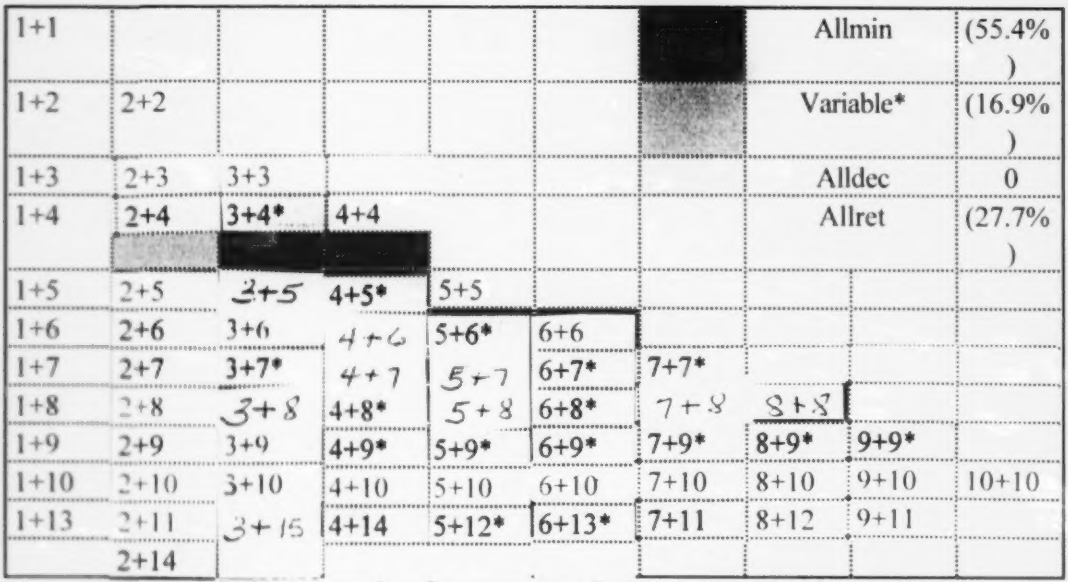

1+1						■	Allmin		(55.4%)
1+2	2+2					▨	Variable*		(16.9%)
1+3	2+3	3+3					Alldec		0
1+4	2+4	3+4*	4+4				Allret		(27.7%)
1+5	2+5	3+5	4+5*	5+5					
1+6	2+6	3+6	4+6	5+6*	6+6				
1+7	2+7	3+7*	4+7	5+7	6+7*	7+7*			
1+8	2+8	3+8	4+8*	5+8	6+8*	7+8	8+8		
1+9	2+9	3+9	4+9*	5+9*	6+9*	7+9*	8+9*	9+9*	
1+10	2+10	3+10	4+10	5+10	6+10	7+10	8+10	9+10	10+10
1+13	2+11	3+15	4+14	5+12*	6+13*	7+11	8+12	9+11	
	2+14								

Figure 3. Problem groups based on strategy use for Steven. *Includes decomposition.

Michael's performance at simple addition was particularly immature. He most dominantly relied on the Min Strategy and on no occasion did he apply a decomposition strategy. Gray (1991) found that children of average-ability at age seven used some decomposition strategies. Michael's lack of decomposition was not surprising given that he only knew (that is, consistently retrieved) two facts: 5+5 and 6+6. He still sometimes relied on the Min Strategy to perform problems that had an addend of ten. His performance was most consistent with that documented by Gray for below-average children, aged between nine and ten.

Richard relied most dominantly on a retrieval strategy and knew most low-tie facts and facts to problems with an addend of one and ten. He still sometimes relied on counting to solve problems with an addend of two. Richard's performance appeared more developmentally mature than that documented for the group of below-average 12 year olds, particularly in the use of decomposition strategies, but still not as mature as that documented for average achieving 12 year olds (Gray, 1991).

Steven relied mostly on the Min Strategy to perform simple addition problems. On problems with a minimum addend of two and problems with an addend of ten, strategy variability between use of a Min and retrieval strategy was still evident. Steven was still either consistently counting problems, or only beginning to retrieve problems that are useful to know and apply as part of a decomposition strategy (tie problems and problems with a sum of 10). Tie facts are usually the first facts to be retrieved (Groen & Parkman, 1972; Ashcraft & Battaglia, 1978). Steven's performance was comparable to that document by Gray (1991) for 10-year-old children of below average ability.

Discussion

Previous studies have established that primary school students with learning difficulties do not move-on to develop a reliance on retrieval for simple addition problems. Findings from this study indicate that for some students, this difficulty persists well on into the secondary years of schooling. The strategies applied by the students in the present study were not different to strategies applied by normal achieving, younger students and so the deficit view of development does not appear applicable. Given the age of these students, to suggest that their performance is consistent with a delayed view of development, without considering the question of whether or not the same asymptotic level of cognitive ability will ever eventually be reached without intervention, is somewhat optimistic. .

Geary (1993) argued that a developmental delay in the acquisition of related conceptual counting knowledge mediated procedural strategy differences exhibited by students with learning difficulties, He also argued that a developmentally different (deficit) pattern of processing mediated retrieval differences exhibited by students with learning difficulties. Geary's explanation of the delay/deficit issue is consistent with the present findings. These students had moved on to develop a reliance on an efficient counting strategy. Their lack of decomposition strategies was more a result of not knowing useful facts, than not applying facts. The greatest difficulty faced by these students was developing a reliance on retrieval.

The question of intervention is beyond the scope of this paper. What is noted is the importance of considering what students already know before launching into intervention procedures. For example, the students in the present study did not retrieve all the tie facts or the facts to problems with a sum of ten. Interventions based on direct strategy instruction of decomposition strategies would therefore be inappropriate. The question of what intervention is appropriate remains open for consideration.

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Beliefs about Mathematics, Mathematics Learning and Mathematics Teaching: Views of Some Primary Teacher Education Students in Singapore and Australia

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Teachers' beliefs about a subject and its learning and teaching are known to affect students' learning. A great deal is known about the beliefs of primary mathematics teachers in Australia and other countries but little is known about how these beliefs compare across countries or whether it is even useful to make such comparisons. This paper reports on a international pilot study in which the beliefs of Australian and Singaporean primary student teachers towards mathematics, mathematics learning and mathematics teaching are measured and compared using a validated instrument. The findings are linked to the TIMSS results in these countries and some tentative conclusions drawn which will inform future research.

In the Third International Mathematics and Science Study (TIMSS, 1998), Australian students faired reasonably well in mathematics, with a middle band result. However, a number of Asian countries, including Singapore, scored much higher than Australia, with Singapore gaining first place at each grade level tested. Naturally, there has been some effort made to compare Singaporean mathematics practices with other countries-including Australia-to ascertain possible reasons for the relative rankings. This paper is one attempt at such a comparison.

Teachers' Beliefs about Mathematics and its Learning and Teaching

It is generally agreed that teachers' beliefs about mathematics, mathematics learning and mathematics teaching play a critical role in determining how teachers help their students develop their mathematics (Barnett & Sather, 1992; Pajares, 1992; van Zoest, Jones & Thornton, 1994) even if the precise link between what teachers say (espoused beliefs) and what they do (enacted beliefs) is not nearly so clear (Bishop & Clarkson, 1998; Sosniak, Ethington & Varelas, 1991; Thompson, 1992). It is recognised that a student's prime, but by no means only, source of mathematical experiences is the classroom (Franke, 1988; National Council of Teachers of Mathematics, 1998) and what occurs in the mathematics classroom influences student beliefs (Relich, 1995). The teacher and, in particular, the beliefs of the teacher, are critical to the classroom implementation of mathematics learning and teaching. All teachers hold beliefs towards this learning and teaching. These beliefs influence and guide teachers in their decision making and implementation of teaching strategies (Baroody, 1987). Indeed, it has been suggested that the investigation of beliefs about learning and teaching may well be the most critical factor in educational research (Pajares, 1992).

Given that teachers' beliefs affect the learning and teaching of mathematics, it is important for Australian data on these beliefs to be compared to those from other countries to help understand the relative rankings in studies such as TIMSS. The pilot study reported in this paper involves two groups of primary student teachers-one from Australia and the other from

Singapore-and their responses to a 20 item survey of beliefs which has been used by the authors in previous studies (e.g., Perry & Howard, 1999; Perry, Howard, & Tracey, 1999; Perry, Tracey, & Howard, 1998). Comparisons of beliefs from these two groups are made and possible curricula and cultural reasons for the differences and similarities found are discussed.

Method

Two groups of second year primary teacher education students were the subjects of this pilot study. In Singapore, the 133 students who completed the survey were enrolled at the teacher education institution in the country either in a three year undergraduate degree or a graduate diploma program. In Australia, the 61 students who completed the survey were enrolled in a four year degree course at a Sydney university. The surveys were completed, in class, at the beginning of the students' second year. In both countries, the survey was presented in English-the language of instruction at the universities.

Results

The student teachers from Australia and Singapore responded as shown in Table 1.

Table 1

Percentage Distribution of Responses[a] from Australian and Singaporean Primary Student Teachers to the Survey Belief Statements

Belief statement	Australian			Singaporean		
	D	U	A	D	U	A
1. Mathematics is computation	21	28	51	32	12	56
2. Mathematics problems given to students should be quickly solvable in a few steps	71	18	12	58	16	26
3. Mathematics is the dynamic searching for order and pattern in the learner's environment	3	23	74	7	15	78
4. Mathematics is no more sequential a subject than any other	39	33	28	35	35	30
5. Mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking	7	23	71	0	20	80
6. Right answers are much more important in mathematics than the ways in which you get them	93	57	2	87	8	5
7. Mathematics knowledge is the result of the learner interpreting and organising the information gained from experiences	3	12	85	7	25	68
8. Students are rational decision makers capable of determining for themselves what is right and wrong.	10	38	53	18	35	47

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[a] Responses: D (disagree); U (undecided); A (agree).

Belief statement	Australian			Singaporean		
	D	U	A	D	U	A
9. Mathematics learning is being able to get the right answers quickly	85	8	7	80	11	10
10.Periods of uncertainty, conflict, confusion, surprise are a significant part of the mathematics learning process	7	18	75	3	14	83
11.Young students are capable of much higher levels of mathematical thought than has been suggested traditionally	5	21	73	7	29	64
12.Being able to memorise facts is critical in mathematics learning	39	31	30	61	20	19
13.Mathematics learning is enhanced by activities which build upon and respect students' experiences	2	8	90	5	9	87
14.Mathematics learning is enhanced by challenge within a supportive environment	2	8	90	5	7	88
15.Teachers should provide instructional activities which result in problematic situations for learners	8	10	82	12	25	63
16.Teachers or the textbook - not the student - are the authorities for what is right or wrong	75	20	5	74	20	5
17.The role of the mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge	15	25	59	31	21	48
18.Teachers should recognise that what seem like errors and confusions from an adult point of view are students' expressions of their current understanding	5	16	79	9	23	69
19.Teachers should negotiate social norms with the students in order to develop a cooperative learning environment in which students can construct their knowledge	0	20	80	8	20	72
20. It is unnecessary, even damaging, for teachers to tell students if their answers are correct or incorrect	59	28	13	43	42	15

Beliefs about Mathematics (Items 1-6*)*

Very few of the respondents from either country agreed that "right answers are much more important in mathematics than the ways in which you get them" or disagreed that "mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking" or that "mathematics is the dynamic searching for order and pattern in the learner's environment". Opinion was much more evenly spread on the items "mathematics is computation" and "mathematics is no more sequential a subject than any other" with the latter item proving not to be particular useful in separating opinion among the student teachers, just as had been the case in earlier studies (e.g., Perry, Howard, & Tracey, 1999). The only real difference to be discerned between the two national groups occurred with the item "mathematics problems given to students should be quickly solvable in a few steps" with 26% of the Singaporeans agreeing and only 12% of the Australians. An independent groups t-test showed that this difference was statistically significant ($t = 2.28$, $p < 0.05$).

Beliefs about Mathematics Learning (Items 7-14)

Large proportions of the student teachers from both countries agreed with the statements "mathematics knowledge is the result of the learner interpreting and organising the information gained from experiences", "periods of uncertainty, conflict, confusion, surprise are a significant part of the mathematics learning process", "young students are capable of much higher levels of mathematical thought than has been suggested traditionally", "mathematics learning is enhanced by activities which build upon and respect students' experiences" and "mathematics learning is enhanced by challenge within a supportive environment". This suggests that these student teachers have beliefs about mathematics learning which concur to a large extent with much of the current reform agenda in mathematics education (Australian Education Council, 1991; Fong, 1999; National Council of Teachers of Mathematics, 1989, 1998; Curriculum Planning Division, 1995). This suggestion is reinforced by the fact that both groups of student teachers largely disagreed with the statement that "mathematics learning is being able to get the right answers quickly".

There were two items about mathematics learning which afforded statistically significant differences. Australian student teachers more likely than the Singaporeans to agree with the statements that "mathematics knowledge is the result of the learner interpreting and organising the information gained from experiences" ($t = 2.54$, $p < 0.05$), and "being able to memorise facts is critical in mathematics learning", ($t = 2.6$, $p < 0.05$).

Beliefs about Mathematics Teaching (Items 15-20)

Both groups of student teachers agreed in large measure with the statements "teachers should provide instructional activities which result in problematic situations for learners", "teachers should recognise that what seem like errors and confusions from an adult point of view are students' expressions of their current understanding", and "teachers should negotiate social norms with the students in order to develop a cooperative learning environment in which students can construct their knowledge", although there were statistically significant differences between the groups on the first ($t = 2.28$, $p < 0.05$) and last ($t = 2.13$, $p < 0.05$) of these. In both cases, the Australian group of student teachers was more likely to agree with the statements. Almost identical aggregate responses from the two different groups of student teachers occurred for the statement "teachers or the textbook - not the student - are the authorities for what is right or wrong", with about three quarters of each response group disagreeing. The statement "it is unnecessary, even damaging, for teachers to tell students if their answers are correct or incorrect" seemed to cause some confusion among both groups with large numbers (28% for the Australian cohort and 42% for the Singaporeans) undecided.

The final statistically significant difference ($t = 2.17$, $p < 0.05$) between the two country groups of student teachers occurs for the statement "the role of the mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge", with more Australians agreeing with the statement than Singaporeans and twice as many Singaporeans than Australians disagreeing with it.

Discussion

As a result of its success in the TIMSS, Singaporean mathematics education has been looked upon with some envy by other countries, particularly Western countries such as the USA and Australia (Menon, 2000; Stigler & Hiebert, 1999). This study represents the beginning of an attempt to map teacher beliefs about mathematics, mathematics learning and mathematics teaching in a number of countries, including Australia and Singapore.

One of the purposes of this pilot study was to identify any difficulties which might arise in the cross-cultural application of the beliefs survey. As well as the perennial problem of the language of the survey not necessarily matching the first language of the respondents - something which can happen in Australia as well as in Asian countries - there is also the possibility that some of the statements have been interpreted in ways which are different from one country to another. While it appears that the survey instrument which was found to be valid for use with teachers in Australia (e.g., Perry, Howard, & Tracey, 1999) seems to be appropriate in eliciting the opinions of student teachers, both in Australia and Singapore, it is intended to interview a sample of the students from each group as a follow up study.

In earlier work with the beliefs survey used in this study, the authors have established the existence of two separate factors upon which most of the items in the survey load for the sample of teachers considered (e.g., Perry, Howard, & Tracey, 1999). These two factors - which have been called 'transmission' and 'child-centredness' - are also confirmed in the student teacher data although not as clearly as for the earlier teacher data. However, they do provide a mechanism for the discussion of differences in the beliefs of the two country groups of student teachers.

There are a great many similarities between the responses of the two groups of student teachers to the belief statements, possibly indicating that the influence of the major studies on mathematics teacher education in Great Britain and the USA (e.g., Cockcroft, 1982; Department for Education and Employment, United Kingdom, 1998; National Council of Teachers of Mathematics, 1989, 1998) has been felt in both Australia and Singapore by mathematics teacher educators and their students. Perhaps these similarities are partly a result of what Nebres (1987) called the 'canonical mathematical curriculum'. He suggests that:

> This canonical mathematical curriculum was simply transplanted to developing countries ...
>
> The result of this long and single-source history of the canonical mathematics curriculum is a very special place for mathematics in the curriculum and a remarkable uniformity of content in mathematics curricula all over the world. This seems strange, of course, considering how diverse the world is in terms of culture and of needs. (Nebres, 1987, p. 14)

Perhaps, the influence of the major reports referred to earlier is further evidence of the implementation of a canonical curriculum, even up to the present day. Whatever is the case, the similarities in beliefs of the two groups of student teachers suggests that there may be an acceptable way in which mathematics education can be, and has been, viewed internationally.

Both groups of student teachers in this study had completed at least a one semester long subject in mathematics education and it must be expected that this would have influenced their responses to the survey. Similarities between the content and approach of these subjects in both universities could be expected to have some effect on the responses of the student teachers.

However, the schooling experiences of the two groups of student teachers has some quite marked differences. For example, in Singapore schools, compared to those in Australia, there is:

- greater time allocated to mathematics;
- an emphasis on examinations and results-including public ranking of schools on the basis of results; and, in many cases,
- a teaching force which has been more successful in mathematics during their own schooling (Menon, 2000).

Perhaps this can help explain some of the differences which were found among the responses from the student teachers from Australia and Singapore.

Even for the statements on which there was a statistically significant difference between the two groups of student teachers, these differences were not large-all were at the 5% level. Nonetheless, the results demand discussion. The Australian teacher education students, as a group, responded significantly more positively than their Singaporean counterparts to the following items:

- Mathematics knowledge is the result of the learner interpreting and organising the information gained from experiences;
- Teachers should recognise that what seem like errors and confusions from an adult point of view are students' expressions of their current understanding;
- Teachers should negotiate social norms with the students in order to develop a cooperative learning environment in which students can construct their own knowledge.

As well, the Australians responded significantly more negatively than the Singaporeans on the statement:

- Mathematics problems given to students should be quickly solvable in a few steps.

These results would suggest that the Australian group of student teachers is tending to be more 'child-centred' and less 'transmission' oriented than their Singaporean colleagues, reinforcing the stereotypical view that Australian teachers 'facilitate' children's mathematics learning while Asian teachers-in this case, Singaporean-actually teach. Menon (2000) suggests that issues such as teachers being appraised on "the number and frequency of exercises and worksheets given to the students" (p. 347) and schools being financially rewarded on their students' performance in public examinations are critical differences between a Western education system, such as in Australia, and the Singaporean system. Such differences could help explain the results described for these four items.

On the other hand, the Australian student teachers, on average, responded significantly more positively to the following statements:

- Being able to memorise facts is critical in mathematics learning; and
- The role of the mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge.

In the previous work using this survey with teachers, these two items have loaded quite heavily onto the 'transmission' factor. It would seem that the Australians are considering mathematics education in a more traditional manner than their Singaporean counterparts. Perhaps the new primary mathematics in Singapore (Curriculum Planning Division, 1995; Ministry of Education, 1997) and the preparation for teaching this which these students have received has had some effect. The curriculum's emphasis on understanding mathematical concepts and problem solving, thinking skills and creativity (Fong, 1999; Kaur, 1999; Menon,

2000) may mean that the student teachers have, at least to some extent, de-emphasised some of the traditional approaches to mathematics learning and teaching. From the other point of view, the fact that the current New South Wales Mathematics K-6 syllabus was written in the mid-1980s and does not have as strong an emphasis on problem solving, thinking and creativity, might suggest a reason why the Australian teacher education students seem more ready to maintain the traditional approaches, at least in terms of their espoused beliefs. Another possible reason could be the form of secondary courses through which the Australian students would have passed. The formal mathematics courses leading up to the Higher School Certificate examination may very well have emphasised memorisation and transmission, while the corresponding courses in Singapore contain more about the development of particular strategies to solve mathematical problems.

The apparently varying directions of the differences between the beliefs of the two groups of student teachers is a good example of the inconsistency shown on individual belief statements which has been mentioned earlier (Sosniak et al., 1991). It may be that the Australian students feel more comfortable in being eclectic in their approaches while the Singaporean students feel more pressure to conform to the approaches suggested by the central policy makers. It may be that there is less direction in the Australian teacher education course than in that in Singapore about appropriate ways to teach mathematics and this is being reflected in the apparent inconsistencies in response.

Conclusion

In this study, we have investigated the beliefs of two groups of primary student teachers about mathematics, mathematics learning and mathematics teaching. Using an established instrument to measure these beliefs, we have found that the two groups are more alike than not and, perhaps more alike than might have been expected, given their different educational experiences. However, there are some significant differences on some of the belief statements. These may be able to be explained through consideration of the structures of curricula and the nature of experiences in schools and teacher education programs.

This is a pilot study and the real work will involve the analysis of responses of practising teachers in a number of countries, including Australia and Singapore, to the beliefs survey. This will be followed by classroom observations, discussions with teachers and analysis of curriculum documents. There is obviously much more work to be done and this has commenced through the partnerships forged with mathematics educators in Australia and several Asian countries, including Singapore.

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Effective Teaching of Decimals: Evaluating Teachers' Practices

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This study arose from a request from teachers for help in evaluating their current methods of teaching decimals. Results reported here are from 14 classes of students, aged 9 through 12, and demonstrate that it is possible to find marked differences in progress of different classes. Students were given a common pre-test and post-test. These results were related to the models and reported teaching procedures used. The main factors leading to students' improvement appeared to be careful planing to meet their needs based on teachers' knowledge of the underlying concepts, the use of a clear model that students could use to visualise decimal division, and careful bridging from visualisation to numerical forms.

There is often a mismatch between educational research that advances theoretical understanding of learning and the day-to-day needs of teachers to know what they are doing effectively. This issue has been emphasised by Viviane Robinson in her book, Problem-Based Methodology (Robinson, 1993). In this book she speaks of dedicating her work to:

> all those educational researchers who have wished that educational practitioners and policy-makers would take more notice of their work, and to all those practitioners and policy-makers who have wondered when educational researchers would produce something that was worth taking notice of. (p. vii)

This comment strikes a responsive chord with both teachers and researchers. The methodology that she proposes is an intensive one, in which researchers and teachers frequently check their understanding of one another's needs and assumptions.

The study reported here arose from a similar concern, although it used briefer consultations than those proposed by Robinson. It started with a request from a principal to help the teachers at her school assess which of the various ways of teaching decimals that they were currently using were most effective. We knew that this was often a difficult topic for students, despite its logical nature. We knew that many different ways were used to introduce this topic to students, yet students continue to have misconceptions in this domain.

Background

Gelman (1999) discussed the fact that all individuals, schooled or unschooled, can carry out additive processes, but that a cognitive shift was needed to understand multiplicative processes. Multiplicative processes include understanding the multiplicative nature of the place value system, the divisions necessary for understanding decimal fractions, common fractions and ratio. These are thought to be the most difficult concepts taught in primary school.

There are several aspects of decimal fractions which students find difficult. They are difficult because they do not follow the rules, or schema, that students have developed for whole numbers, nor are they the mirror image of this system reflected at the decimal point (see for example, Baturo, 1997; Irwin, 1997; Stacey & Steine, 1998). If students try to generalise from their understanding of whole numbers, they are likely to be incorrect. They must appreciate that decimal fractions are parts of a whole, and that the numbers that are used for counting can be subdivided. They need to understand the relationship of decimal fractions and the language used to describe them, in that the unit that a decimal fraction is a proportion of is always implied, not stated. They need to understand the multiplicative character of the number system, perhaps for the first time, in which the value or a number in each place is 10 times what it would be if it were in a place to the right and vice versa. This is different from the way in which whole numbers may be understood using an overflow image, in which each place gets filled up when you get to 9 (or 99 or 999), and then you use another column to the left to write the next number.

A wealth of students' misconceptions can be traced to their failure to understand these basic qualities of decimals. For example, it is common for students to believe that there is a "oneths" column to the right of the decimal point. They may believe that adding another tenth to 0.9 yields 0.10. They may believe that 4 hundredths is written as 0.400. They may believe that decimal fractions work in a manner that is a mirror image of whole numbers, so that 0.0023 is larger than 0.23. All of these misconceptions are common. They all reflect an imperfect understanding of what a decimal fraction is and how it is shown in writing.

Several methods for helping students understand the basic nature of decimal fractions have been demonstrated to be effective in comparative studies by educational researchers. Some emphasise a strong visual image, while others emphasise conflict between students' existing belief and a known example.

Swan (1983, 1990) taught two parallel classes of 12-13 year-old students using different methods. In both classes he provided students with a number-line model for understanding the meaning of decimal notation, and encouraged students to visualise this when doing simple addition and subtraction or comparing the size of numbers that included decimal fractions. One class was taught by what he called a "positive only teaching style". In this class concepts were explained and methods for obtaining correct answers taught using the number line. Students then practised what they had learned on similar problems. The other class was taught using a "conflict teaching style". This teaching had four phases. Initially, students were given problems that were likely to expose their misconceptions. Next they were asked to repeat the problems using the number line model that had been shown to the other group. This was followed by a class discussion of the errors and misconceptions exposed by the contrasting results of the first two phases. Finally there was a consolidation phase in which students practiced similar problems, as had the other class. The group taught by the conflict method was reported to be more difficult to teach because of the extent of their debates, but the students made significantly more gain between pre-test and post-test than did the other class.

Another study that provided a method for teaching decimal fractions was that of Wearne and Hiebert (e. g. Hiebert & Wearne, 1989; Wearne 1990). Their focus was also on students' conceptual understanding. In laboratory and classroom studies of fourth, fifth and sixth grade students in the United States, they introduced decimals through the use of place value blocks. The large cube was used to represent a unit, the flat block to represent a tenth of this, a long block to represent a hundredth of the unit, and the small cube to represent a thousandth of the large cube. They found that students were able to show markedly greater understanding of numerical representation of decimal fractions after instruction using this concrete model. For example, in one study of fourth grade students, 20% of the students gave accurate answers on interviews before instruction, and eight weeks after the instruction period 93% of responses were correct on measures using the materials and 75% of responses on a transfer task with numbers only were correct (Wearne & Hiebert, 1989).

A third method, useful for helping students overcome their misconceptions, was developed by Irwin (e.g., 1997). This program used students' everyday knowledge to help them overcome misconceptions. Students worked in pairs to solve a number of problems that involved slightly unusual forms of decimals yet in familiar contexts. For example, they were asked to add the cost of $4.95 and 91.9 cents for a particular outing, or how many Samoan tala they would get for $NZ10 if the exchange rate was $NZ1 to $S1.5989. In these cases, application of simplistic rules that they had learned for handling whole numbers like "adding a 0" to multiply by 10, or "lining up the decimal point" did not work, and their knowledge of the setting made them realise that their schemas must be modified. Students who worked through these cognitive conflicts made significantly more progress on a post-test than did students who worked on similar problems without contexts.

Other important ways of improving instruction have been developed by researchers, including the interactive computer program of Stacey and diagrams of Baturo. Aspects of all of these methods are used in schools in New Zealand, although many teachers will not be familiar with the underlying research. There are several other methods used that are recommended in textbooks or teaching guides. However, while educational research focuses on the value of single teaching procedures, classrooms usually use a wide variety of procedures. They rarely provide careful comparison studies like those in the cases given above. While researchers want to demonstrate the value of a particular method, teachers change their methods or models frequently, according to what they perceive as the needs of their students.

This study attempted to capture the wealth of procedures used in classrooms by teachers who were not restrained by the need to prove the value of a particular method, but who still wanted to know if their teaching was effective.

Method

Participants

The data presented below came from 14 classes in four schools that are members of the University of Auckland Consortium of Schools. They are in the West, South, and Central parts of Auckland, and covered a range of economic backgrounds, from Decile 1 (low family income) to Decile 7 (moderate family income). The students represented the cross-section of ethnic groups in this city: Maori; Pacific nations; Pakeha/European; Indian; Asian; and others. The classes were all composite Year 5 and 6 (ages 9 and 10) or composite. Year 7 and 8 classes (ages 11 and 12). Some classes were cross-grouped for mathematics, making them relatively homogeneous in achievement, and others were heterogeneous in attainment.

The teachers varied in their ethnicity, place of teacher education, and years of experience, although all had been teaching for at least three years.

Procedure

The procedure for evaluating teaching effectiveness and class progress was negotiated with the teachers. It was decided that all teachers would give the same test as a pre-test and a post-test. This was based on the Chelsea Diagnostic Test of Place Value and Decimals (Hart et al., 1985), but did not include all of the items. This test has levels that are described as: (0) little understanding of whole number values; (1) understanding of whole numbers; (2) understanding of tenths; (3) understanding hundredths and thousandths; (4) the relation of value to adjacent places; (5) more complex relationships between places; and (6) decimals as the result of division and as infinitely divisible. The selection of items used yielded a maximum raw score of 37, leaving out one scored item from each of Levels 5 and 6 of the original test. This test is intended to be diagnostic, and presents several different models for representing decimals, if the concept is understood. The ordering of levels on the original test was pragmatic, reflecting the development of students in the English sample. Because this assessment started with understanding of whole numbers, it was suitable for students who still had a weak grasp of whole numbers as well as those with more advanced skills of decimals.

Teachers taught a unit on this topic that was usually of three weeks' duration, with days lost for other activities. Teachers decided what and how they would teach, but they agreed to keep records of their planning, the changes they made, samples of students' work and any homework. Some teachers planned together and others planned individually. At the end of the teaching unit each teacher was interviewed.

Interview questions covered the selected achievement objectives and learning outcomes, use made of information from the pre-test, the main models, resources and activities used to teach this unit, what they thought were the most difficult aspects of decimals for students to grasp, and questions about the teachers' satisfaction with the unit, their perception of the students's learning, and their confidence in teaching this unit.

This report concentrates on the progress made by the students, and provides snapshots of the teaching provided for the classes of those that made most and least progress.

Results

Every teacher taught differently, even those who had planned together. They all used different resources although they sometimes obtained these resources from common sources. They taught the unit for between 2 and 4 weeks, with most teaching for three weeks. In every case, mathematics was not held on some of the days because of other school events. Thus the time devoted to the unit varied between 8 days and 15 days. There was no relationship between the number of days for which the subject was taught and the results for the class.

All but one teacher used textbooks and worksheets, but nine different texts were used. Models included dividing chocolate and carrots to demonstrate tenths, using place value blocks, drawing area diagrams and number lines, using calculators for discovery, and games for practice. Some teachers saw themselves as the main resource used. Half of the teachers reported feeling confident about teaching decimals, with the less confident teachers usually teaching the classes with lower pre-test scores.

Every class progressed in average level on the test when assessed on the post-test, as did the vast majority of students. Progress was noted in the mean increase in level of understanding of this test, the mean increase in raw score, and in the percent of students who increased at least one of the Chelsea levels on the test (e.g., from understanding whole numbers to understanding tenths). Since classes started with different mean levels of understanding, the results have been divide into three groups: those with relatively low, medium or relatively high initial understanding.

Average Attainment and Improvement of Students in Classes Assessed.

In three classes, the majority of students failed to show an adequate understanding of whole numbers on the pre-test. The class with the highest percent of students moving up a level also had the highest mean increase in level and the highest increase in raw score.

Class B in this group made the most progress. Their teacher chose to teach only whole numbers and tenths. She used models that enabled students to visualise tenths, using chocolate that was divided, the number line, and moving on to grids and calculator work with fraction equivalents (1/2 = 0.5). She did very little exploration of decimal numbers without models that demonstrated their meaning. This teacher did not feel confident about teaching this topic at more complex levels but, by keeping instruction at a level suitable for the students, advanced their understanding.

Table 1

Classes that Started with a Low Level of Understanding of Place Value

Class (Years)	Initial Mean Level	Mean increase in level	Mean increase in raw score	Percent of students increasing by 1 or more levels
A (5/6)	0.14	0.34	1.23	23%
B (5/6)	0.20	0.84	6.40	84%
C (5/6)	0.32	0.77	5.27	60%

The teacher whose class made the least progress (A) appeared to have pitched teaching at too high a level for these students. He was confident in teaching more advanced students and was still coming to grips with the understanding of these students.

The seven classes in Table 2 started with an initial understanding of whole numbers, but limited understanding of decimal fractions.

Table 2

Classes that Started with an Understanding of Whole Numbers but were not Strong on any Decimals

Class (Years)	Initial Mean Level	Mean increase in Levels	Mean increase in raw score	Percent of students increasing by 1 or more levels
D (5/6)	0.55	1.55	9.83	79%
E (5/6)	0.75	0.04	1.75	08%
F (7/8)	0.75	0.55	2.70	45%
G (5/6)	0.79	1.58	8.58	76%
H (5/6)	0.94	1.91	9.38	75%
I (5/6)	1.00	1.56	9.38	75%
J (7/8)	1.12	3.94	23.00	100%

These classes ranged from making little progress on this test, to making a great deal of progress, with the majority making substantial progress. This suggests that students with an initial grasp of whole numbers (at Level 1) are in a better position to make major gains than are students without this grasp. Class J made the most remarkable progress, with the majority of the class moving from Level 1 (understanding of whole numbers) to Level 5 (understanding of the relationship of places in decimals, multiplying by 10 and 100, etc). These were older students whose initial understanding was relatively uniform. The teacher's main purpose was to give his students a visual representation that would enable them to understand decimals and operate with them. The model that he emphasised was the number line. He used this model in class every day, starting classes with whole-class lessons on the placement of decimal numbers on a number line. For example, he would ask students to place a number between 3.45 and 3.46, or he would ask them to put some numbers, with a different number of decimal places, around a number like 1.8. He made up worksheets that gave students practice in thinking about decimals as places on an infinitely divisible number line. He also covered other activities, including transfer from the model to numerical form and adding and multiplying decimals, but students were expected to think about these activities in relationship to this one major visualisation. Their post-tests showed that they had incorporated this visualisation, but used their own words to write about it. Main factors that contributed to their success were likely to have included his thoughtful use of errors on the pre-tests, his own pedagogical content knowledge (he has a Diploma in Mathematics Education), his careful preparation, his strong emphasis on a visual model that had infinite divisions, and use of this model to solve numerical problems.

The teacher of class H had a class that was far more diverse initially, a fact that is masked by using class means. She had taught the older students in the previous year, and they scored between Level 1 and 4 on the pre-test. The younger students were new to her, most scored at Level 0. Her aims were to assure that students understood the difference between parts of numbers and whole numbers and had a firm understanding of the multiplicative nature of place value. Initially she taught the Year 5 and 6 students separately, explaining the difference between wholes and parts to the Year 5 students using area models drawn on the white board. As the unit progressed, she taught the class as a whole although exercises were sometimes different for the two groups. Her emphasis was always on the mathematical concepts behind numeration. She talked of the decimal point as a fence that separated whole numbers from fractional numbers and that multiplying or dividing by 10 or 100 could involve "jumping over that fence", not "moving the decimal point". The post-test results for the younger students showed that in addition to moving up 1, 2, or 3 levels on the test they also understood some items at the highest levels. For example, none of them made errors when asked how to write eleven tenths. She took advantage of the apprenticeship model in which the students who were less advanced learned from their more competent classmates (e.g., Rogoff, 1990).

The teacher of class E, which made the least progress, was the one who taught the unit for the longest period. She appeared to have taught too wide a range of concepts, and used a wide variety of models and resources, without enabling the students to consolidate their understanding. She was not confident in this topic, while the teachers of the two classes discussed above were both confident.

The four classes in Table 3 started with a general understanding of at least tenths. Three of these were classes of older students, and one was a selected group of younger students with above average attainment. In one of these classes most students already understood more complex relations between places in decimals.

Interesting teaching happened in Class K, which started with a full range of scores, from 0 to 6, and nearly all students moved up at least one level. Their teacher followed the general framework for moving from concrete models to diagrams to numbers. The teacher of class N had an accelerate class most of whom scored at level 5 initially. He did not specifically teach decimals, but integrated the topic into ongoing work. The post-tests of many students looked as though they had been taken in a rush, without much care, an impression confirmed by this teacher.

Table 3

Classes that Started with a Higher Average Understanding of Decimals

Class (Years)	Initial Mean Level	Mean increase in Levels	Mean increase in raw score	Percent of students increasing by 1 or more levels
K (high 5/ 6)	1.60	1.36	7.12	82%
L (7/8)	1.73	1.5	7.70	75%
M (7/8)	1.79	1.08	7.90	73%
N (high 7/8)	3.89	0.55	-0.82	45%

Summary and Discussion

All of these teachers taught from a common, mandatory curriculum yet their teaching methods varied widely. They drew on a broad range of models and resources, as is typical of current practice. No two classes were taught in the same way. Even those teachers who had planned jointly used different models, resources, and activities. However, there were similarities among the most successful teachers.

Those teachers whose students made the most progress in this unit shared several characteristics. (1) They thought carefully about what their students needed to learn; (2) they planned carefully for their students' needs; (3) they used a model which enabled students to visualise what decimals were; and (4) they were careful in bridging from this visualised model to the numerical form. They reported making the topic fun, and expected their students to work, for example, through doing homework. On the other hand, none of these most successful teachers put heavy emphasis on place value columns, as do many teachers in New Zealand schools.

Most of the teachers whose students made good or outstanding progress had a good understanding of place value as a multiplicative concept, although they did not speak of it this way. They were familiar with the difficulties that students often have in this domain. Usually a teacher's confidence in an area goes along with competent teaching. It is interesting that the teacher of class B was an exception in this regard, although she considered herself to be more confident after her successful teaching. She thought carefully about what her less advanced students needed to know next, and by limiting her teaching to this, enabled her class to progress. Other less confident teachers all appeared to have attempted to teach too much, using too many models. It is a good reminder that one can be a good teacher of primary school mathematics without being a confident mathematician, as long as one thinks carefully about what needs to be learned and make sure to know that part well.

Meetings were held with the teachers in each school to report back on the findings of the research. Teachers reported finding these meeting very constructive. Some group problem solving went on, as teachers whose classes had had varying levels of success talked about improving their practice. They reported that the evaluation had met their needs.

Although no teacher mentioned any of the educational research on this topic, several taught in ways that incorporated aspects of this research. No teacher mentioned using cognitive conflict, but their comments on students' usual difficulties and the methods that they used suggested that they did understand students' confusions and the need to modify their schemas developed for whole numbers. Several mentioned the use of place value blocks for presenting the relationship of wholes, tenths, hundredths, and thousandths. However, none of the teachers who used this model found it particularly helpful. They found that the students were confused by the fact that all blocks were said to represent different values from those used in learning about whole numbers. Teacher J, who made more extensive use of the number, appeared to have used it primarily in Swan's sense of "positive only teaching style" (Swan, 1990). The practice of the successful teachers did confirm the importance of visualisation (e.g., Presmeg, 1997), solid pedagogical content knowledge (Schulman, 1986) and learning through apprenticeship (Rogoff, 1990).

What this research has done is identify some teachers who have been very successful in teaching a difficult topic. They have done so in conditions that are more complex than those of much educational research. They lead us to ask if the characteristics that have been identified from these classes would also be identified in a more regulated study. It is through this interplay of teachers' practice and researchers' findings that a more complete understanding of teaching and learning is likely to occur.

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Numeracy for Preservice Teachers: Focusing on the Mathematics and its Discursive Powers in Teacher Education

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To be considered numerate beyond 2 000 preservice teachers need to know more than some basic mathematical concepts and skills; they need to know and value the mathematics and know themselves to be competent and agentic in its articulation. Teacher education programs attempt to realise these ideals but are hampered by what I consider to be untenable assumptions regarding agency for preservice teachers; that rational reflection leads to progressive change, and that knowledge is power. These assumptions are based on humanist versions of a person where agency is taken to be a feature of any adult, human being. In this paper I argue rather for a poststructuralist understanding of preservice teachers as discursively constituted through power relations, both in schools and in tertiary education, where agency, or the lack of it, lies in the constitutive power of discourse. I explore what this might mean for mathematics teacher education beyond 2 000.

It has become quite commonplace for mathematics teacher educators and researchers to bemoan the appalling lack of mathematical knowledge and skills, and poor attitudes, of students entering teacher education programs (Carroll, 1998; Clarkson, 1998; Department of Employment, Education and Training, 1989). This is exacerbated by the fact that the preservice teachers do in fact know a lot about what mathematics is and how it should be taught that is not thought to be useful considering the more investigative or inquiry based teaching methods deemed appropriate in a postmodern world (Foss & Kleinsasser, 1996). While I appreciate and accept all that this research has to offer in making clear some of the parameters within which teacher educators work, the time now seems right to make use of this data in as positive a manner as possible to perhaps improve the effectiveness of teacher education programs, which have in the past been shown to have little effect on preservice teachers' beliefs and practices (Foss and Kleinsasser, 1996; Raymond, 1997). In this paper I argue that a view of agency for preservice teachers as discursively constituted, rather than as unproblematically flowing from (re)constructed knowledge and critical reflection, might facilitate an easing of, and perhaps even an eventual moving beyond, a rather serious case of institutional cramp currently thwarting everyone's best efforts in teacher education.

The *Discipline Review of Teacher Education in Mathematics and Science* (DEET,1989) encapsulates the key theoretical framing and assumptions of contemporary practice in teacher education; that active individuals having constructed knowledge through rational and critical reflection in supportive contexts will unproblematically implement this knowledge when teaching. Throughout this paper I argue that this thesis is based on traditional, though untenable, assumptions regarding the nature of knowledge and the human subject; as well, it ignores the power relations that inhere in all learning encounters, and their constitutive force. It may be that as teacher educators attempt to teach preservice teachers the mathematical concepts, ideas and relationships not properly understood in school, and the various theories their students will need to inform classroom teaching, that they unintentionally reproduce entrenched notions of the unquestionable authority of the teacher/lecturer and of mathematics as a difficult and confusing subject. My concern is that traditional power relations may endure which (re)form at least in part, the teachers of the future.

The Humanist or Discursively Produced Subject?

Humanist discourses suggest that any adult, sane individual *has* identity. This identity is unified, rational and coherent. The use of language is seen to be transparent and used by individuals to describe and analyse the real world and make choices based on rational thought. Through a process of socialisation an individual internalises norms and values and ways of operating which become an integral part of that individual. Currently, most teaching and teacher education programs are founded on these particular notions of the learner. In poststructuralist theory, which is developed in large part in contrast to humanist understandings of the individual, the person is the effect of a production, produced in power relations in many overlapping and intersecting discourses throughout life. The experience of being a person is captured in the notion of subjectivity (Davies, 1991). Subjectivity is constituted in discourses and is not under rational nor conscious control; it is contradictory in that one discourse that contradicts another does not undo one's constitution according to the first discourse. Rather, as Davies (1996, p. 17) makes clear, all discourses through which one is constituted leave lingering legacies, desires, that are not easily erased or replaced:

> Old discourses exist amongst/with the new. Ways of knowing and desiring overlay each other, bump into each other, inform each other. Like the palimpsest of writings on an old parchment, where the old was partially rubbed out and the new overlaid on the old, the old can still be seen, and shapes, at least in part, how we see the new.

Preservice teachers, and teacher educators for that matter, have constituted subjectivities and desires that have been formed in discourses of parenting, child care, health and safety, classroom management, school mathematics, child development and teacher professionalism, to name just a few, that inform pedagogical practice. Constituted subjectivities are fragmentary, malleable and changing, in contrast to the knowable identity of humanist understandings of the individual; in this case there is no essential rational self to which teacher educators, curriculum planners and policy writers can appeal.

Central, too, to poststructuralist theorisations of the individual is the interesting notion of desire. Desires are *imagined possibilities*, signifying pleasurable and rewarding states of being within a discourse. Davies (1991, p. 43) states that: "Desires are integral to the various discourses through which each person is constituted and are not necessarily amenable to change through rational analysis". In teacher education, it may be that preservice teachers choose a certain course of action based on what they consider to be rational analysis, though in the end, in the classroom, this may be subverted by constituted desire. For example, a preservice teacher may write or speak of the importance of collaboration and group work in learning mathematics though, in practice, may defer to constituted desires to care and nurture, and possibly control children, and end up implementing classroom practices where the students sit at individual desks so that s/he can keep tabs on what each child is doing and make sure all are constantly engaged. Schuck (1999) reports that her students' desires to give the children they teach a different experience of mathematics from that they had in school, and their desire to show empathy and have fun with children, obviate their teaching mathematics competently. I see it as a matter of some regret that preservice teachers have desires to foster a certain kind of nurturing relationship with children though many of them appear not to have the desire to share with children the wonder and power of mathematics as a field of study.

In poststructuralist theory all knowledge is discursively constituted; thus both *what* is taught in teacher education programs and *how* it is taught matter enormously as each is co-constitutive of the lives and future teaching practices of preservice teachers. Though a poststructuralist notion of knowledge construction can tell us little about the physiological cognitive processes that comprise mathematical construction, it can, through the concepts of constituted subjectivity and agency, inform our pedagogical endeavours. Future teachers need to be competent and agentic in the classroom and, since agency lies in the room for movement that arises when discourses collide and contradict each other, there is much that might be attempted in preservice teacher education and in teacher development programs.

Preservice Teachers

As previously mentioned, there is a more or less general acceptance of the fact that preservice teachers, especially those in primary and early childhood programs, have serious gaps in their knowledge of mathematical ideas, patterns and relationships (DEET, 1989). However, a research literature is developing which demonstrates that these students do indeed come to preservice teacher education with an extensive pedagogical knowledge. However, as Crawford and Deer (1993, p. 119) note this unquestioned social knowledge leads to resistance to change and state: "It seems to us that one reason for the glacial rates of change in educational practice is that *everybody* knows what teachers do and *everybody* knows about learning in school". Schuck (1996), too, reveals that most of her students in preservice teacher education see the role of the teacher as revolving around telling facts, providing demonstrations and explanations, and checking exercises for correctness. Foss and Kleinsasser (1996) add the sobering rider that these constituted beliefs are not greatly affected by teacher education programs and preservice teachers may not reconceptualise their roles as future teachers of mathematics. One reason for this might be that teacher educators subscribe to overly optimistic notions of agency; that the ability to know, and implement, an alternative mathematics flows unproblematically from rational, reflective thought and knowledge (re)construction.

In keeping with recommendations made in the *Discipline Review* (DEET, 1989) teacher educators have tended to see their role as, in part at least, turning around the lack of mathematical knowledge, poor attitudes and beliefs about mathematics so that their students will be better equipped to teach a mathematics appropriate to a largely unknown future. Emphasis is placed on getting students actively involved in revisiting mathematical concepts and on critical reflection around challenging and confrontational dilemmas in teaching which will hopefully facilitate a (re)construction of prior and present beliefs. Crawford and Deer (1993, p. 119), for example, state that preservice teachers "need to unlearn old beliefs and attitudes before they can begin the process of learning to put theory into practice". This reflective approach may also centre on issues of class/race/gender discrimination and the hope that preservice teachers will become more culturally aware and that this will positively influences their later teaching practice. These practices reflect the belief that rational collaborative analysis and reflection will lead to progressive change in teaching later. However, Brodkey (1992) reminds us that critical reflection has no negotiable currency beyond itself; preservice teachers do not necessarily teach according to what they have deemed to be important in teacher education programs (Foss & Kleinsasser, 1996; Raymond, 1997).

After all, who are these students and mathematics educators who inhabit and give life to the discursive regime in teacher education? Are they rational, primarily cognitive individuals who can and will freely choose to teach according to theories and policies presented and reflectively critiqued in lectures and tutorials? Or are they the constituted subjects of poststructuralist theory, for whom their knowing about mathematics and how it operates is not merely cognitive and rational, but is visceral, emotional and often unconscious? Lather (1991) tells us that in our action is our *knowing*; here she refers to all that we have become, all that we know and know ourselves to be through our total experiences of living life thus far. Much of it has to do with past positioning within power/knowledge relationships and is beyond the bounds of cognitive (re)construction. Thus preservice teachers may cognitively accept that it would be good if, for example, learners of mathematics were able to actively construct their own knowledge, though in practice later may defer to deeper, constituted (and constituting) knowledges that unconsciously inform them of what is doable/pleasurable/authoritative in a particular schooling site. A preservice teacher (cited in Tillema and Knol, 1997, p. 31) talks about the power that past constitutive experiences have to influence her future practice:

> My own experiences are important to me, no matter what I learn here. They are the ones that have left a deep mark. I remember a very nice female teacher who treated us to sweets when we learned a lesson well and a male teacher who could tell exciting stories in history class. I would like to be that way; it gives you a comfortable feeling to try to be that way because you know it worked out so well when you were there.

Although the preservice teacher might readily agree that there is more to history than exciting stories and that one should not need to be rewarded with sweets for learning, this rational, cognitive analysis is forgotten in the face of constituted desire to be a certain type of teacher. If allowed, poststructuralism can tell of the power of constituted subjectivity and desire over human rationality and reason.

Does Knowledge bring Power?

Over the past decade or so mathematics education programs have been influenced by *constructivist* notions of the importance of the active construction of knowledge (DEET, 1989). Framing the suggestion that powerful knowledge can be constructed through active engagement is the idea that such personally constructed individual knowledge is sound and durable and can be applied in diverse contexts. Ernest (1994, p. 1) states:

> What the various forms of constructivism all share is the metaphor of carpentry, architecture or construction work. This is about the building up of structures from pre-existing pieces, possibly specially shaped for the task. The metaphor describes understanding as the building of mental structures ...

Although the social or collaborative construction of knowledge has been emphasised by Ernest (1994) and others, an underlying social-psychological understanding of the learner takes for granted a rational and autonomous individual constructing this knowledge. I have already argued that this formulation of the individual learner is untenable in that what one knows has intellectual (mathematical) and social (subjectivity) aspects that together may or may not have empowering potential. A poststructuralist understanding of the individual is that the *social*, including the effects of power relations, is *in* the person not cognitively internalised as in humanist conceptions. Where teachers and teacher educators ignore the fact that learning mathematics involves participating in a social world where established ways of reasoning and acting are not neutral and set conditions on learning, they are able to defer to notions of individual pathology or lack to explain why appropriate cognitive constructions do not occur, especially in the case of minorities, women and the poor.

A poststructuralist view of the social/discursive construction of knowledge recognises that as students construct mathematical knowledge, they are simultaneously themselves produced as capable numerate agents, or not. Significantly for preservice teachers many of them will have passed the set examinations in school, yet they may ultimately doubt themselves and their ability to teach mathematics well. That is, they know the mathematics but know themselves to be unable to teach it to others (Ball, 1990). This may be because in order to compete successfully in the discourse of school mathematics they had to abandon or place on hold their already constituted and embodied self, which "while pleasurable because of the illusion of mastery that it brings, is also painful, a denial of the self" (Walkerdine, in Paechter, 1998, p. 66). I am referring here to instances where students engage with mathematical content, complete the exercises and pass the exams though their desires to make sense of lived experience are not met. In mathematics methods subjects, too, a similar situation may arise; students are eager to pass exams with high marks so the opinions they express and the views they hold of what the lecturers are presenting to them will always be influenced by power/knowledge relations. For example, in examinations the preservice teachers are not likely to extol the virtues of the *basic facts* and *sums* if all semester the lecturer has concentrated on investigatory and inquiry approaches to teaching patterns and relationships in mathematics. Thus agency for preservice teachers is quite illusory even in teacher education, especially where lecturers remain blind to relationships of power influencing all that they, and their students, do and say. As Freire has endlessly reminded us, and does so again in the following quote (cited in Price & Ball, 1998, p. 263), the effects of power relations in pedagogy must be acknowledged:

> When teachers call themselves facilitators and not teachers, they become involved in a distortion of reality. To begin with, in de-emphasising the teacher's power by claiming to be a facilitator, one is being less than truthful to the extent that the teacher turned facilitator maintains the power institutionally created in the position. That is, while facilitators may veil their power, at any moment they can exercise power as they wish.

Numeracy for Preservice Teachers

Doing mathematics in school or in the wider community can no longer be seen to be learning and applying disparate facts, skills and procedures in a specific context; doing mathematics in a postmodern era is a social practice, in which numerate behaviours are produced and/or suppressed (Walkerdine, 1988). Numeracy reflects the agency one has with/in the mathematical discourse in a context; *with* the content and *in* the discursive power/knowledge relations. Numeracy is not a gift, a thing or an attribute of a person, it is discursively constituted, fleeting, transitory and contextual. Many of the preservice teachers currently entering teacher education consider themselves numerate in that they have passed their final exams at school; however, it may be that successful performance in traditional mathematics classes mainly derived from memorising formula and performing procedures (Ball, 1990). The constitutive powers of school mathematics have taught them that mathematics comprises disparate and unconnected facts and skills and that teaching involves mainly transmission. They have not come to know mathematics as a social practice where powerful mathematical ideas and structures are used in various social contexts for individual and social betterment; nor have they come to know themselves as agentic, confident users of its powerful ideas. Many of them are not well placed to value and implement investigative pedagogies leading to the construction of mathematical knowledge and inquiring habits of mind appropriate to teachers, learners and workers in the new millennium.

When teacher educators, myself included, engage in talk and writing along the lines of "they just don't know the mathematics", or "they have poor attitudes and/or lack confidence" we are designating a perceived student lack that we then attempt to remedy in our programs. We imagine that agency lies in teachable attitudes and skills and that we can give the students the necessary knowledge, as power, to go out there and teach well. If however, we imagine that agency lies in the constitutive power of discourse, or to put it another way, that in the preservice teachers' actions in the classroom lies their constituted *knowing* about mathematics teaching and learning, then our task becomes one of interrupting this already constituted knowing by counterposing an alternative discourse. The effect of this alternative discourse will of course, like all interventions, be problematic and many, especially those preservice teachers who already consider themselves appropriately numerate for teaching, may resist "because the old discourses are still more convincing and desirable to them and can readily be used to destabilise the new, or because the new can readily be reworked to become the old, since any new discourse is always overlaid on the old, and does not replace it" (Davies, 1996, p. 210).

An alternative discourse that could be appropriately constitutive and relevant for teachers of the future might be one where doing and learning mathematics is shown to be a social practice, with power relations that variously position participants. It is important that they recognise that agency with mathematical (content) knowledge, as in reasoning, conjecturing, problem solving, is not a personal quality or attribute but is constituted by the discursive practices of the community. What I am suggesting here is that students will not reason purposefully and engage in processes of conjecture and hypothesis formation if all the teacher asks is that they complete a page of multiplication algorithms, for example. Agency, which interestingly has not been a formative feature of most preservice teachers previous experiences in school, but which they are meant to make possible for others, involves the discursive constitution of a learner as one who can and should speak the powerful truths of the discourse, and who can and should bring one's own multiple meanings to bear on the operation of the discourse. Thus, and most importantly, a sense of oneself as valued participant and inquirer may become constitutive of learners and teachers of the future; depending on the extent to which teacher educators themselves take up the challenge, and spark the desire in others, to teach in more empowering and equitable ways.

In Teacher Education

All discourses have constitutive power and all discourses can empower or defeat. Educators can never take a *supportive* learning context for granted and one way for preservice teachers to become aware of the coercive force of discourse is to analyse the activities and practices of past schooling, and those in teacher education, for their oppressive or liberatory tendencies. Preservice teachers need to experience a new discursive practice where "looking with new eyes" is stressed and highly regarded and hopefully constitutive of the preservice teachers' (and teacher educator's) future practice. Davies (1991, p. 46) states:

> The speaking/writing subject can move within and between discourses, can see pecisely how they subject her, can use the terms of one discourse to counteract, modify, refuse or go beyond the other, both in terms of her own experienced subjectivity and in the way in which she chooses to speak in relation to the subjectivities of others.

Thus traditional mathematics teaching (at school, and perhaps in teacher education) is held up against a discourse that reveals its empowering and oppressive potential. Perhaps preservice teachers will learn to speak of, and desire, agency for themselves and their students as they come to realise that having fun and empathy for learners does not necessarily result in teaching practices that are mathematically and personally empowering.

And then of course there is the mathematics. It is highly unlikely that teachers could be effective and competent if they do not know the mathematical content they are to teach. My view is that preservice teachers need to know at least some mathematics well; the onus is upon teacher educators not merely to (re)present lots of disparate facts and skills but to ensure that preservice teachers come to value mathematics, not only as a tool through which to get a job, but for its inherent structure, patterns and relationships and integrity as a field of study. The extent to which this is realised depends to some extent on *what* is taught in teacher education and *how*; because the students are coming to see themselves as teachers, there has to be the *imagined possibility* that they could teach the particular concepts or ideas well. It may be useful, rather than to put the students into potentially oppressive situations of having to construct mathematical patterns, for example, that these are actually pointed out to them and they then have to speculate on how and why they occur.

Preservice teachers often want practical things that they can use when teaching, so they could compile modules such as "representing fractional amounts", "mathematical patterns", "patterns in nature", "making connections", "exploring base ten" and "mathematics, a social practice", for example. Each module could be accompanied by a cover sheet listing the mathematical content and how the activities might be made interesting and relevant to children. The students might share their ideas and it is important that they learn to speak about what they have discovered and how they will explore these ideas in the classroom. Strangely enough, many of them have not previously spoken mathematical terms, nor learned to explain aloud mathematical ideas. The aim is always to make learning mathematics less threatening and to have preservice teachers view mathematical knowledge as accessible and useful; and to regard themselves as agentic and appropriately numerate for teaching. The extent to which these aims are realised will be the subject of intense debate with the preservice teachers hopefully recognising the power relationships at play in all interactions. Ultimately, the aim is to have preservice teachers recognise that all teaching/learning interactions, whether they feature the use of textbooks, worksheets, group work, transmission, or games, have empowering and liberatory potential. The difficult part is to know where and when each resource and type of interaction is most appropriate. Adhering to poststructuralist notions of the discursive construction of knowledge, better interactions are those that take account of who students are, their desires, and their hope to make sense of their mathematical, and for student teachers, pedagogical worlds.

Conclusion

The pedagogical ideas canvassed in this paper are speculative and so far untried in mathematics teacher education. The ideas are based on the (constituted) notion that teacher educators who ignore subjectivity and desire do so at their peril! Preservice teachers have lived many years of classroom mathematics; they have come to know teaching and learning mathematics cognitively, emotionally and physically. This constituted knowledge of mathematics is powerful, and though it may be debated and challenged in teacher education programs, is not so easily erased. It seems clear that new theorisations of the human subject and of knowledge as discursively constructed may go some way towards properly addressing this important and challenging issue in mathematics teacher education programs.

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Preface

The Twenty-third Annual Conference of the Mathematics Education Research Group of Australasia (MERGA) was held in Fremantle, Western Australia, 5–9 July, 2000. The Conference attracted papers from researchers in Australia, Canada, Israel, New Zealand, Singapore, South Africa, and the United States of America. Contributions from a large number of established researchers—many with substantial international reputations—as well as a significant number of beginning researchers were included in the program.

The Proceedings of the Conference have been prepared after a refereeing process which conforms with current practices for ensuring that the quality of the individual papers is of a high standard, and constitutes scholarly research which is providing an addition to knowledge. The Proceedings are published by MERGA and are available for purchase from the MERGA Publications Officer, whose address is given elsewhere in this publication.

Research studies reported at the Conference encompassed a variety of mathematics education related topics and made use of a wide range of research methodologies and theoretical perspectives. Published presentations were organised into Keynote Papers, Symposia, Research Papers, and Abstracts of Short Communications. In accepting research papers for the Conference, every effort was made to be as inclusive as possible of the research needs and interests of members of the mathematics education research community.

Refereeing processes for the Conference were overseen by an Editorial Committee comprising the Co-editors Jack Bana (Edith Cowan University) and Anne Chapman (University of Western Australia); and David Haimes, John Malone, and Len Sparrow (Curtin University of Technology). MERGA members were invited to act as referees, and the responsibilities of this role was made clear to those who accepted. Several experts outside the MERGA membership were also invited to referee papers submitted.

Authors who wished to have a paper considered for the Conference were required to submit the full text of their paper to double-blind peer reviewing by two referees. As far as was possible the research interests of referees were taken into account when papers were allocated for review. Referees were asked to heed guidelines for published papers consistent with those set out in the call for papers and to provide their written comments in a standard format under the following suggested headings:

- Statement of problem/issues and discussion of its significance
- Literature review and theoretical framework
- Methodology and data analysis (where applicable)
- Discussion
- Conclusions and implications
- General comments on the paper

At least one of the Co-editors and at least one other member of the Editorial Committee read all referees' reports. Where only one of the two referees recommended acceptance, the paper was given to a third referee and in normal cases the majority decision prevailed. In a few cases several members of the Editorial Committee read all the reports and also the original paper before making a final decision.

Some research reports were submitted as Early Bird papers, which were refereed and accepted in the usual way. In cases where revision of the paper was recommended, authors were invited to resubmit, and the resubmitted articles were re-refereed in the usual way, often by the original referees.

Some research reports were submitted as entries for the Practical Implications Award. Refereeing of these papers was organised by the Vice-President (Research) of MERGA, and involved being reviewed by at least four referees chosen by the Vice-President (Research).

Similar criteria were used for choosing these referees, and for recommending whether a paper should be accepted for presentation as a research paper if it was not selected for the Award. Several research reports were accepted for presentation at the Conference in two Symposia. Abstracts for short communications were reviewed by the Editorial Committee to ensure their relevance to the Conference.

The purposes of adopting the methods indicated above were to facilitate high quality feedback to authors and achieve the best possible assurance that the Conference Proceedings conformed to MERGA research guidelines. It was especially hoped that beginning researchers would benefit from the feedback received.

On behalf of the Editorial Committee I wish to thank all those who generously agreed to act as referees for the research papers submitted. Their support has ensured that the quality of the Conference Proceedings is of a standard befitting the mathematics education research community.

Jack Bana
Editorial Committee, Convenor
Perth, Australia
June, 2000

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Gary	Asp	University of Melbourne
Paul	Ayres	University of New South Wales
Jack	Bana	Edith Cowan University
Mary	Barnes	University of Melbourne
Dawn	Bartlett	NSW Dept of Education
Bill	Barton	University of Auckland
Andy	Begg	University of Waikato
Fred	Biddulph	University of Waikato
Alan	Bishop	Monash University
Janette	Bobis	University of Sydney
Gard	Brekke	TFN Institute, Norway
Peter	Brinkworth	Flinders University
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Roger	Brown	Swinburne University of Technology
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Classroom and Teacher Effects in Mathematics Achievement: Results from TIMSS

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Recent work on classroom and school effects in Australia has suggested that teacher effects account for up to 50 per cent of variation in mathematics achievement. The present study used data from the Third International Mathematics and Science Study (TIMSS) to look at classroom and school differences in mathematics achievement at both primary and secondary school levels. It found that while classroom differences account for about one-quarter of the variation in student achievement, little of this was due to teachers. Most of the classroom variation was due to compositional and organisational factors. This has important implications for policy regarding the improvement of mathematics achievement in schools.

Introduction

How much difference do teachers make to variations in achievement in mathematics? The work of Hill and his colleagues suggests that teachers may make a huge difference (Hill, 1994; Hill & Rowe, 1996; Hill, Rowe, Holmes-Smith, & Russell, 1996; Rowe & Hill, 1994). In their three-year longitudinal study of educational effectiveness known as the Victorian Quality Schools Project they examined student, class/teacher and school differences in mathematics and English achievement. The study began with 13,909 primary and secondary students and 931 teachers in 90 schools. Using multi-level modelling procedures to study the interrelationships between different factors at each level—student, classroom, and school—the authors found in the first phase of their study that at the primary school level 46 per cent of the variation in achievement in mathematics was due to differences between classrooms, and at secondary school level the rate was almost 39 per cent. Follow-up work found that between-class differences were also important in student growth in mathematics achievement. Differences in achievement progress located at the classroom-level ranged from 45 to 57 per cent (Hill, Rowe, Holmes-Smith, & Russell, 1996; Rowe & Hill, 1998).

In explaining the large classroom-level differences in student achievement in mathematics, Hill and his colleagues highlighted the role of teacher quality and teacher effectiveness. They contended that while not fully confirmed they had "evidence of substantial differences between teachers and between schools on teacher attitudes to their work and in particular their morale" (Hill, 1994) and this supported the view that "it is primarily through the quality of teaching that effective schools make a difference" (Rowe & Hill, 1994). In further work that examined the impact of teacher professional development on achievement they again argued that differences between teachers helped explain much of the variation in mathematics achievement (Hill, Rowe, Holmes-Smith, & Russell, 1996; Hill & Rowe, 1998).

However, alternative explanations for the large classroom-level differences were also provided by Hill and his team. They pointed to the possibility that classroom-level pupil management practices such as streaming and setting accounted for the class effects. This was not pursued by the authors who stated that in all of the schools they surveyed the classes were of mixed ability (Hill, 1994; Rowe, & Hill, 1994). Another possibility was an under-adjustment for initial differences, that is, they did not control adequately for prior achievement differences. A further explanation considered was the possibility of inconsistency in teacher ratings used in the measure of student achievement in mathematics.

This possibility was also deemed by Hill and his colleagues as unlikely to have had a major bearing, though its influence was not ruled out. However, the authors did not use, or argue for the use of, more objective, independently assessed mathematics tests.

Do teacher quality and teacher effectiveness account for classroom-level variation in mathematics achievement or are other factors more important? The current study aims to look at this question using data from the national samples of students, teachers and schools participating in the Third International Mathematics and Science Survey (TIMSS). It examines patterns of mathematics achievement in both primary and secondary schools and partitions variance using multi-level modelling procedures to estimate the amount of variance that can be explained at the student, classroom and school levels. By introducing different classroom and teacher variables it is possible to test the extent to which factors linked to teachers and those linked to classroom organisation and practice influence mathematics achievement.

The results of this work are important. If Hill and his colleagues are right and differences in mathematics achievement are heavily influenced by variations in the quality of teachers and teacher effectiveness then there are major policy implications for schools and school systems in terms of changing the provision and quality of teacher training, taking more care in teacher selection practices, re-shaping and investing more heavily in teacher professional development, and reforming the way schools deploy teachers and monitor their effectiveness. Alternatively, if other features of classrooms and schools explain more of the variation then schools and school systems may not obtain expected changes in mathematics achievement by targeting teachers.

Data and Method

Data

TIMSS—the Third International Mathematics and Science Study—was sponsored by the International Association for the Evaluation of Educational Achievement (IEA) and was conducted in 1996 (Lokan, Ford, & Greenwood, 1996). It set out to measure, across 45 countries, mathematics and science achievement among students at different ages and grades. In total, over half a million students from more than 30 000 classes in approximately 15 000 schools provided data. Not only were comprehensive mathematics and science tests developed for the study, there were also questionnaires developed for students, their teachers and their school principals. Prior to the development of the tests, an extensive analysis of textbooks and curriculum documents was carried out. Mathematics and science curriculum developers from each country also completed questionnaires about the placement of and emphasis on a wide range of mathematics and science topics in their country's curricula. Together the data provide a unique opportunity to examine an extensive range of contextual variables that influence mathematics and science achievement.

TIMSS investigated mathematics achievement at three stages of schooling with the following target populations:

- Population 1: adjacent grade levels containing the largest proportion of nine-year-old students at the time of testing;
- Population 2: adjacent grade levels containing the largest proportion of thirteen-year-old students at the time of testing; and
- Population 3: the final year of schooling.

In Australia, at the Population 1 level, over 11 000 students were tested from more than 540 classes in 179 primary schools (Lokan, Ford, & Greenwood, 1997). At the Population 2 level, almost 14 000 students were tested from almost 600 classes in 180 schools (Lokan, Ford, & Greenwood, 1996). At both levels, the number of schools was intentionally over-sampled from the smaller states to derive more reliable state estimates.

This study utilises data from the Australian samples of Population 1 and Population 2 students. For Populations 1 and 2, the original TIMSS design specified a minimum of 150 randomly selected schools per population per country, with two classes randomly selected to participate from each of the adjacent grade levels within each selected school. However, due to the cost of collecting such data, most countries were unable to achieve this position, and Australia, the United States and Cyprus were the only countries which selected and tested more than one class per grade level per school. The importance of the sampling design used in Australia is that it enables differences between schools to be separated from differences between classes within schools. In this way we are able to analyse school and classroom differences. The final sample numbers are presented in Table 1.

Table 1
The Sample Sizes

	Population 1 (Age 9)	Population 2 (Age 13)
Students	11248	13922
Classrooms	542	599
Schools	179	180

Method

This study aims to look at the effects of classrooms and teachers after controlling for student-level and school factors. An appropriate procedure for doing this is hierarchical linear modelling or HLM (Bryk & Raudenbush, 1992). This procedure allows modelling of outcomes at several levels (e.g. student level, classroom level, school level), partitioning separately the variance at each level while controlling for the variance across levels.

In the present study the interest is on variability within and between classrooms and schools. Two main sets of analyses were undertaken to measure the levels of variation. The first set was based on Population 1 (primary school) data and modelled mathematics achievement at age 9 years. In the analyses several models were tested each adding successively a new group or layer of variables. The first involved fitting a variance-components model to estimate the amount of variance due to the effects of students (level 1), within classrooms (level 2), within schools (level 3) by running the models without any explanatory variables. The second model introduced a group of student background variables comprising sex, socioeconomic status, family size, ethnicity and number of books in the home. The third model added a set of mediating variables to the student background variables. The mediating variables included results on a standardised word-knowledge test, and attitudes towards mathematics. The fourth model contained grade- or year-level and a set of classroom composition variables relating to mean word-knowledge score, and mean socioeconomic status (SES). The next model added a set of teacher variables including the sex of the teacher, age, qualifications, and scores on six scales related to attitudes and practices in mathematics teaching. The final model added a school-level factor, the mean SES of the school.

By examining changes in the size of the variance components estimates after the addition of each group of variables it was possible to measure the effects of teacher, classroom and school-level factors that influence mathematics achievement. In this way it was possible to estimate the extent to which factors linked to teachers rather than classroom composition and organisation shape differences in mathematics achievement.

The second set of analyses was based on data from Population 2 (secondary schools) and modelled achievement at age 13 years. The same sequence of models was applied, except that scores on a scale measuring student views on the importance of mathematics were added to the model introducing the student-level mediating variables. Additional variables were also added on classroom composition to identify whether classes were in the top-band if they were set or streamed, or if the classes were in schools which did not set or stream in mathematics. To the classroom teacher variables was added a measure of the years of teaching. Additional variables included in the school-level model were school size and a school-level policy to set or stream in mathematics. Table 2 lists and describes the variables included at each level.

Table 2
Student, Classroom and School Variables

	Population 1	Population 2
STUDENT LEVEL		
Student background variables		
Sex	Student's gender	Student's gender
Books	Number of books in student's home	Number of books in student's home
Family size	Number of people living in student's home	Number of people living in student's home
Ethnicity	A composite of student's birthplace, birthplace of parents and language spoken at home	A composite of student's birthplace, birthplace of parents and language spoken at home
Socioeconomic status	A composite variable representing family wealth	A composite variable representing family wealth
Student mediating variables		
Word knowledge	Verbal ability as measured by the word knowledge test	Verbal ability as measured by the word knowledge test
Attitude to mathematics	A composite variable reflecting the student's attitude towards mathematics.	A composite variable reflecting the student's attitude towards mathematics.
Importance of mathematics		A composite variable reflecting the perceived importance of mathematics to the student.
CLASSROOM LEVEL		
Grade-level and classroom composition variables		
Grade or Year level	Grade or year level of class	Grade or year level of class
Mean word knowledge	Average score on Word Knowledge test for the class	Average score on Word Knowledge test for the class
Mean SES	Average SES for the class	Average SES for the class
High band		Whether student is in one of the two the top streams in mathematics
No band		Whether classroom is non-set or non-streamed
Classroom teacher variables		
Age	Teacher's age	
Gender	Teacher's gender	Teacher's gender
Education. qualifications	Teacher's qualifications	Teacher's qualifications
Years teaching		Number of years teaching
Factor 1	Problem-solving approach to teaching	Problem-solving approach to teaching
Factor 2	Discipline oriented approach to teaching	Discipline oriented approach to teaching
Factor 3	Process oriented approach to teaching	Process oriented approach to teaching
Factor 4	Eclectic approach to teaching	Eclectic approach to teaching
Factor 5	Algorithmic approach to teaching	Teacher satisfaction with job
Factor 6	Teacher satisfaction with job	
SCHOOL LEVEL		
School variables		
School size		Measure of the number of students in the school.
Policy to stream		Whether the school has an explicit streaming policy
Mean SES	Average SES for the school	Average SES for the school

Results

Table 3 presents the results of the HLM analyses for students in primary schools. The variance components estimates are presented in column 2. The third column presents the percentages of variance (intraclass correlations) in mathematics achievement located at each of the levels—student, classroom and school. The final column contains the percentages of variance explained at each level after controlling for the different groups of variables.

As a first step, a fully unconditional (null) model was tested. This model, the equivalent of a one-way ANOVA with random effects, estimates variances in the outcome variable at the student, classroom and school levels. The results suggest considerable variation in mathematics achievement at the student and classroom levels. Approximately two-thirds (66.4 per cent) of the estimated variation occurs at the student-level. However differences between classrooms also account for a substantial amount of variance—24.5 per cent. This figure is lower than that reported by Hill (1994) for class/teacher effects in primary schools (44 per cent) but is still substantial. Differences between schools accounted for the remaining 9.1 per cent of variance. This suggests a moderate though significant level of variation between schools.

The next step in the analysis involved adding the student-background predictors (SES, gender, ethnicity, family size, number of books at home) to the model of mathematics achievement. This allowed differences between classrooms and schools to be adjusted for differences at the individual level. The results presented in column 4 show that differences in the background characteristics of students accounted for 10.1 per cent of the estimated variance at the student-level, 9.4 per cent of the variance between classrooms, and 49.3 per cent of the variance at the school-level.

Table 3
Variance in Mathematics Achievement Explained by Three-level HLM Models: Population 1, TIMSS

	Variance	Variance between levels	Variance explained at each level
		%	%
Variance within classrooms (level 1 variance)	5774.3	66.4	
After controlling for:			
Student background variables	5190.2		10.1
Student mediating variables	3750.3		35.0
Variance between classrooms (level 2 variance)	2126.8	24.5	
After controlling for:			
Student background variables	1926.3		9.4
Student mediating variables	1132.6		46.8
Grade-level and classroom composition variables	418.1		80.3
Classroom teacher variables	360.3		83.1
Variance between schools (level 3 variance)	792.7	9.1	
After controlling for:			
Student background variables	402.1		49.3
Student mediating variables	230.0		71.0
Grade-level and classroom composition variables	128.5		83.8
Classroom teacher variables	127.4		83.9
School-level variables	120.9		84.8

Adding the student mediating variables (word knowledge scores, attitudes towards mathematics) in the next step substantially increased the percentages of explained variance at each level. When achievement is adjusted for the student background and mediating variables the amount of variance explained at the student-level increases to 35 per cent. At the classroom-level the amount of variance explained increased to 46.5 per cent, and at the school level the amount rose to 71.0 per cent. These results suggest that intake characteristics account for most of the variance at the school-level, but also a major part (almost half) of the variance between classrooms.

The next step involved the inclusion of the grade-level and classroom-composition variables—mean word-knowledge, mean SES. It produced substantial increases in the percentage of variance accounted for at the classroom-level. The between-classroom variance explained jumped from 46.8 per cent to 80.3 per cent. At the school-level, the amount of variance explained increased from 71.0 to 83.8 per cent. It shows that much of the variation in maths achievement in TIMSS at the primary school level was linked to the grade-level of a student and to the social and ability composition of the classroom.

Teacher effects would appear to be quite small, at least based on the changes that occur after adding in the available teacher variables—age, sex of the teacher, qualifications, and views on mathematics teaching. This group of variables increased the explained variance at the classroom level by about 3 per cent (from 80.3 to 83.1 per cent). The school-level variables also added little to the explained variances.

Table 4 presents the results for Population 2 (students and classes in secondary schools). The variance components or null model results show that the amount of variance due to differences between classrooms was larger than at the primary school level—27.8 per cent as against 24.5 per cent respectively. The effects of school were also greater (15.2 per cent for secondary schools compared to 9.1 per cent for primary). This suggests that intake factors were less influential in secondary schools than in primary schools, at least in terms of achievement in mathematics. Classroom and school differences were larger in the secondary school sample, a finding that for mathematics achievement ran counter to the results of Hill (1994, p. 4) who found larger effects in the sample of primary school students and classes.

Table 4

Variance in Mathematics Achievement Explained by Three-level HLM models: Population 2, TIMSS

	Variance	Variance between levels	Variance explained at each level
		%	%
Variance within classrooms (level 1 variance)	5129.7	57.0	
After controlling for:			
Student background variables	4698.0		8.4
Student mediating variables	3754.9		26.8
Variance between classrooms (level 2 variance)	2495.8	27.8	
After controlling for:			
Student background variables	2131.0		14.6
Student mediating variables	1371.4		45.1
Grade-level and classroom composition variables	435.2		82.6
Classroom teacher variables	425.7		82.9
Variance between schools (level 3 variance)	1368.1	15.2	
After controlling for:			
Student background variables	761.0		44.4
Student mediating variables	547.5		60.0
Grade-level and classroom composition variables	105.4		92.3
Classroom teacher variables	95.7		93.0
School-level variables	92.8		93.2

The student background and student mediating variables when included in the HLM models did not help explain as much variance among students as they did in the primary school sample. After including both groups of variables, the percentage of variance in mathematics achievement explained at the student-level was 26.8 per cent (about 10 per cent less than for the primary school students). However, the two groups of variables help explain almost half (45.1 per cent) of the variance at the classroom level and 60 per cent of the variance at the school level.

The classroom composition variables used in the models involving the secondary school sample included measures for setting or streaming as well as the measures used for the primary school sample (mean word knowledge, mean SES, year-level). After adjusting for the grade- or year-level and the classroom composition variables, the percentage of variance explained at the classroom level increased almost 40 points—from 45.1 per cent to 82.6 per cent. Variance explained at the school level also increased substantially—from 60 to 92.3 per cent.

These results suggest that the large classroom effects were due mainly to intake factors, year-level and to student grouping practices. Indeed, when the teacher variables were added to the models, there was little change in the percentage of variance explained at the classroom level. The findings support the view that classroom differences in mathematics achievement are largely due to the students in the classrooms rather than their teachers. School-level pupil management practices such as setting or streaming contribute to the classroom effects by shaping classroom composition.

The results also suggest that composition and intake factors account for most of the between-school differences.

Conclusion

The findings from the current work suggest that some of the difference in mathematics achievement between students is indeed at the classroom level. However, this difference is not due mainly to variations in the quality of teachers or teacher effectiveness, but rather to classroom composition and year-level. About a quarter of the variation in mathematics achievement for students in both primary and secondary schools is due to differences between classrooms. Of this variation at the classroom level, over 80 per cent is explained by grade- or year-level and compositional factors such as the mean SES background of students, the mean level of language skills of students, and whether or not the classes are streamed or set.

These findings do not support the view of Hill and his colleagues that it is the differences in quality of teachers and teacher effectiveness that accounts for much of the classroom variation in mathematics achievement. Rather they support an alternative explanation, that the types of pupil grouping practices schools employ shape the classroom learning environments in ways that affect student progress and student achievement, and it is these kinds of differences that more significantly influence classroom effects. By this, it is not suggested that the quality of teachers does not matter or that all teachers have the same effectiveness. Indeed in a further paper using TIMSS results we identify several teacher-related factors that contribute significantly to differences in mathematics achievement (Fullarton & Lamb, 2000). What the TIMSS results suggest simply is that the organisational and compositional features of classrooms have a more marked impact on mathematics achievement.

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3-Dimensional Geometry: Assessment of Students' Responses

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The authors formulated three research questions in association with the development of a test in 3-dimensional geometry in response to Mariotti's study on the complexity in mental imaging using nets of solids. This paper reports on the coding of responses to a question on nets, using two frameworks, the SOLO Taxonomy and the van Hiele levels of understanding. The findings show that students' descriptions of nets can be coded in both frameworks, thus supporting the three research questions.

Introduction

An important part of growth in insight for 3-dimensional geometry is visual imaging. Through imaging, the development of understanding of the relationships within a figure is promoted. Researchers continue to be interested in understanding the nature of growth in students' insight into 3-dimensional geometry, and in the role played by visual-based processes (Despina, Leikin & Silver, 1999; Gutiérrez, 1996; Mariotti, 1989; Mitchelmore 1980). Gutiérrez (1996, p.1-9) defines visualisation in mathematics as the kind of reasoning activity based on the use of visual or spatial elements, either mental or physical, and that it contains four main elements, mental images, external representations, processes, and abilities of visualisation. He considers that visualisation is one of the main bases of cognition (p.1-3), while Mitchelmore (1980, p.83) argues that "it is of great value to be able to visualize and represent three-dimensional configurations and to comprehend the geometrical relations among the various parts of a figure." Students' ability to decompose and recombine images, that is the ability to break down a visual image into simpler parts and them recombine those parts into new images, is an important component of imagery (Brown & Wheatley, 1997

Two ways in which the ability to translate between 3-dimensional solids and their 2-dimensional representations can be demonstrated are, first, through the 2-dimensional geometrical drawing of 3-dimensional solids, and, second, through the development of 2-dimensional nets of 3-dimensional solids. A net has been described by Borowski and Borwein (1991) as a diagram of a hollow solid consisting of the plane shapes of the faces so arranged that the cut-out diagram could be folded to form the solid. The first method of representation, the 2-dimensional geometric drawing, was explored by Mitchelmore (1980), and the second, students' ability to recognise and design nets, was investigated by Mariotti (1989) and Despina, Leikin and Silver (1999).

Mariotti (1989), in her attempt to identify specific didactic variables related to the utilization of mental images, hypothesised that there are two levels of complexity in the manipulation of mental images:

> a first level when primary intuitions are sufficient: the image is global, it is not necessary to coordinate intermediate processes to solve the problem:
> a second level when the primary intuitions are not sufficient any more: an operative organization of images is required to coordinate them according to the composition transformation. (p.260)

In her study, she found that "constructing the correct net of the solid implies coordination of a comprehensive mental representation of the object with the analysis of the single components (faces, vertices and edges)" (p. 263), and that it is possible to construct a hierarchy of difficulties in relation to the elaboration of mental images (p. 265). Despina, Leikin and Silver (1999, pp. 4-248), in their examination of the problems related to finding different nets of a cube, found that students experienced great difficulty when attempting to work systematically.

While Mariotti (1989, p. 263) argued that "constructing the correct net of a solid implies coordination of a comprehensive mental representation of the object with the analysis of the single components (faces, vertices and edges)", Despina, Leikin and Silver (1999, pp. 4-242), in investigating students' ability to transform the cube to different nets, found that "there is little information with respect to how students coordinate and analyze the components of a 3D solid when transforming it into a 2D net or vice versa."

Research Design

The authors formulated three research questions related to Mariotti's study:

1. Can Mariotti's two levels of difficulty be demonstrated in students' descriptions of nets?
2. Do Mariotti's levels correspond with recognised frameworks of insight?
3. Is there a hierarchy of difficulties within Mariotti's second level?

To investigate these research questions and others, the authors developed a test, the responses to which could be analysed for information about students' understanding of 3-dimensional geometry. The questions covered the solid form and its cross-sections, as well as nets of 3-dimensional figures. This report considers the coding of the students' responses to one of the questions in which students expressed their understanding of a net. Two frameworks, the SOLO Taxonomy (Biggs & Collis, 1982) and the van Hiele levels of understanding (van Hiele, 1986) are used in the coding of the nature of thinking displayed by the students in their responses.

Background

The SOLO Taxonomy (Biggs & Collis, 1982) has been identified by several writers (e.g., Pegg & Davey, 1998) as having strong similarities with the van Hiele Theory, despite having some philosophical differences. It is concerned with evaluating the quality of students' responses to various items. A SOLO classification involves two aspects. The first of these is a mode of functioning, and, the second, a level of quality of response within the targeted mode. Of relevance to this study are the three modes, ikonic, concrete symbolic, and formal. Within each of these modes, students may demonstrate a unistructural, multistructural or relational level of response.

In the *ikonic mode* students internalise outcomes in the form of images and can be said to have intuitive knowledge. By contrast, in the *concrete symbolic mode,* students are able to use or learn to use a symbol system such as a written language and number notation. This is the most common mode addressed in learning in the upper primary and secondary schools. When operating in the *formal mode*, the student is able to consider more abstract concepts, and to work in terms of 'theories'. A description of the levels that occur within the modes is given:

The *unistructural level* of response draws on only one concept or aspect from all those available.

The *multistructural level* of response is one that contains several relevant but independent concepts or aspects.

The *relational level* of response is one that relates concepts or aspects. The relevant concepts are woven together to form a coherent structure.

The targeting of the concrete symbolic mode for instruction in primary and secondary schools, and the implication that most students are capable of operating within the concrete symbolic mode (Collis, 1988) has resulted in the exploration of the nature of student responses within the mode. This has led to the identification of at least two unistructural (U) - multistructural (M) - relational ® cycles within the concrete symbolic mode (Pegg & Davey 1998). The most noticeable characteristic of the cyclic form of the levels is that the relational response (R1) in the first cycle becomes the unistructural element (U2) in the second cycle. This cyclical nomenclature has been used in the coding of concrete symbolic responses for this study.

Pierre van Hiele's (1986) work developed the theory involving five levels of insight. A brief description of the first four van Hiele levels, the ones commonly displayed by secondary students and most relevant to this study, is given for 3-dimensional geometry (Pegg's (1997) differentiation between Levels 2A and 2B is used):

Level 1. Perception is visual only. A solid is seen as a total entity and as a specific shape. Students are able to recognise solids and to distinguish between different solids. Properties play no explicit part in the recognition of the shape, even when referring to faces, edges or vertices.

Level 2A. A solid is identified now by a single geometric property rather than by its overall shape. For example, a cube may be recognised by either its twelve equal edges or its six square faces.

Level 2B. A solid is identified in terms of its individual elements or properties. These are seen as independent of one another. They include side length, angle size, and parallelism of faces.

Level 3. The significance of the properties is seen. Properties are ordered logically and relationships between the properties are recognised. Symmetry follows as a consequence. Simple proofs and informal deductions are justified. Families of solids can be classified.

Level 4. Logical reasoning is developed. Geometric proofs are constructed with meaning. Necessary and sufficient conditions in definitions are used with understanding, as are equivalent definitions for the same concept.

Van Hiele saw his levels as forming a hierarchy of growth. A student can only achieve understanding at a level if he/she has mastered the previous level(s). He also saw (i) the levels as discontinuous;that is, students do not move through the levels smoothly, (ii) the need for a student to reach a 'crisis of thinking' before proceeding to a new level, and (iii) students at different levels speaking a 'different language' and having a different mental organisation.

Design and Initial Analysis of Responses

Among the questions in the 3-dimensional geometry test designed by the authors were six questions investigating students' understanding of nets. These questions probed for identification, description, explanation of relations and construction of nets. Initially, the test was given to over 1000 students from all except the final year (i.e., from Years 7 to 11) of four secondary schools in a rural city in New South Wales, Australia. This paper considers the responses to one of the questions on nets, namely:

Describe in as much detail as possible what is a net of a solid.

Responses to the question were grouped according to the depth displayed in each answer. In an attempt to find some broad descriptors for the coding, the responses were initially grouped into three sections. Descriptions of each group, together with examples of responses matching the descriptors are given below.

Group A Responses

Responses in this group identified students who considered the net solely as a global image. There was no indication of awareness of a process of transformation between the solid and the net, or of properties or parts of the solid.

- what the solid is before it is a solid (A1)
- the 2D thing of a solid (A2)
- a solid flat (A3)
- a flat drawing of a solid (A4)
- is drawn like below (diagram) (A5)
- the outside of a solid (A6)

Group B Responses

In these responses, students showed awareness of the process of conversion from the 2-D to the 3-D image and/or vice versa, by folding/unfolding, putting together/taking apart, or by spreading out flat. However, there was no indication of awareness of the properties or parts of either the solid shape or the net.

- the unfolded 3 dimensional shape (B1)
- something that can fold in to make an object; for example, this is a pyramid net of a solid (diagram) (B2)
- the solid taken apart and flat (B3)
- a shape spread out ... when you fold it back together it forms that shape (B4)
- the combination of 2D shapes that you need to form a 3D shape (B5)
- the pattern made before it is put together ... the outline of the solid (B6)
- the shape and parts of an enclosed space, when it is taken apart and laid flat to make a shape (B7)
- a plan of a solid that has been opened (B8)
- what I have drawn on the right (diagram) ... this net makes a triangular prism (B9)

Group C Responses

Responses in this group contrasted to those in group B in that in addition to the process of transformation between a solid and a net, the students showed in their answer that they were aware of at least one part (faces, edges and vertices) of a solid. Included in these responses were those which indicated the need for edges to correspond.

- a description of the solid with its faces, edges, and vertices in a flat layered out position ... it shows the different faces on sections of the shape (C1)
- the shape and all its faces folded out (C2)
- a solid kind of flattened out ... a plane shape with sides on it (C3)
- the plan (like a building plan) that shows all the sides in one picture (C4)
- a 3-dimensional shape flattened out to make a 2-dimensional shape ... the edges are separated to make it flat (C5)
- the faces laid out flat ... you can see only the framework for a shape (C6)
- the outline of the skeleton of a solid ... it is a 2D depiction of the solid as if it were

unfolded ... from the net, each line represents an edge and each area represents a face ... when make into a 3D solid, the vertices form as a result of joining the edges and faces together (C7)

- a 2-D representation of a solid ... when all sides are joined together it must make a solid, so these 2-D shapes must be in the correct position on the net ... the sides of the 2-D net must be correct lengths so that when they are joined together there must be no uneven or left over edges (C8)

It is considered that the above grouping supports the first research question; that is, it demonstrates Mariotti's two levels of difficulty. Group A corresponds with the first level in that the responses indicate solely a global perception of a net, whereas, groups B and C, in demonstrating awareness of the process of transformation between the 3-dimensional and 2-dimensional shapes, correspond with her second level.

Final Coding

The responses within the above initial groups were coded with relation to the SOLO Taxonomy. The first consideration was to classify the responses according to their targeted mode of functioning. Group A responses were considered to illustrate students who were perceiving a net intuitively as a visual image. Hence, these responses were classified as being of the ikonic mode. The responses in the latter two groups, B and C, were both considered to be perceiving the net as a symbol or representation of the solid, with group C perceiving the net additionally as corresponding to the parts of the solid. Responses belonging to both these groups were classified as being of the concrete symbolic mode. No responses were considered to display the formal mode of functioning.

The nature of the responses for group A was not considered further. However, groups B and C responses, both deemed as concrete symbolic in nature, were further classified for the cyclical levels within the mode. It was considered that responses in group B, those displaying awareness of transformation between the 3-dimensional solids and 2-dimensional nets, but not identifying any correspondence between the parts of the solid and the shape of the net, were in the first cycle. These responses were coded for the level demonstrated, according to whether they indicated a single process in the transformation; for example, unfolded/flattened, or whether they considered the transformation as a multiple process;that is, those responses which indicated either that the process was reversible, or that it consisted of two or more actions; for example, cut and unfolded. Responses of the first type were coded as being unistructural in the first cycle (U1). In the examples, it is considered the responses B1, B5, B8, and B9 are unistructural, first cycle. Those of the second type were coded as being multistructural in the first cycle (M1). Responses coded as M2 from the examples are B2, B3, B4, B6, and B7. R1, or relational responses of the first cycle become the unistructural element (U2) in the second cycle.

Responses which identified part(s) or property(ies) of a solid were deemed to belong to the second cycle of levels. Identification of a single part or property indicated a response as being unistructural (U2), for example, responses C2, C3, C4, C5, and C6. Reference to more than one property or part of the solid classified a response as multistructural (M2), for example, response C1. If a response also included information concerning a correspondence between the faces or edges of the solid and the sections of the net, or of the matching of edges in the reconstruction of the solid from the net, the response was coded as relational (R2). Responses which matched this coding are C7 and C8.

Finally, the codings were related to the van Hiele descriptors. It was considered that the ikonic responses indicated a visual understanding, corresponding to van Hiele Level 1. Responses which demonstrated an appreciation of the physical process involved in the transformation between a solid and its net only were considered not to show awareness of properties, yet were exhibiting more than visual awareness. Hence, these responses have been coded as transition; that is, between van Hiele Levels 1 and 2A. Responses which indicated awareness of a single property or part of a solid, and were coded as U2 were considered to demonstrate van Hiele Level 2A understanding, while responses which were similar but showed also awareness of more than one property or part of a solid were considered to demonstrate van Hiele Level 2B understanding. Finally, it was considered that responses which included a reference to the relationship between the faces and/or edges of the solid and the net demonstrated understanding at van Hiele Level 3.

A summary of the coding of the nature of thinking displayed in responses to the question 'Describe in as much detail as possible what is a net of a solid' is given in Table 1.

Table 1
Coding of the Nature of Thinking Displayed in Responses on Nets of Solids

Description	SOLO mode/level	van Hiele Level
Visual only		
a flattened shape	Ikonic	1
Perceiving the process		
a shape folded/unfolded	CS/U1	transition
the above plus a linkage (cut, take apart, make up)	CS/M1	transition
identifying part or parts of the net	CS/R1	transition
Considering the shapes		
edges/faces matching or some property of the net	CS/U2	2A
combination of the above	CS/M2	2B
relational position of the faces	CS/R2	3

The coding of the responses using the two frameworks, the SOLO Taxonomy and the van Hiele levels of understanding, not only confirms the first research question, that Mariotti's two levels of difficulty can be identified in students' descriptions of nets, but also supports the second and third research questions. Mariotti's first and second levels correspond respectively with the ikonic and concrete symbolic modes in the SOLO Taxonomy, and also with van Hiele Level 1, and van Hiele Levels 2 and 3. Further, the coding within the two frameworks clearly demonstrates that Mariotti's second level can be partitioned into a hierarchy of degrees of insight. This is shown in the two cycles of levels in the SOLO Taxonomy and in the correspondence between Mariotti's second level and van Hiele's Levels 2 and 3.

Conclusion

In this study, the authors have investigated further the insight required in visual reasoning in relation to nets of 3-dimensional solids. Three research questions developed from Mariotti's study have been demonstrated to hold true. This now provides the theoretical evidence necessary to explore the issues further.

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Mathematics and Gender: Beliefs they are a Changin'

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In this paper, we present findings from a new instrument, "Who and Mathematics", devised to measure the extent to which mathematics is now stereotyped as a gendered domain. A large sample of Victorian grade 7-10 students participated in the study. Gender trends elicited from the responses to each of the 30 items comprising the instrument are examined and gender differences among the respondents are reported. Many of the results were inconsistent with previous findings in the field. Students' gendered beliefs about mathematics and the relevance of mathematics as an area of study appear to be changing.

Introduction

Gender differences in mathematics learning have attracted much attention over the past 25 years or so, from researchers, practitioners, and policy makers (see Leder, Forgasz, & Solar, 1996; Forgasz, Leder, & Vale, in press for extensive reviews). During the 1970s and 1980s much emphasis was placed on areas in which females appeared disadvantaged: enrolment in the most advanced mathematics subjects, courses which needed such subjects as prerequisites, and achieving well-above average scores. Frequently reported findings are summarised in Table 1.

In contrast to the historical emphasis on the perceived disadvantages faced by girls, more recently considerable attention has been placed on the educational disadvantages faced by boys. Views of boys' disadvantage, even in the traditionally male preserves of mathematics and science, are receiving increasing media publicity and coverage (e.g., Colebatch, 2000; Gough, 2000) The impact of gender on performance and participation in mathematics continues to be of concern to the community.

Mathematics Learning: Affective Factors

It is widely accepted that "affective issues play a central role in mathematics learning and instruction" (McLeod, 1992, p. 575). Of the instruments used to assess student attitudes to mathematics, the Fennema-Sherman [F-S] Mathematics Attitudes Scales [MAS] (Fennema & Sherman, 1976) have been used particularly frequently (Walberg & Haertel, 1992).

The MAS consist of "nine, domain specific, Likert-type scales measuring important attitudes related to mathematics learning" (Fennema & Sherman, 1976, p.1). The contents of the subscales can be gauged from their headings: *Confidence in Learning Mathematics; Mother Scale; Father Scale; Mathematics as a Male Domain; Teacher Scale; Attitude toward Success in Mathematics Scale; Usefulness of Mathematics; Mathematics Anxiety Scale; and the Effectance Motivation in Mathematics Scale*. The Likert format makes the scales easy to administer and score. An extensive meta-analysis of mathematics education research studies concerned with affective variables and gender revealed that "mathematics as a male domain" had the largest effect size for gender differences (Hyde, Fennema, Ryan, Hopp, & Frost, 1990). The authors reported that gender differences in perceptions of mathematics as a male domain had declined during the 1980s. Nevertheless, some differences continued to be evident in the most recent of the work they reviewed.

Arguments for Re-examining the Fennema-Sherman Mathematics as a Male Domain [MD] Scale

The MAS were published in 1976. The assumptions underpinning the development of the MD were noted by Fennema and Sherman (1976):

> The less a person stereotyped mathematics, the higher the score. This is done to fulfill the purpose of the scale development as it was assumed that the less a female stereotyped mathematics as a male domain, the more apt she would be to study and learn mathematics. (p. 7)

The corollary of this assumption is that low-scoring females believe mathematics to be a male domain and would thus be less likely to study and learn mathematics. Given the prevailing Western societal views of the 1970s, when the scale was developed, it is not surprising that no allowance was made for beliefs that mathematics might be considered a *female* domain. However, low scores on the MD can no longer be interpreted as necessarily reflecting the stereotyping of mathematics as a *male domain*. Recent research studies have indicated that significant numbers of people, both males and females, who reject the notion that mathematics is a male domain do so because they believe that women are higher achievers in mathematics than men. Substantive evidence is presented in Forgasz, Leder, and Gardner (1999). Clearly, the scale is in need of revision.

Developing a New Scale

We developed two forms of a new survey instrument: *Mathematics as a gendered domain* and *Who and Mathematics*. The aim of both versions of the instrument was to measure the extent to which students stereotype mathematics as a gendered domain; that is, the extent to which they believe that mathematics may be more suited to males, to females, or be regarded as a gender-neutral domain.

An important difference between the two versions was in the response formats used. For the *Mathematics as a gendered domain* scale, a traditional Likert-type scoring format was adopted – students indicated the extent to which they agreed (or disagreed) with each statement presented. A five-point scoring system was used – strongly disagree (SD) to strongly agree (SA). A score of 1 was assigned to the SD response and a score of 5 to SA. This version of the instrument consisted of 48 items. There were three subscales: *Mathematics as a male domain, Mathematics as a female domain,* and *Mathematics as a neutral domain.* The 16 items making up each subscale were presented in a random order.

An innovative response format was adopted for the *Who and Mathematics* version of the instrument. Thirty statements were presented. For each statement, students had to select one of the following responses:

BD – boys definitely more likely than girls
BP – boys probably more likely than girls
ND – no difference between boys and girls
GP – girls probably more likely than boys
GD – girls definitely more likely than boys

In developing the items, we drew on the F-S *Mathematics Attitudes Scales* and on previous research findings about gender issues in mathematics learning. We devised items related to ability, career, general attitude, environment, peers, effort, and task. Feedback on the items was obtained from 10 volunteer mathematics educators and some two dozen volunteer grade 7 to 10 students. Various items were omitted or further modified on the basis of reactions obtained from these groups. In the first trial, approximately 400 Grade 7-10 students from Victorian schools completed each questionnaire. The effectiveness of the different items and formats was examined statistically. Possible gender and grade level differences were also explored. Selected results from this trial have been summarised in Forgasz, Leder and Barkatsis (1998; 1999).

In preparation for the second trial, psychometrically unsatisfactory items were deleted from the original questionnaires and others added to produce the second version of the instruments. These modified questionnaires were administered to approximately 1600 students from eight schools situated in the metropolitan and country regions of Victoria. At each school, approximately half of the students completed each version of the questionnaire.

In the remainder of this paper we report gender trends elicited by the *Who and Mathematics* instrument. The specific items retained for the second trial of the questionnaire are listed in Table 1. Selected findings from the second trial of the *Mathematics as a gendered domain* scale are described in Forgasz and Leder (in press).

The "Who and Mathematics" Instrument

Sample Size

The sample size for trial 2 was 861 (402 females, 436 males, 23 unknown). By grade level there were: grade 7: 188; grade 8: 215; grade 9: 251; grade 10: 182; grade unknown: 25.

Analyses

In order to interpret the response patterns to items, the categories were scored as follows: BD = 1, BP = 3, ND = 3, GP = 4 and GD = 5.

The data were entered into a database and analysed using SPSS_{WIN}. Mean scores were calculated for each item. Thus, a mean score <3 meant that, on average, the students believed that boys were more likely than girls to match the wording of the item; mean scores >3 that they believed girls were more likely than boys to do so. Independent groups t-tests, by gender, were used to explore for gender differences in the responses to each item.

Results

Gender Trends in Response Patterns

Anticipated directions for students' responses as inferred from previous research findings and findings from the study (bold italics) are shown in Table 1. As can be seen in Table 1, there were only eight items for which the responses were consistent with previous findings. These items were largely related to the learning environment and to peers. For example, boys were believed to be asked more questions by the teacher (Item 3), to distract others from their work (Item 16), to tease both boys (Item 21) and girls (Item 30) who did well in mathematics, and to like using computers to solve problems (Item 24). That students' beliefs on so many items were inconsistent with previous findings implies a fairly recent change in gendered perceptions related to mathematics education. In the past, boys were generally believed to have more natural ability for mathematics than girls, and were considered to enjoy mathematics and find it more interesting than did girls. The findings in the present study reveal that, on average, students now consider boys more likely than girls to give up when they find a problem too challenging (Item 4), to find mathematics difficult (Items 27 & 18), and to need additional help (Item 20). Girls were considered more likely than boys to enjoy mathematics (Item 6) and find mathematics interesting (Item 29).

Table 1
Predictions Based on Previous Research and Findings from the Study (Italics)

Item	Pred	*Find*	Item	Pred	*Find*
1 Maths is their favourite subject	M	*F*	16 Distract others from maths work	M	*M*
2* Think it is important to understand the work	F	*F*	17* Get wrong answers in maths	F	*M*
3* Are asked more questions by the maths teacher	M	*M*	18 Find maths easy	M	*F*
4* Give up when they find a maths problem too difficult	F	*M*	19* Parents think it is important for them to study maths	M	*nd*
5* Have to work hard to do well	F	*M*	20* Need more help in maths	F	*M*
6 Enjoy mathematics	M	*F*	21 Tease boys if they are good at maths	M	*M*
7* Care about doing well	M/F	*F*	22* Worry if they don't do well in maths	M/F	*F*
8* Think they did not work hard enough if don't do well	M	*F*	23* Are not good at maths	F	*M*
9* Parents would be disappointed if they don't do well	M	*F*	24 Like using computers to solve maths problems	M	*M*
10* Need maths to maximise employ opportunities	M	*M*	25 Teachers spend more time with them	M	*nd*
11 Like challenging maths problems	M	*nd*	26* Consider maths boring	F	*M*
12 Are encouraged to do well by the maths teacher	M	*nd*	27* Find maths difficult	F	*M*
13 Maths teacher thinks they will do well	M	*F*	28 Get on with their work in class	F	*F*
14* Think maths will be important in their adult life	M	*F*	29* Think maths is interesting	M	*F*
15* Expect to do well in maths	M	*F*	30* Tease girls if they are good at maths	M	*M*

Note. Statistically significant gender differences were found for items marked with an asterisk (see Figure 1)

Gender Differences in Response Patterns

The mean scores for each item, by gender, are shown in Figure 1. Itcan be seen that male and female students were consistent in their beliefs about which group (boys or girls) was more likely to match the wording for 22 of the 30 items. The extent of agreement varied considerably on several of these items – the statistical significance of gender differences on specific items was assessed by independent groups t-tests. For example, the female students were more convinced than the males that girls "think it is important to understand the work" (Item 2, $p<0.001$) and "worry if they do not do well in mathematics" (Item 22, $p<0.001$), and that boys "teased girls who were good at mathematics" (Item 30, $p<0.001$). Males were more convinced than females that boys "need more help with mathematics" (Item 20, $p<0.001$) and "give up when they find a mathematics problem too difficult" (Item 4, $p<0.01$).

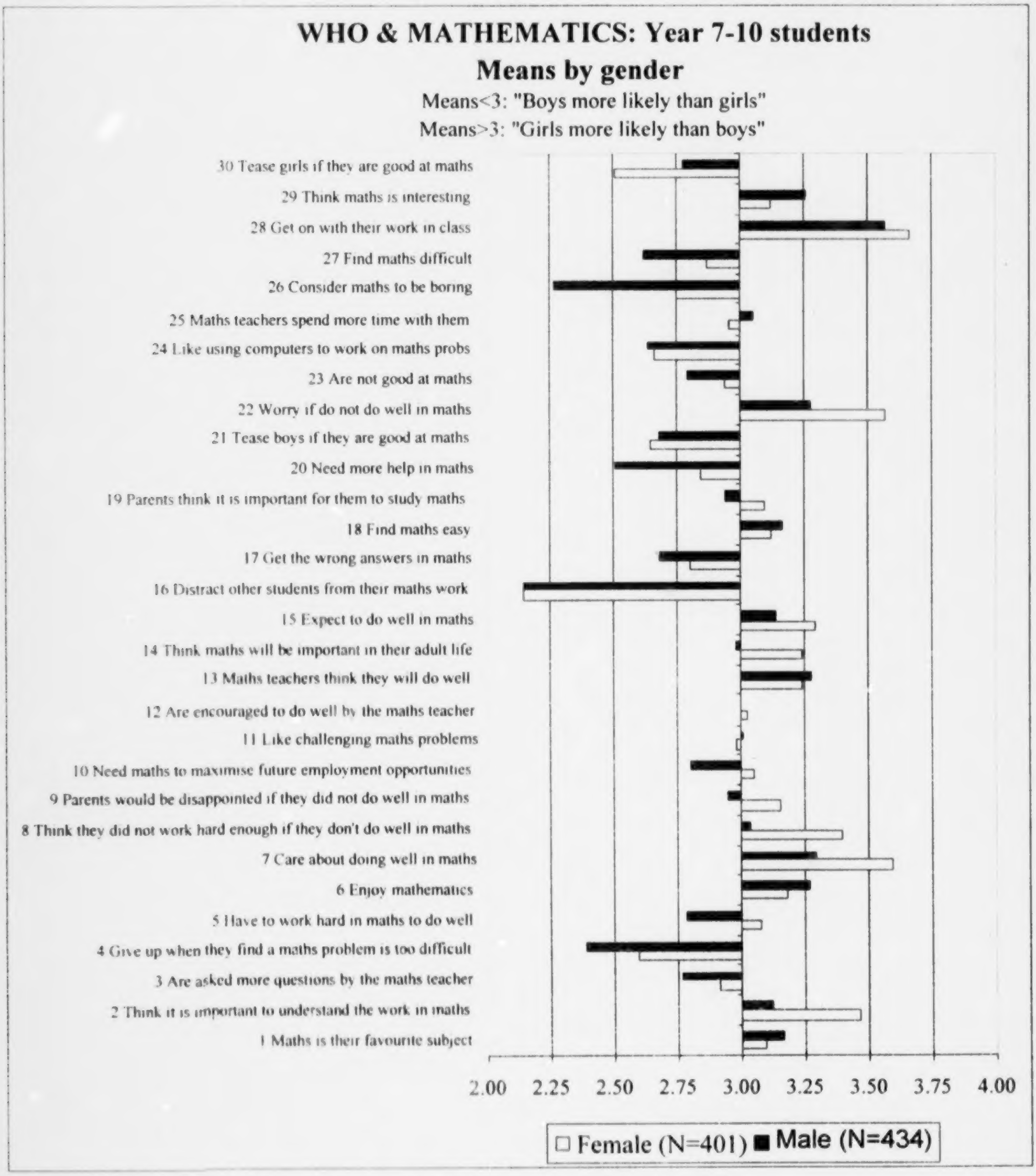

Figure 1. Who and Mathematics. Item means by gender

There were several items on which the male and female students' beliefs supported their own sex. (Independent groups t-tests were again used to determine the statistical significance of gender differences found). For example, the females believed it was girls and the males believed it was boys who "have to work hard to do well in mathematics" (Item 5, $p<0.001$), "who need mathematics to maximise their employment opportunities" (Item 10, $p<0.001$), whom "parents think it is important to study mathematics" (Item 19, $p<0.01$) and about whom parents "would be disappointed if they did not do well in mathematics" (Item 9, $p<0.001$). There were no statistically significant differences for items on which there was cross-gender support. There were, however, several significant gender differences on items for which one group's beliefs were in the direction of either girls or boys whilst the other group considered there was no difference between boys and girls. For example, the female students believed that girls were more likely than boys to "think they did not work hard enough if they did not do well in mathematics" (Item 8, $p<0.001$); the males considered that there was no difference between boys and girls.

Concluding Comments

Although the *Who and Mathematics* is a new, and previously untried instrument, considerable care went into its construction. Some of its items were based on the widely used Fennema-Sherman *Mathematics Attitudes Scales*, others on the results of previous research on gender and mathematics learning. The initial instrument was modified after trials with a small sample of mathematics educators and students from the age groups at which the questionnaire was aimed. A first trial of the instrument with a sample of some 400 students selected from a diverse set of schools led to further modifications, and in particular rejection of psychometrically unsatisfactory items. Thus the changing beliefs expressed by the students in our sample about the importance of mathematics and perceptions of learners of mathematics were elicited by a carefully devised and trialed instrument which contained new items as well as, where appropriate, items used and validated in earlier work. The attitudes and values elicited by the students in this study challenged historical stereotypes about mathematics and perceptions of learners of mathematics and were consistent with the now prevalent perceptions of boys as the educationally disadvantaged group. The statistical significance of many of the differences found warrants some generalisations of the findings beyond the specific sample in the study. However, further research is needed to determine to what extent these findings are replicated in other Australian states, among subgroups of the Australian school population (e.g., extremely high achievers, students from a non-English speaking or socio-economically disadvantaged background) and students in other societies.

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Informal Assessment Questions Used by Secondary School Mathematics Teachers

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Considerable emphasis is placed on teachers' informal assessment of students, yet little has been written about the assessment that teachers actually use in their classrooms and its effect. This paper presents a framework for analysing teachers' use of informal assessment questions. A sample of the teaching of two secondary school teachers is presented to demonstrate the usefulness of this framework. The analysis of these two lessons demonstrates the possibility of discriminating lessons based on teaching procedures from those emphasising more complex conceptual relationships. The potential value of the framework is discussed.

Background

There would be few dissenters to the view that assessment is a vital component of teaching and learning mathematics. However, defining what is implied by assessment would produce a wide range of ideas and opinions. One of the most common understandings of assessment is that suggested by Niss (1993, p. 3):

> ...assessment in mathematics education is taken to concern the judging of the mathematical capability, performance, and achievement – all three notions to be taken in their broadest sense – of students whether as individuals or in groups, with the notion of "student" ranging from Kindergarten pupils to Ph.D. students.

In spite of this wide ranging definition, some teachers still promote only summative assessment, often meaning paper–and–pencil tests or written examinations, while others also espouse the value of formative, informal or diagnostic assessment. In addition, there is now a growing level of agreement that assessment methods should go beyond formal written forms, whether formative or summative, and enter the informal domain (Schloemer, Cain, & Kenney, 1994). However, while there are a number of frameworks for formal assessment (e.g. *Bloom's Taxonomy* – Bloom, 1956; *SOLO Taxonomy* – Biggs & Collis, 1982 and *Authentic Assessment* – de Lange, 1995) there are few if any for informal classroom assessment. Informal assessment can be defined as all classroom procedures that are not followed by written records, which would include observation of students' work, discussion and question posing, either at the individual or group level.

These informal assessment procedures are common in most mathematics classrooms, with teachers making decisions about their students both during and after lessons. However, without a clear framework it is difficult to ascertain either the foundation or the effectiveness of such informal assessment. Research has shown that the classroom practice of teachers emphasises different assessment procedures (Dennisse, Sharon, & Charlene, 1997). The interactions between teachers and students in the classroom have become a focus of teachers' professional development, along with teachers' content and pedagogical knowledge. All are increasingly becoming objects of research (e.g. Britt & Irwin, in press; Bromme, 1994; Chinnappan & Thomas, 1999).

The question arises as to what aspects of content and pedagogical knowledge are likely to be primary influences on teachers' informal assessment, and how one can analyse their influence. Skemp's (1985) idea that we construct 'reality' by engaging in mental building and testing of a schema is a useful way of thinking about our conceptual knowledge. Such schemas have been postulated to help us to categorise when problem solving (Sweller, 1992) and to play an important role in teaching, according to the model presented by Chinnappan and Thomas (1999). One of the roles of schemas is described as assisting teachers to discriminate between various mathematical representations of the same concepts, described by Kaput (1987) as involving a correspondence between aspects of two 'mathematical' worlds, the represented and the representing. On the basis of this teachers can build suitable conceptual learning experiences into their lessons rather than taking for granted switches between representations. Following mathematics across these changes of representation forms one of the four basic activities of mathematics described by Kaput (1992) as: syntactically constrained transformations within a notation system, with or without reference to any external meanings; translations between notation systems and co–ordination across these systems; construction and testing of mathematical models, which amounts to translation between aspects of situations and sets of notation; and consolidation or crystallisation of relationships and/or processes into conceptual objects or cognitive entities that can be used in relationships and/or processes at a higher level of organisation.

Teachers who are aware of these activities, and in particular view the keeping intact of concepts across translations between notation systems as a vital part of their classroom activity, should have pedagogical schemas (Shulman, 1986) which encourage informal assessment of student conceptual learning across the representations. Hence we were interested in developing a framework which would enable us to analyse whether teachers' informal assessment procedures (specifically, informal assessment questions) were based on pedagogical schemas. This paper presents this framework and analyses its application in the classrooms of two teachers.

Guiding Principles

In order to develop a framework for informal assessment, some of the issues of taxonomy and authentic assessment given above were seen as essential. Particular emphasis was given to the importance of ideas stemming from Skemp and Kaput on conceptual development through linking different representations, and to Bloom's first category of *Knowledge*. The principles below were seen as essential in this development.

Principle 1: Informal assessment rests on the importance of the interaction between teacher and students in the teaching and learning process.

Principle 2: Informal assessment seeks to inform teachers of the quality and organisation of students' procedural and conceptual knowledge. This then informs their teaching

Principle 3: Informal assessment should cover the main types of mathematical activity in schools (see Kaput, 1992, as noted above*)*.

Principle 4: Informal assessment should take in to account the nature of the subject matter presented in school mathematics and the usefulness of integrating real world contexts as discussed in de Lange (1995).

The Framework for Informal Assessment Questions

The framework for informal assessment questions (FIAQ) presented here was developed using the principles above, in conjunction with the views of experienced teachers. It should be noted that we have taken a broad interpretation of the word 'question' throughout, including in it requests requiring a student response. The FIAQ has identified five levels in the secondary school mathematics teachers' informal *assessment regime* Assigning levels to informal assessment questions depends on the current situation of the class and the background of the students; what is a higher–level question for one class may be a review question for other students. Sometimes a lower level question may be more difficult than a higher level question, but the degree of difficulty of informal questions has not been a factor determining the hierarchy of levels. Rather, the hierarchy of levels is related to the richness in areas such as making connections, transformations between representation systems, and problem solving in real-world contexts. Such inter–representational connections and transformations are common in mathematics, but links are not always maintained. For example, when moving from a symbolic form of a function to a graphical form it is important to connect sub–concepts such as the dependent and independent variables in each representation, as well as the primary concept of function. Such linking can then assist in the connecting of procedures in each domain, for example in finding the zeros of the function.

Table 1
The Framework for Informal Assessment

Level	Description	Example
1	Questions on subject matter that do not require any mathematical operation or transformation	How do you define a 'set'? State Pythagoras's theorem.
2	Questions of one step or a few steps that do not require any transformations	What is –1 minus –2? Solve $2x - 6=0$.
3	Questions requiring transformation from one representation system to another without interpretation or extrapolation and/or connecting two or more sub concepts, without reference to a practical situation (3.1) or with reference to a practical situation (3.2).	What is the standard form of 145.28? (3.1) The population of New Zealand is said to be 3.6 million. Write this in standard form. (3.2)
4	Questions requiring transformation from one representation system to another with interpretation or extrapolation and/or connecting two distinct concepts without reference to a practical situation (4.1) or with reference to a practical situation (4.2).	Now you found that, x is less than -2 and x is greater than 3, for this inequality. How do you represent this answer in a notation system that we used in sets? (4.1) Explain the trend in a (given) time series graph of ice cream sales (4.2).
5	Questions requiring students to make generalisation with or without judgement.	Most proofs in geometry. Given that $n(A)=25$, $n(B)=20$, and $n(A\cap B)=5$, use a Venn diagram to find $n(A\cup B)$ and hence find a relationship among $n(A)$, $n(B)$, $n(A\cap B)$ and $n(A\cup B)$.

The five levels of the Framework for Informal Assessment Questions asked by secondary school mathematics teachers in their regular classroom are given in Table 1. Levels 1 and 2 in the framework examine the use of definitions or steps in procedures, while levels 3 to 5 refer to these connections across representations. Hence, the framework is considered to be capable of differentiating lessons that are primarily procedural, from those that are primarily conceptual.

The Framework for Informal Assessment in Action

To date the FIAQ has been used to analyse the lessons of 10 teachers, from New Zealand and Sri Lanka in classes where the students ranged from years 9 to 12. The class dialogues were transcribed to identify the teacher's informal assessment questions and then these were assigned levels according to the framework. There was a high level of agreement between the two experienced teachers who assigned levels to the teachers' informal assessment. Two of these lessons (teacher A and teacher B, who are both from New Zealand) have been selected to demonstrate the FIAQ. Extracts from teacher A's lesson, which was later identified as procedural, and teacher B's lesson, which was identified as conceptual, are given below. Minor grammatical errors in the dialogue, have been corrected. Most of teacher A's questions fell in categories 1 and 2, while Teacher B used a range of questions, covering all five of the levels of the FIAQ. Figure 1 shows the number of informal assessment questions in each of the five categories, for these two teachers.

Samples of Teacher Dialogue

Teacher A was teaching a Year 9 class in a lesson on rounding. The following portion of dialogue took place close to the beginning of the lesson. It can be seen that the major portion of the teacher's informal assessment fell into levels 1 and 2 described above. These were either questions that did not require any mathematical operations or transformations, or ones that required only one or two operations. There were no requests requiring transformations between representations, or requests for generalisations.

T: Do you know what a decimal number is? **L1** [A question on subject matter]

S: Yes.

T: What type of number is called a decimal number? **L1** [A question on subject matter]

S: Any number, which has a decimal point.

T: (Writes that statement on the board.)

T: Any example for a decimal number? **L1** [A question on subject matter]

S: (Say many decimal numbers.)

T: (Writes on the board.)

S: 9.9999.

T: How do you show that? **L1** [A question on convention]

S: 9.9 dot.

T: What does that mean? **L1** [A question on convention]

S: 9.9 recurring

All these level 1 informal questions are limited to asking about subject matter of varying kinds, such as definitions and conventions. Another portion from the same lesson:

T: (Writes on board) 35.5283 How many decimal places are there in this number? **L2**

S: Four. T: Four.

T: How do you round this number to one decimal place? What is the answer? **L2**

S: 53.5

The two questions above are designated level 2 questions because students have to follow a pre-learned procedure in order to respond to them. There was only one level 3 question in this lesson, as follows.

T: What is the difference between rounding to decimal places and rounding to significant figures? **L3.1**

This question is assigned to level 3 because it requires the students to make connections between two methods of rounding numbers, namely to decimal places or significant figures. These may be considered as two different representations of the number.

The second example is from Teacher B. This was a lesson on time series, with a Year 12 class and the portion selected came near the end of the class period. The students were given a table showing monthly electricity consumption of an energy conscious family over three years and this provided a context for the mathematics. Analysis of the teacher's questions showed a wider range of levels of questioning than teacher A, including requests for transformations across representations, and one request for a generalisation.

T: So, the question (b) says 'what's the point of doing this? What's the point (of doing moving averages)? **L1**

S: To smooth the data.

T: (Repeats the student's answer) It probably needs a better explanation than that. To smooth the data and what else does it eliminate? **L1**

S: Noise

T: Noise. Very good answer there is, 'to eliminate noise and smooth the data. Question c says 'the family has tried to become more energy-conscious over the 3-year period. Have they succeeded? Comment. **L4.2**

The first two questions above illustrate well the need to take into account the background of the students when assigning the question levels using the FIAQ. In both cases they are only level 1 questions since the purpose of moving averages has been taught in the previous lesson. In contrast, the final question above is at level 4.2. To respond to this students have to link a graphical and a verbal representation, interpreting the data from the graph in the practical context.

In the following discussion the teacher asked a similar question in a different way.

T: If you are more energy conscious, what would be happening to the number of units? **L3.2**

In this case the students have to transform data from a tabular representation to a verbal representation in a practical context, but without interpreting it. Three other examples of questions assigned to this level are given in the passage below.

S: Using fewer of them.

T: (repeats the answer) From your table it appears that the mean of three is decreasing, but from your graph it's easy to see. The graph of the moving mean is this. It's decreasing. So, it tells you that 'yes, they've become more energy-conscious. Had you not done this graph, the original graph looks likes this (Shows the previous graph of row data). Can you tell from this, if the family has become more energy-conscious? **L3.2** [Transformation between representations, but without interpretation]

S: Yes.

T: (Smiles) Yes. It's a bit dicey, isn't it? You can see: moving means tell you more. When there is a lot of noise, the moving mean is a really good thing to do. So, I think the family has become more energy-conscious and in comments you can say, because…(pauses) because why? **L3.2** [Transformation between representations, but without interpretation]

S: The moving mean is decreasing.

T: Decreasing. Dropping. OK. Now we move on to d. Individual seasonal effects need to be calculated. Because electricity usage is a seasonal thing. We use more electricity in the winter than in the summer. Give me any other example for a seasonal thing. **L 3.2**

S: Ice cream sales.

This final question requires students to look for another situation where the 'seasonal effect' is prominent. Therefore they have to link the concepts (not distinct) 'seasonal effect' and 'variation' with an appropriate practical situation. Had they done similar examples before, then this question would have been at level 1.

During the lesson, this teacher asked one level 5 question, which required generalisation. When discussing moving averages, she wanted to get the idea of 'mean of three' as a student's response. She asked, "Has anybody come up with an excellent idea to line up moving average against a month?" A student responded, "Find the mean of three x values". That question was at level 5, since it was about a generalisation, the concept of *mean of three* which is a general idea that can be used to smooth time series graphs.

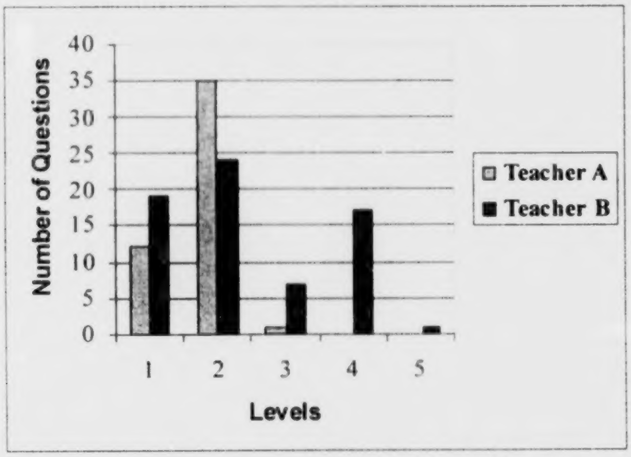

Figure 1. The number of informal assessment questions asked by teachers A and B at each of the levels of the FIAQ

Figure 1 presents a summary of the number of questions in each category asked by each teacher. While Teacher A's questions were limited almost entirely to Levels 1 and 2 of the FIAQ, Teacher B asked questions across all 5 categories. It should also be noted that Teacher B asked nearly 50% more questions than did teacher A (68 questions versus 48 questions). This quantity is in itself an indication of the interactive nature of the class.

Discussion

These two samples provide examples from one lesson in which teacher's questions were primarily at level 1 and 2 of this framework and another from a lesson that included questions from all levels. They demonstrate the difference between a lesson that appears to be primarily procedural and a lesson in which the relationships between procedures and concepts are being elicited (see Kaput, 1987; Skemp, 1985). Because this framework can demonstrate this distinction, it can be of considerable help to teachers who are trying to evaluate their own practice. It can enable them to see if they are asking questions that provide information on all aspects of students' understanding. This information can then be used together with an analysis of teachers' content and pedagogical knowledge in their own professional development.

There are some things that this framework for analysing informal assessment questions cannot tell. It gives evidence of the informal assessment that can be seen in teachers' questions but it does not cover a variety of other informal assessment procedures other than questions or requests that also add to a formative and diagnostic view of students' understanding. This other informal assessment could include observing students' written work, and noting students' correct or incorrect answers or pauses. It could include non-verbal behaviour of either the teacher or the students that helps provide information on students' understanding.

In addition, on its own it does not tell us about the effectiveness of this informal assessment. To know that would require a fine-grained analysis of both the teachers' and the students' understanding at various times, such as that which might be provided by stimulated recall.

These other aspects may be important, but are less easy to capture than are teachers' questions. An analysis using the FIAQ gives important information about the thrust of a teacher's assessment. From this analysis it is possible to tell if, in a particular lesson, the focus has been on procedures, as defined in this framework, or if the focus has been wider, both exploring students' knowledge of the relationship of concepts and procedures and encouraging them to construct these relationships.

With Kaput, we believe that there are many types of knowledge fostered in mathematics classrooms. Some lessons may need to be primarily procedural. The teacher's question in these lessons might be expected to be categorised as level 1 or 2. Such lessons might cover particular content matter, or be found at certain times in a sequence of lessons when procedures are being taught or reviewed. It might be appropriate in other mathematical topics, or at different points in a sequence of lessons to emphasise the inter-relationships and generalisations that are the focus of Levels 3, 4, and 5 of this classification framework.

We see two major benefits of this framework. Firstly, it can help examine the practices of different teachers who espouse or practice different assessment procedures (Dennisse, Sharon & Charlene, 1997). Most teachers want to improve students' understanding of the relationships between different representations, procedures or concepts as their students develop more inclusive schemas in mathematics. Their professional development is aided by a close analysis of their own teaching. Using this analysis framework gives them a basis for reflecting on their informal assessment, to see if it meets their own goals as well as it might (see Britt & Irwin, in press).

Secondly, it would be very useful to analyse the teaching of entire units of mathematics, to explore the balance of questioning at the various levels of the Framework for Informal Assessment Questions. This research could inform debate on the order of procedural or conceptual questioning in use. It could be used to explore generalisations about the need to teach procedures in isolation before teaching how they relate to concepts and each other, versus teaching these procedures in the context of higher order concepts as found in solving complex problems.

Both of these uses would enable us to know more about the nature and importance of informal or formative assessment questions. As stated initially, this is seen as a very important aspect of teaching (Schloemer, Cain, & Kenney, 1994) but because it is difficult to capture it has received less attention than has summative assessment. It has been possible to build a framework for analysis of informal assessment, based on the work of several writers (Bloom, 1956; de Lange, 1995, Kaput, 1987, 1992; Skemp, 1985). This framework provides an important step forward in the analysis of informal, formative assessment.

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Grade 1 Children in Problem-Posing Contexts: Problem Solving Prior to Completing the Task

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With support from undergraduate university students, Grade 1 children (aged 6-7 years) were challenged to pose and then solve their own open-ended tasks. The way in which the children engaged in problem solving prior to formulating or posing a problem was explored. An overview of the types of problems posed by the class is presented, along with two detailed case studies. The case studies show that the children were able to identify and discuss the type of mathematics content and strategies they would need to employ in order to successfully complete the task.

Problem posing is an important companion to problem solving and lies at the heart of mathematical activity (Kilpatrick, 1987). It has been used to refer both to the generation of new problems and to the reformation of given problems (Silver, 1994). In the first instance, "the goal is not the solution of a given problem but the creation of a new problem from a situation or experience" (Silver, Mammona-Downs, Leung & Kenney, 1996, p. 294).

Silver (1995) identified four types of problem-posing experiences that provide opportunities for children to engage in mathematical activity. He argued that problem posing could occur *prior* to problem solving when problems were being generated from a particular situation, *during* problem solving when the individual intentionally changes the problem's goals or conditions, or *after* solving a problem when experiences from the problem-solving context are modified or applied to new situations. The way in which children engage in problem solving prior to constructing or posing a problem was investigated in the present study.

Constructing Problem-posing Environments

Some researchers (Ellerton, 1986; Mamona-Downs, 1993) have found that, for motivational purposes, it is helpful to have someone in mind when designing problems. In the present study, the friend would be someone in the class below or above that of the problem poser. As well as generating a problem for someone to solve, the Year 1 problem generators would also be asked to articulate why such a problem would be appropriate for the respective problem solver. The feedback obtained from this stage of the process fosters a reflective component of the problem-solving process (Lowrie, 1999; Silver et al, 1996).

Ellerton (1986) found those encouraging students to write problems for a friend was a useful way of understanding that person's mathematical ability. In such problem-solving situations the problem poser is forced to consider the individual for whom they are designing the problem. As Stoyanova (1998) commented:

> ...there is a strong acceptance among researchers and educators of the notion that students' ability in posing quality problems provides a useful indication of potential mathematical talent. (p. 172)

The very fact that a student must consider the mathematical ability of another person when engaged in free problem-solving situations requires reflection and careful planning. In order to complete the task successfully, the problem poser might not only focus on the underlying structures of the problem but also the extent to which the problem solver will be able to interpret the components of the problem. Such metacognitive thinking processes encourage mathematical power. Some educators (eg., Kulm, 1994; Leung, 1996) have argued that learners as assessors complement and even enhance a teacher's understanding of students' mathematical ability. Such assertions are also explored in this paper.

It could also be argued that problem-posing situations allow children to have some control over the curriculum content and the type of learning activities presented in the classroom. Furthermore, the tasks or the activities children construct may provide insights into their beliefs or attitudes they have toward mathematics and the way in which mathematical knowledge is developed.

The Purpose of the Study

The central concern of the study was to investigate the extent to which young children were able to determine the type of mathematical understandings and problem-solving strategies they would need to use in order to solve a problem they had posed. The following research questions were posed:

- Were the children able to identify mathematical activities that may be required to complete the problem?
- Could the children outline the type of problem-solving strategies they would use to solve the task?
- To what extent could the children highlight possible difficulties that may arise when completing the task?

Method

A cohort of fourth-year undergraduate students (n = 25) in their final semester of a Bachelor of Education course were matched, on a one-to-one basis, with Grade 1 children from a school in a large rural city. Each student teacher worked with one of the Grade 1 children for one hour per week for five weeks. During the first two weeks of the study the children were administered a diagnostic assessment task in order to establish a profile of their mathematics ability. The student teachers also gained an understanding of the children's likes and dislikes over this two-week period and developed a strong rapport over this period of time. An awareness of the children's mathematical ability and their preference for investigating particular content areas certainly helped the student-teachers assist the children to pose their own problems.

During the next three weeks of the investigation the children were encouraged to pose their own problems and solve these tasks with the help of the student teacher. It is quite difficult for young children to design appropriate problems without a substantial amount of practice (Ellerton, 1986), specific instruction (Leung, 1996) or guided questioning (Lowrie, 1999). The rationale behind matching a student teacher with each child was to provide support with problem construction. Further, some children required assistance in constructing problems that were challenging but still "solvable" in the given time period. On occasions the student teachers had to use their professional judgement with respect to the degree of input they had in the problem-posing process. It could be argued, however, that the same balancing act occurs in most teaching-learning situations. After the first two sessions the student teachers were required to develop a profile of their Grade 1 child. The profile included information about the child's:

- Performance in space, measurement and number content strands;
- General problem solving ability;
- Affective dimensions of learning (including motivation, task persistence, beliefs about mathematics); and
- General areas of interest.

The profiles were quite detailed and it was apparent that the reflective nature of this activity helped the student teacher establish a comprehensive understanding of the Grade 1 child's mathematical progress.

The Problem-posing Sessions

For each of the three problem-posing sessions, the student teacher was responsible for assisting the Grade 1 child to pose problems that both were open-ended in nature and suitable for the individual's needs. A topic or scenario was negotiated with a problem or series of problems generated from the theme. Once the collaborative team was happy with the task the student teacher attempted to establish whether the Grade 1 child was able to determine the type of understandings and strategies that would be required to complete the task. Importantly, the Grade 1 children knew that they were able to seek assistance from their "teacher" to solve the task. This added another dimension to the study because the children were not inhibited by their inability to complete computations or solve multi-step problems. Thus, the Grade 1 children were challenged to consider the types of strategies and methods they would need to use in order to complete the task without being restricted by a lack of content-specific knowledge.

A series of questions were posed to elicit this information before each child began the task. These questions varied depending on the type of problem posed but included some of the themes presented below.

- What might we collect to build this?
- Where will we get these materials?
- What is going to be the hardest thing to do?
- Will we need to do some mathematics?
- When do you think you will need help from me?
- Where should we begin?

The following section identifies some of the problems posed by the Grade 1 children and investigates in more detail the way in which two children responded to these questions before attempting to solve their problem.

Results

General Observations

Not surprisingly, most of the problems generated initially by the children were quite closed and could be solved within a few minutes. With some guidance, the children began to pose problems that were more open in nature and were generally reflected in a *design and make* or *project* form. In total, nine (36%) of the children posed problems that were solved over a three-week period. Table 1 identifies some of the extended problems posed by the Grade 1 children. It also presents responses from the question *Will we need to do some mathematics?*

The children had posed problems that were diverse in nature and appeared to represent areas of personal interest. Mathematics content involved measurement concepts of time, duration, sequencing, length and mass; space concepts of position, shape and structure; and number concepts of money. Although the problems were open-ended the children were still able to recognise the type of mathematical understandings that where embedded in the problem. This was not to say that they knew fully how to go about solving the problem but that they recognised the type of problems they faced within the context of the entire problem.

Table 1
Types of Problems Posed by the Grade 1 Children and Their Response to the Mathematics Required to Solve the Problem

Description of Problem	Student's Response
Designing the school playground	Measure how big and little things are
	Measure how wide and long things are
	Make sure we put things in the right place
	Count the sides of the sandpit
	Step out how far things are away from other things
Listing chores to be done on a farm	Knowing how long it takes to feed all the animals
	Knowing when to feed the animals
	Writing down the time
Becoming a bank manager for a day	Be able to count the money
	Know where everyone works
	Know what time to open and close the bank
	Know where all the money is kept
Build a playground	Make sure the swings and slippery dip are strong enough
	Make sure everything is the right size
	Have really big trees
	Make sure things are not too close together
Making a board game	Make sure all the squares are the same size
	Have a "miss a turn" or "roll again" in every 12 squares
	Make sure some of the special cards are good and some not good

After the children had highlighted components of the problem that required mathematical thinking, they were encouraged to consider whether some of these activities would be more difficult to complete and to identify where they may need additional help. In some instances the children were able to identify the approaches they would need to employ without too much prompting whereas on other occasions more in-depth probing was required. Generally, the children had "grand" ideas for completing the activity. Some of these approaches were quite ambitious and not practical. In such instances the student teacher encouraged the child to propose another solution. Table 2 provides examples of particular elements of posed problems that the children considered to be difficult to solve and approaches that could be used to complete the task.

It could be argued that these responses were insightful and reflective. Although solutions to these problems would not be obtained for up to three weeks, the children were able to discuss openly approaches they would need to use and in addition identify elements of the problem that may require modifying. Moreover, they were able to identify aspects of the problem that could not be solved without assistance from the student teacher.

Table 2
Components of a Problem Identified as Being Difficult to Solve and Suggested Approaches to Complete Such Activities

Description of Problem	Difficult Elements of Task	Approaches That Could be Used
Designing the school Playground (See Figure 1)	Measuring the height of the big tree	You could stand on a ladder with a very long measure. We could measure the shadow on the ground.
Listing chores to be done on a farm (See Figure 2)	Knowing how long it takes to feed all the animals	We could ask a farmer We could make sure that the more animals we have the longer it takes to feed them.
Becoming a bank manager for a day	Be able to count the money	We could use a calculator but it might be better if you helped me with this
Build a playground	Make sure the swings and slippery dip are strong enough.	We could make the model of you (the teacher) first because you are heavier than me. Then we could make sure you can sit on the swing. If you can I can.
Making a board game	Have a "miss a turn" or "roll again" in every 12 squares.	We could put a cross on each number twelve square and then make sure we have one inside each cross

Analysis of the Students' Responses

Case 1: Jane's playground. Jane decided that she wanted to construct a model of the school playground (See Figure 1). Interestingly, she recognised that it would be too difficult to make a model of the entire playground "because we wouldn't have enough time". The student teacher working with Jane helped her modify the task to include a section of the playground that contained a boundary fence, the sandpit and some large trees.

Jane thought that the best way to begin constructing the model was to position all the important objects on a large piece of paper (which later became a piece of masonite). With respect to positioning these objects, Jane wanted to "step out" how far each landmark was from the other. Not surprisingly, she did not know how to transfer this information accurately onto the paper but did not see this as a problem because "you have to put it where it looks right." Jane did not have an understanding of scale (after all she was in Grade 1) but appreciated that her model needed to represent the 3D world because "that tree will have to be the biggest thing in my model." She also realised that the tree had to be taller than the sandpit was long.

Despite the fact that Jane was not able to develop a scale she felt that it was important to measure everything. Her greatest concern was working out how to measure the large tree. Initially, she thought that her teacher could stand on a ladder with a large tape and she could read the measurement on the ground. Before they had a chance to "act out" her proposal she conceded that the school did not have a ladder that tall. She then hypothesised that they could measure the shadow of the tree in order to ascertain the height of the tree. She was not convinced that this was valid but felt that it must have been a good idea based on the student teacher's reaction.

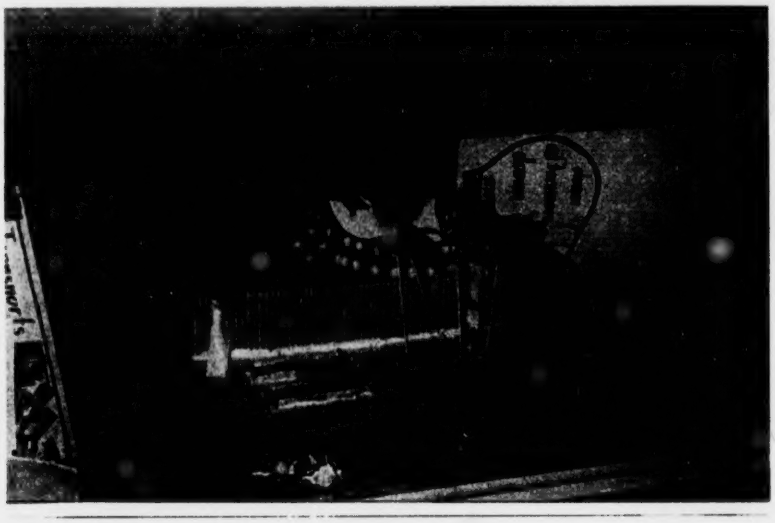

Figure 1. Jane's playground.

Jane was actually creating new problem-posing situations to solve within the context of the original problem she had designed. The open-ended nature of the task provided her with the opportunity to set personal outcomes as the problem evolved.

Case 2: Peter's farm. When Peter originally generated a problem to solve he proposed to build a farm. After discussing the context of the problem with his teacher he modified the task so that it involved describing what chores a farmer and his helper would be required to undertake each day. With some support, Peter recognised that his problem required him to develop timelines of events for the respective characters.

Figure 2. Peter's farm.

Peter decided that he needed to think about what animals were going to be on his farm before he could work out who was going to feed the animals. Thus, Peter demonstrated that he was able to select and monitor relevant problem-solving approaches in a multi-level context. Peter maintained that the most difficult component of his problem was determining the length of time it would take to feed the animals. Importantly, he established a set of criteria that would determine the time it would take to feed the animals with "the more animals we have the longer it takes to feed them." With respect to determining which person would feed particular animals he felt that the farmer should look after all the large animals (eg., cows) while the helper could look after the smaller animals like chickens. Like Jane, Peter was able to develop structures within the open-ended context of the problem that required him to solve additional problems and at the same time established a solution path for the initial problem.

Conclusion

The Grade 1 children described in this study were able to design their own open-ended problems and then collaboratively solve the problem over a three-week period. The children were able to identify mathematical concepts associated with the task and identify areas where they might need assistance. The children were able to consider components of the problem that would be difficult to solve and approaches that could be used to overcome this complexity. Although the children received assistance on a one-to-one basis during the activity, several of the children demonstrated an ability to reason metacognitively.

Several implications emerged from the study.

- Young children are able to generate their own problem-posing contexts if assisted to refine the scope of their investigation.
- Opportunities for engaging in effective problem-solving processes occur prior to children attempting to solve the task.
- Young children are able to modify open-ended tasks if they are assisted to reflect upon the approaches and understandings they require in order to complete the task successfully.

- Some of the most effective open-ended tasks are activities that involve *design and make* or *project* components.

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Knowledge and Strategies Students Employ to Solve Open-Ended Problem-Solving Activities

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This paper examines the type of specific understandings and general strategies students of different ages (Grade 3 and Grades 5/6) employ to solve an open-ended mathematics problem. The task required the students to construct a map that would allow a friend to get from a specific destination to their house. Although the Grade 6 students had a more sophisticated understanding of scale and proportion than the Grade 3 students there was not a great deal of difference between their worksamples.

Increasingly, open-ended problem-solving tasks are expected to be part of teaching and assessing in primary schools. Open problems required students to make their own decisions, plan their own routes through tasks, choose methods, and apply mathematical knowledge (Boaler, 1998). These tasks may have many or no specific answers, a variety of solutions or possible representations, and/or more than one interpretation of the problem. Such problems provide opportunities for students to engage in mathematical experiences at a higher level than do typical routine tasks (Lowrie, 1998a). Researchers are using a variety of terms to indicate these tasks including: novel problem solving (English & Warren, 1994); extended mathematical tasks (Leder & Forgasz, 1997); non-routine problem solving (Lowrie, 1996, 1998a; Taplin, 1994); and goal-free problems (Sweller, 1992). Open-ended tasks allow students of different ability levels to work on the same problem and achieve different outcomes. Very good open-ended tasks can generate a variety of mathematically valid responses that differ in the quality of understanding displayed (Clarke, Clarke & Lovitt, 1990). In the present study student's responses are assessed in relation to the mathematical knowledge and general knowledge of the processes (in this case maps) that are displayed in their solution.

Knowledge and Understandings Used in Open Tasks

Open-ended tasks allow the problem solver to have some control over the curriculum content and the type of learning activities presented in the classroom. Furthermore, the tasks or the activities children construct may provide insights into the beliefs or attitudes they have toward mathematics and the way in which mathematical knowledge is developed. The very nature of these tasks ensures that the problem solver will use different types of knowledge and a variety of strategies to complete a given task. Although numerous studies have found that children are encouraged to use a diverse range of approaches when solving open-ended tasks (eg., Lowrie, 1998b; Sullivan, Bourke, & Scott, 1995) few studies have attempted to monitor the extent to which mathematical understandings and knowledge are developed and enhanced when students solve such tasks. Mayer (1992), for example, argued that whilst open-ended problems have been studied widely over the last thirty years the emphasis has been on the diversity of responses and not on the kinds of mathematical knowledge which are evoked through such experiences.

The ability to access relevant information is central to success in problem solving (Chi, Glaser & Rees, 1982). In most open-ended situations the problem solver needs to access a combination of specific knowledge relevant to the problem and general knowledge that is more easily transferred from other problem-solving situations (Glaser, 1991). The opportunities to find alternate solutions, justify solution strategies, and reflect on one's actions from immediate feedback are also critical aspects of the problem-solving process (English & Halford, 1995). In this study we assess the extent to which students use specific mathematics knowledge of proportion and scale and general understandings of map representation to complete an open-ended investigation.

The Purpose

The central concern of the study was to investigate the influence students' general understanding of maps and specific knowledge of scale had on their ability to complete an open-ended task. The following research questions were posed.

- What impact did a student's knowledge of scale and proportion have on the quality and accuracy of the final product?
- Did the students' age (Year 3 to Year 6) influence the way in which they interpreted and represented the problem?
- Did the students' locality (city or rural) influence the way in which they interpreted and represented the problem?

Method

The students

The investigation took place in two primary schools in New South Wales. The first site involved 21 Year 3 students from an independent girl's school in Sydney. At the second site students from a Year 3 class (n = 26) and a selective Year 5/6 Opportunity Class (OC) class (n = 29) participated in the study. This school was located in a large rural city. It could be argued that the composition of the sample was diverse in nature, with the children coming from distinct cultural and socio-economic backgrounds. It was anticipated that the students would bring quite different experiences to the problem-solving situations when considering their age and place of residence[1].

The Activity

The activity was conducted over a week (of four or five mathematics lessons) and consisted of three interrelated sections. These three components included a) a questionnaire; b) an open-ended problem-solving task; and c) a reflective response. The questionnaire was designed as an assessment tool to determine the type of knowledge and understandings the students had acquired on concepts associated with maps. The students were encouraged to describe instances when they had used maps, consider people who relied on using maps and identify important characteristic of maps. The questions were carefully scaffolded in order to encourage the children to remember as much as they could about maps including concepts of scale and proportion.

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[1] The problem-solving activity required the students to design a map of their local area. The way in which the map needed to be represented would be different for children living in the city or rural areas. Moreover, the children's understanding of what constituted a map would have been influenced by their experiences in using particular types of maps.

The open-ended task was presented in the following scenario.

> **Pretend a friend from out of town is coming to play with you on the weekend. Your friend knows how to get to your school but has never been to your house. Unfortunately you are unable to meet them at school so you decide to post your friend a map that gives directions on how to get from school to your house. The map should allow your friend to find your house easily.**

The students were encouraged to consider the mode of transport most commonly used to get home from school as a starting point for designing an appropriate route. Many of the city students lived a considerable distance from school with some children's journey involving several modes of transport.

The third component of the activity was completed after the students had completed their map. This reflective response encouraged the children to describe parts of the problem they found easy or difficult, what they had learnt about the process, and whether or not they would do things differently if presented with a similar problem in the future. Many of the questions were designed in order for the students to think at a metacognitive level.

Data Coding

Three sets of data—including the questionnaire, the problem-solving task and the reflective response—were used to monitor outcomes for the activity. The criteria identified in Table 1 are categorised under the headings of a) general knowledge of maps; and b) specific mathematics understandings.

Table 1
Criteria Used to Assess Students' Responses

General knowledge of maps	Specific mathematics understandings
Landmarks	Understanding of scale
Key	Directions- N, S, E, & W
Symbolic representations	Coordinates
"Birds eye" [plan] view	2-D representation

Results

Questionnaires

When asked to describe what they knew about maps a wide range of responses were obtained. Students were aware of a diverse array of maps in the form of street directories, world and country, treasure, weather, railway, mind / concept, planet, orienteering and wildlife maps. Most were able to explain that maps were useful for providing directions to specific locations and to help find travel routes.

The greatest difficulties which the students expected to encounter while trying to draw their maps were trying to fit everything in on the page and trying to remember street names. The Grade 5/6 teacher commented that with her students, there appeared to be very little carry over of what they said they knew about scale to actually applying it to their maps (only one Grade 5/6 student included a scale on their map). In describing these difficulties the older children openly discussed scale and proportion. Some of the younger children demonstrated informal notions of proportion when commenting that it would be difficult to fit everything on the page. Interestingly, the younger children were not as worried about precision in their drawings as much as the older students. The Grade 5/6 teacher noted that the class was full of "perfectionists" with many of the children enjoying a "healthy rivalry" with peers. This would be expected in a class of very talented students.

The Problem-solving Task

All of the maps produced by the students were analysed and specific features identified and collated in Table 2. All maps drawn were from a "birds eye" or plan perspective view.

Table 2
Incidence of Significant Map Features

Features	Year 3 City School	Year 3 Rural School	Year 5/6 Rural School
	n = 21	n = 26	n = 29
Use of grids	24%	0%	0%
Use of streets	90%	96%	100%
Naming of streets	72%	54%	97%
Use of key	0%	12%	93%
Use of scale	0%	0%	4%
Use of proportion	12%	15%	48%
Use of direction	33%	42%	86%
Compass directions	0%	4%	6%
Landmarks			
– Houses	52%	92%	83%
– Railways	24%	0%	21%
– Other	81%	100%	34%
More than 1 page	43%	4%	7%

From the above data, it can be seen that five students from Grade 3 at the city school used grids (see Figure 1) on their maps and nine made use of more than one page. A possible explanation could be that these students had prior exposure to street directories. This is in contrast to the children from the rural school whose parents no doubt did not make frequent use of a street directory. This is a good example of how children bring their prior knowledge/experience to school tasks. The use of a key (see Figure 2) by three of the rural Grade 3 students was interesting since it had not been taught specifically at school. Once again an example of possible prior knowledge/experience was being used in a school context.

A large number of students from the Grade 5/6 class included street names on their maps (see Figure 3) perhaps suggesting a developing awareness of a common feature found on most maps. Another interesting observation was that the city students were less likely to include houses in their maps than the other students were. This may be due to the fact that the majority of these students lived a long distance from their school and concentrated their attention on bus/railway routes and other significant landmarks to find their way in suburbia. Only one student made a rough draft before drawing the map. He explained that "I needed to tell myself where to start and finish. I put myself in the car and drove there." The three teachers commented that most students started from one point (eg., the school) and simply progressed to the destination.

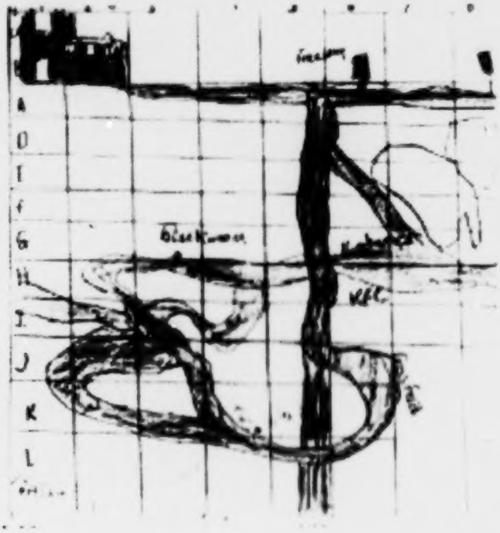
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Figure 1. Example of a map drawn by a city Year 3 student

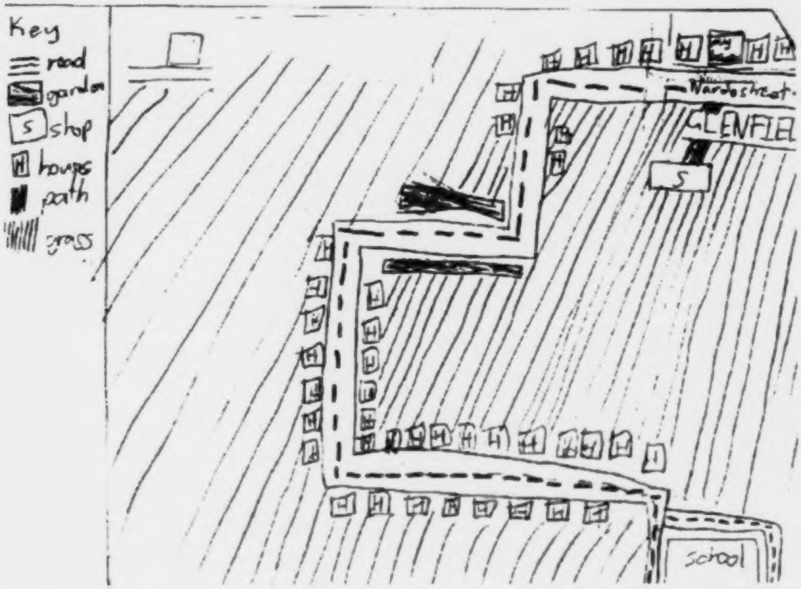
Key
road
garden
S shop
H houses
path
grass
S
H
GLENFIEL
school

Figure 2. Example of a map drawn by a rural Year 3 student

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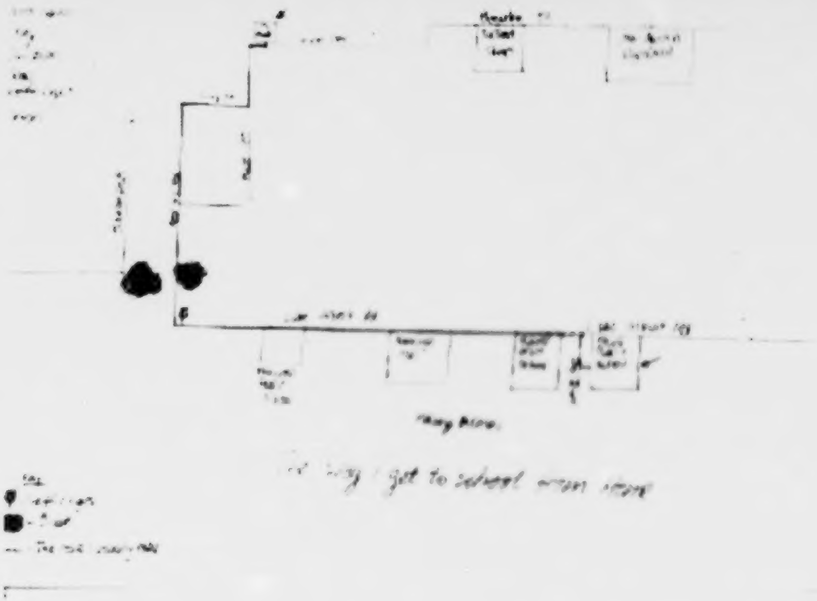

Figure 3. Example of a map drawn by a Year 6 student.

Reflective Responses

Having completed their maps, the students were asked a number of questions to ascertain what they had learnt from doing the exercise. A number commented on the need to learn the names of the streets they use as they travel between their homes and school. This suggested that the students were concerned about the specific content rather than the actual process of designing the map. Some students expressed the need to first try to visualise a rough copy of a map in their head. Others suggested that they needed to be more accurate. Seven (14%) of the Grade 3 students demonstrated an understanding of informal scale; however these younger children were not able to explain why scale could have helped make their map more accurate. In contrast, almost all of the Grade 5/6 students commented that they should have calculated a scale before attempting to draw a map. Over 50% of these students recognised that some of the distances would need to be guessed but accepted that this would still be far more accurate than their original drawings.

The knowledge students had acquired about maps was similar—with the exception of understandings associated with scale—despite the fact that some children were three years older than other students. In most instances, the Grade 5/6 children could not, however, transfer their knowledge of scale into this new problem-solving context. Similarly, the Grade 5/6 children employed the same type of processes to complete the problem even though they appeared to have more sophisticated processes at their disposal.

Conclusions

As anticipated, the Grade 5/6 students had a more sophisticated understanding of scale and proportion than the younger children did. Interestingly, this effected their ability to complete the task to their satisfaction because they were worried about constructing something that was not accurate. On the other hand the Grade 3 students did not have such a well grounded understanding of these mathematics concepts and so were much more satisfied with their final product because they thought it served as a good *informal* measurement which looked reasonably accurate.

The Grade 5/6 students were able to articulate what they had learnt from the process and were able to offer insightful comments about ways of making their maps more accurate in the future. Moreover, their maps contained an increased degree of symbolic representation. Overall, however, their maps were not significantly different from the more basic "mud maps" produced by the Grade 3 students. Despite the fact that the older students would have been exposed to the teaching of specific concepts such as space and scale, we felt that fundamentally the ability of all three groups of children to carry out the mapping task was essentially similar. Overall most of the maps produced by the Grade 3 students were quite accurate and could be used to find directions between A and B. No doubt these children were able to link their evolving mapping skills to prior experiences and knowledge gained outside the classroom context. These students were able to draw on their life experience rather than relying on what they learnt from school. When the children were presented with new open-ended problem-solving experiences, such as drawing a map, they reformulated their existing cognitive framework only if the new knowledge was connected to knowledge already in their memory. Children must be encouraged to actively construct knowledge into their existing framework for meaningful learning to occur.

We were able to detect subtle differences between the maps produced by the Grade 3 students from the city and rural school. Some of the maps drawn by the students from the city school resembled a street directory—five contained grid lines and nine were drawn on a number of pages that were attached together. This was in contrast to the rural students' maps that did not have such features. It would be reasonable to suggest that the students who lived in the city were more likely to be exposed to such maps than their rural counterparts. Thus, the lived experiences of the students influenced strongly their representations.

It could be argued that this open-ended task did not provide the older students with the flexibility and scope to demonstrate the specific mathematics knowledge they possessed. However, these students were able to verbalise what they would do differently if given a similar task in the future in a much more sophisticated manner than were the Grade 3 students. They were able to reflect upon the problem-solving strategies they had used and recognised that they should have considered scale and proportion more closely in their representations. Consequently, this investigation shows that the problem-solving process involves much more than an analysis of the completed product (worksample). Although the Grade 5/6 students were not able to represent their knowledge and understandings through the mapping activity, the open-ended investigation enabled them to learn much from the process.

Implications

Most differences in the worksamples could be attributed to normal developmental processes of abstract thinking, spatial awareness and estimation skills rather than an ability to apply mathematical knowledge to new situations. The manner in which the children represented their work was influenced strongly by experiences gained outside the school context. This study has shown that we need to be very clear about the purpose of open-ended problem solving and its associated worksamples. Consequently, we would argue that:

- A worksample (ie., the product) is not sufficient as evidence of student outcomes in a particular content area. The completed task should be put in the context of what school and life experiences the student brings to the task and the metacognitive reflections the student is able to convey about the physical product; and
- Open-ended tasks may not provide opportunities for students to represent what they know about a particular content area if the novel nature of the task limits their ability to apply understandings to such new contexts.[1]

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Mental Computation, Number Sense and General Mathematics Ability: Are they Linked?

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Number sense is generally regarded as an important aspect of general mathematics knowledge, while mental computation performance is frequently thought to link to number sense ability. With current national numeracy testing placing an emphasis on number sense, this study explores the relationship between number sense, mental computation and general mathematics performance of primary students. Results indicate that performance in these three areas was often disparate, and raise some interesting questions about number sense and mental computation and assessment of mathematics curriculum.

Introduction

Number Sense and Mental Computation

In the video *Real Maths, School Maths* (Lovitt & Clarke, 1992) a child (Aidan) aged about 10, is seen dealing with two computations. First, he is shown a chocolate bar and is told it costs forty-five cents; he is asked to calculate mentally what change he would get, if he bought one chocolate bar with a five-dollar note. He handles the calculation confidently and efficiently, and explains his strategy clearly. On a second occasion he is seen seated at a desk with paper on which the identical subtraction (500 - 45) is written in vertical form. Aidan expresses doubt whether it is a subtraction or a division, looks uncomfortable, performs the written subtraction incorrectly, and is at a loss to explain his method. Aidan's mental computation displays, and draws on, number sense that is markedly lacking in his written computation.

Although the term "number sense" is relatively new to the language of mathematics curricula, its emphasis on understanding and meaningful learning is commonplace in the literature of mathematics education (Brownell, 1935; Hiebert, 1984; Plunkett, 1979). Number sense refers to a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for managing numerical situations (McIntosh, Reys & Reys, 1992). It is characterised by a desire to make sense of numerical situations, by looking for links between new information and previously acquired knowledge, and by an innate drive within the learner to make the forming of these connections a priority (Reys et al., 1991). Those with well-developed number sense have strongly connected knowledge and in learning they appreciate the importance of making links between new and previously acquired knowledge. They approach problems involving number by seeking to make sense of the whole situation and by using the multiple representations and connected knowledge that they have.

Mental computation, or the ability to conduct mental numerical calculations without recourse to external assistance (e.g., pen and paper), is also regarded as a valuable skill for all Australians (Australian Education Council, 1991). Research has indicated that people use a variety of (usually self-devised) strategies to assist mental computation (e.g., Bana & Korbosky, 1995; Swan, 1991; McIntosh, De Nardi, & Swan, 1994). There is evidence to suggest that at present students are often not directly exposed to such strategies at school but rather are left to devise for themselves more or less efficient strategies (McIntosh, Bana, & Farrell, 1995). Such strategies appear to derive from an individual's flexibility in handling numbers, and the development of useful strategies applied in efficient ways. Strategy development for mental computation, therefore, can be seen to derive to some degree from an individual's number sense.

National Numeracy Benchmarks

National standardised testing, particularly in Grade 3 and Grade 5, is a feature of the current school situation in Australia (Cuttance & Stokes, 2000), born out of the national education goal that children will be literate and numerate upon exiting primary school (MCEETYA, 1999). National literacy and numeracy benchmarks are being developed to provide a framework for reporting student performance at particular levels (Curriculum Corporation, 1999). The National Numeracy Benchmarks are categorised into 3 strands of Number Sense, Measurement and Data Sense, and Spatial Sense, and are incorporated into the standardised tests for numeracy developed in each state (Cuttance & Stokes, 2000). There is clearly a growing emphasis on the development of number sense within the mathematics curriculum. Indeed, documents calling for reform of school mathematics in many industrialised countries have emphasised the need for students to develop number sense (e.g., Cockcroft, 1982; Emanuelsson & Johansson, 1996; National Council of Teachers of Mathematics, 1989).

Implications for this Study

The advent of national testing of literacy and numeracy in the primary school (currently Grade 3 and Grade 5) and the development of National Numeracy Benchmarks that place an emphasis on "sense" about number, space, measurement and data, give rise to a number of questions pertaining to comprehensiveness, reliability and accuracy in determining children's mathematics ability. Primarily, the question is, would a general mathematics test be sufficient to assess a student's number sense as defined in the literature? A further dimension to mathematics performance and number sense occurs if mental computation is also considered. If mental computation performance is enhanced by number sense, then where, if at all, does the assessment of mental computation occur within assessment practices at school, system or national level? The exploration of such issues was the basis for this study.

This study was designed to begin to probe the relationship between the mental computation ability, number sense, and general mathematics of Grade 3 and Grade 5 students, and to raise questions about assessing mental computation and number sense, particularly in the current climate of increased emphasis on system-wide assessments and of national testing in Grade 3, Grade 5 and beyond. The following exploratory questions were posed at the outset of this research: Is mental computation ability closely linked with number sense, as some believe, but more loosely connected with general mathematical ability? Is a mental computation test assessing something different from a general mathematics test or a test of number sense, or will a class of children or individual children perform equally well on each?

The Study

Separate pencil-and-paper tests of mental computation (MC), number sense (NS) and general mathematics (MA) were given to a total of 58 Year 3 and 60 Year 5 students in two Tasmanian primary schools. Tests were administered during students' usual mathematics time over three days. The mental computation and number sense tests each contained thirty items at Grade 3 level and forty items at Grade 5 level. The general mathematics tests were devised by taking items from Diagnostic Mathematics Profiles (Gough 1999). Validity of the two general mathematics tests was obtained by subjecting them to the scrutiny of two experts and by the agreement of the four class teachers involved that the tests reflected the range and level of mathematics being taught in their classrooms. Reliability tests (Cronbach-Alpha) for internal consistency for the Grade 3 and Grade 5 tests of mental computation, number sense, and general mathematics gave the following results: Grade 3 - MC 0.91, NS 0.68, MA 0.75; Grade 5 - MC 0.93, NS 0.84, MA 0.90. Apart from the Grade 3 number sense test, and to some extent the Grade 3 Mathematics tests, the tests appear to be extremely reliable. The difficulty of constructing adequate pencil-and-paper tests of number sense at grades as low as Grade 3 has been discussed elsewhere (McIntosh, Reys, Reys, Bana & Farrell, 1997).

On the basis of test results, students were selected for individual interviews to explore their solution strategies and thinking. Students were chosen either because their results on the three tests were markedly similar, or because they were markedly dissimilar.

Results and Discussion

For Grade 3, the mean scores for MC, NS and MA were 78%, 67% and 77% respectively. For Grade 5, the mean scores for MC, NS and MA were 65%, 58% and 67% respectively. In Table 1, mean scores for each of the tests, together with standard deviations, minimum and maximum scores are presented.

Table 1

Summary of Results as Percentages for Grade 3 (n=58) and Grade 5 (n=60) on the Mental Computation (MC), Number Sense (NS) and Mathematics (MA) Tests

	Grade 3				Grade 5			
	x	SD	min	max	x	SD	min	max
MC	78	20	20	100	65	24	3	97
NS	67	16	20	95	58	19	0	93
MA	77	15	21	96	67	18	20	93

Correlation tests (by Fisher's r to z) showed that the scores on each of the three tests were closely associated with each other. To ascertain the extent to which performance on one test predicted performance on another, simple regressions (r^2) were calculated. The results are shown in Table 2.

At both levels there appears to be an appreciable amount of the variance in the mental computation scores (Grade 3: 0.36, Grade 5: 0.33) which is not predicted by the mathematics scores. This is even more marked in the case of the number sense scores (Grade 3: 0.56, Grade 5: 0.45). While it is not possible to make confident assertions on the basis of these very small samples, MA scores appear to be a better predictor of MC than NS for both grade levels, and MC scores appear to be a better predictor of NS at Grade 5 than at Grade 3. This result is not surprising given the fact that the Grade 3 NS test had a much lower reliability score than that for Grade 5 NS.

Table 2

Simple Regressions (r^2) at Grade 3 (n=58) and Grade 5 (n=60) between Scores on the Mental Computation (MC), Number Sense (NS) and Mathematics (MA) tests

	MA:MC	MA:NS	MC:NS
Grade 3	0.64	0.44	0.59
Grade 5	0.67	0.55	0.72

Interviews

Eight students from each Grade level were selected for individual interview on the basis of their test scores. For brevity, interview data for three Grade 5 students (Student 5A, 5B and 5C) and three Grade 3 students (Student 3A, 3B and 3C) will be presented here.

Grade 5 Students

Student 5A scored highly on both the mental computation (91%) and general mathematics (93%) tests, but relatively low on the number sense test (67%). Interview data revealed that, for mental computation, this student executed written algorithms mentally at a very rapid rate, and the written algorithm performed mentally was the predominant strategy used. For example, when asked to explain her strategy for mentally calculating 72 ÷ 3, the student replied: "I put down a division sort of thing, well how you write down on paper, how you work it out with that kind of one and the curvy bit and I just did it in my head instead of doing it on paper." When asked to elaborate on the process, the student replied: "I had 72 under the line and then there was 3...I went 3 into 7 is twice and I put down the 1 next to the 2...and ...3 into 12 is 4 so 24."

This response indicates the student's capacity to hold a great deal of symbolic information in her head at once, and to visualise the written operation to achieve the correct answer. Interestingly, the student had difficulty dealing with certain number situations if she had not previously encountered them, evident in her response when she was asked by the researcher to mentally calculate 450 ÷ 15: "Okay, I haven't learnt these yet...I don't know, I haven't done double numbers divided."

However, when given the "double numbers divided" problem of 3500 ÷ 35, the student immediately noticed the relationship between the numbers by stating that the answer was 100 because: "... 3500 divided by 35, well they're sort of the same number so I took off ... 2 zeros is 100 so ... 100 times 35 is 3500". For items relating to number sense, there appears to be evidence that Student 5A has not become aware of the effects of some operations. For example, one of the number sense items asked students to determine whether, upon calculating 29 ÷ 0.8, the solution would be less than 29, equal to 29, greater than 29, or impossible to tell without calculating. Student 5A's response was as follows: "Well...in the test I kind of wasn't really sure and put the last answer" [D. Impossible to tell without calculating].

Another item on the number sense test related to estimation with students asked to choose the best estimate for 45 x 105 from three choices (4000, 4600, 5200). The following transcript appears to suggest that Student 5A is reluctant to use (or at least to articulate the use of) estimation as a strategy to assist with calculation. (In the transcript, the letters S and R are used to identify the dialogue of the student and researcher respectively).

S: Probably about 4600 or 5200 or something
R: So either of those two?
S: Yeah, maybe
R: Can you be more specific or would you rather work it out?

S: I'd rather work it out.

On a similar item, where students were asked to estimate 18 x 19, Student A2 replied:

S: Probably about one hundred and eighty

R: ...and what are you doing in your head?

S: Just guessing.

From interview questions relating to items on the general mathematics test, it appeared that Student 5A's understanding of fractions, measurement, space and chance and data assisted her to do well on this test. She also executed written algorithms flawlessly. However, on one particular item where students were asked to share $42 equally between 4 people, Student 5A totally disregarded the context of the question and simply performed the standard written division algorithm, as indicated by her response to her solution:

S: For this one I did 42 divided by 4 and I know my times tables so it'd be 10 and 2

R 10 remainder 2?

S: Yes

In Student 5A's case, there appears to be a high link between mental computation ability and mathematics ability. However, Student 5A's competent calculation skills, both written and mental, appear to have overridden any need for this student to explore the effects of operations on numbers. The student does have number sense in terms of "seeing" particular relationships between numbers, but other aspects of number sense are not evident.

Student 5B scored highly on all three tests, but her number sense test score was the highest (90%) for all Grade 5s tested (MC 89%, MA 97%). Interview data revealed that this student used a range of strategies for mental computation, assisted by being able to see relationships within and between numbers, as is evident from her solution processes to the following mental computation items. For 72 ÷ 3, Student 5B saw 72 as being a combination of 60 and 12, and explained her strategy as "I just did 3s into 60 and then how many 3s go into ...12 and then add them together." For 165 + 99, the student noticed the link of 99 to 100: "Well because 99 is close to 100 you just add that and when I've got the answer to that I'd take 1 off again". For items on the number sense test, the student could explain the effect of various operations on numbers, and could analyse solution strategies to particular calculations. This student's number sense clearly assisted her with mental calculations and number situations, but her less than perfect result on the mental computation test appears to be the result of inaccuracy in application of some strategies.

Student 5C scored at about the class average for mental computation and number sense (74% and 70% respectively), but had a relatively low general mathematics test score (62%). Interview data revealed that this student used visualised written algorithms for mental computation involving decimals, multiplication and division, but with less accuracy than Student 5A. This student also appeared to rely on "rules" for particular mental calculations (e.g., 3500 ÷ 35 = 100 because "35 only goes into 35 once with the two zeros on the end"; and 4200 ÷ 60 = 700 because " 6 goes into 42...7 times and then add the zeros which is 700"). In the number sense test, this student could explain the effect of operations on numbers and could analyse strategies for calculations. On the general mathematics test, the student's poor fraction knowledge appeared to hinder high performance, and also there was evidence of limited understanding of measurement concepts associated with mass. In this instance, the student's mental computation ability does not appear to be related to her number sense ability as the student does not apply a range of strategies for mental computation. The similarity in scores on each of these two tests actually appears to be quite unrelated when the interview data are analysed.

Grade 3 Students

Like Student 5A, Student 3A's mental computation and general mathematics scores were similar (90% and 83% respectively), but number sense score was relatively low (60%). Student 3A was found to have some very good mental strategies, for example in describing his solution process for 74 - 30, he stated "I just took 4 away ... 40 and then I put the 4 back on so that was 44". In the number sense test, there was some evidence of difficulty with strategy analysis for estimation (he was unable to identify a strategy from a given list that would determine whether correct change had been given). Interview questions pertaining to items from the general mathematics test revealed some good conceptual understanding of fractions, measurement and space.

Like Student 5B, Student 3B's MC and NS scores were similar, but MA score was considerably different. Unlike Student 5B, Student 3B's test scores indicated a weaker mental computation and number sense (both 50%), but a relatively high mathematics ability (88%). Interview data revealed that, when performing mental computation, this student saw possible solution strategies, but tended to employ a sometimes inefficient and inaccurate counting on and counting back strategy. For example, in the following interview extract, the student is describing her solution process for 36 - 9:

S: I counted...take 9 from 36

R: You were thinking very hard then, can you just tell me what was happening in your mind when you decided it was 27?

S: I was counting, but I was thinking of another way of doing it

R: Were you counting backwards?

S: Yes

R: How did you know when to stop?

S: I was using my fingers

R: And the other strategy you were thinking of?

S: You can take 10 equals 26 and...add on 1 equals 27

In the number sense test, Student 3B could describe appropriate strategies for checking calculations, but had difficulty on items relating to estimating sizes of collections and numeration. Items relating to the general mathematics test indicated good conceptual understanding of concepts associated with measurement, space and chance and data.

Like Student 5C, Student 3C scored the highest on NS for all Grade 3 students (95%), and her mental computation and general mathematics test also were high (87% and 92%) respectively. Interview data revealed that this student employed a wide range of strategies for mental computation, and knew about the effects of operations on numbers. Her strong conceptual knowledge assisted with general mathematics test items. The profile of this student is similar to Student 5C, where the strong sense of number relationships (an aspect of number sense) appears to lead to the application of a variety of mental strategies for mental computation. Also like Student 5C, Student 3C did not rate the highest on the mental computation test, but the level of "sense about numbers" related to high scores on both the number sense test and the general mathematics test.

The Grade 5 and Grade 3 children who performed well on tests of both mental computation and general math but not so well on tests of number sense appear to have developed useful and efficient strategies for managing mental situations, which is a component of number sense (McIntosh, Reys, & Reys, 1992), but appear not to have a general understanding of number and operations (McIntosh, Reys & Reys, 1992) or an awareness of the relative effect of operating on numbers (NCTM, 1989). These students' conceptual knowledge of various topics within the mathematics curriculum assisted in high scores for general mathematics tests. It is tantalising to suggest that high levels of competence in accuracy for mental computation exercises and general mathematics tasks has negated the need for these students to develop thinking about the processes of computations and operations.

The Grade 5 and Grade 3 students that scored the highest on the number sense test were seen to utilise a range of strategies for mental computation, (although their mental computation test scores were not the highest of all subjects in this study), and they appeared to have a greater awareness of numbers and the effects of operations on numbers. There appears to be a link between number sense and mental computation in terms of strategy knowledge. For the Grade 5 and Grade 3 students with similar MC and NS scores but relatively different MA scores, there appear to be other factors at work impeding their accuracy in mental computation (such as poor application of strategies, difficulty with holding a lot of numbers in the mind to complete calculations mentally, lack of confidence with large or unfamiliar numbers), and also performance on general mathematics tests (such as poor conceptual knowledge of the topic, or misconceptions).

Concluding Comments

This was an exploratory study to consider some aspects of the relationship between school mathematics, mental computation and number sense. It is not possible to generalise from the results of such a small sample. Nevertheless some interesting points for discussion about mental computation, number sense and general mathematics ability can be raised.

In this study, it appeared that the three tests were testing somewhat different things. Students' results on the three tests often were disparate, and in some cases quite remarkably so. Tentative results reported here tend to suggest that the three tests are not as closely connected as might have been expected. If all three areas (general mathematics, mental computation and number sense) are considered important, indeed essential, parts of mathematics at school, then it appears that mental computation and number sense need to become integral components of curriculum and assessment procedures, at class, school and system levels. Otherwise, the curriculum may be distorted by playing down the importance of number sense and mental computation, and students may be either advantaged or disadvantaged if there is failure to assess important aspects of mathematics.

From interview data, it was apparent that students with good mental strategies were not always the students that performed the highest on mental computation, but that often they had a good understanding or "sense" of numbers and their relationships. In all cases, good conceptual understanding of particular mathematics topics was seen to contribute to high scores on the general mathematics test, and good conceptual understanding assisted in understanding particular number sense and mental computation items (particularly fraction items).

In conclusion, this study suggests that students who may score highly on mental computation tests and general mathematics tests may not be developing a "sense" of numbers. And students who do not score highly on written tests of mental computation, number sense and general mathematics may still have quite good strategies for mental computation and a lot of "sense" about numbers. Fostering the development of number sense and conceptual understanding of numbers and operations, and probing further to ascertain whether a student's accuracy in mental computation is the result of successful mental application of written strategies or based on a more flexible range of mental strategies, may strengthen the relationship between mental computation, number sense and general mathematics ability.

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Designing Constructivist Computer Games for Teaching about Decimal Numbers

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This paper reports trials of two computer games designed to enhance learning about decimals. In two exploratory studies, students used especially designed computer games that focussed on aspects of decimal understanding. These games were effective in challenging children's misconceptions about decimals. Students developed strategies for dealing with decimals, assisted by teachers. This article considers the nature of teaching assistance that could be programmed into the games to further strengthen their usefulness as a tool for increasing understanding about decimal numbers.

Solid understanding of decimal number concepts is important in many aspects of everyday life, such as when dealing with money and measurement. Learning about decimals is also important for successful academic performance in mathematics and other subject areas at high school and beyond. The Victorian Curriculum and Standards Framework suggests that decimal numbers should be taught in late primary school, from Grade 4 onwards. Around 25% of grade 4 students quickly develop a good understanding of decimals. However, only about 70% of year 10 students in schools in our region have achieved mastery of the decimals concept (Steinle and Stacey, 1998).

International and other Australian research shows that this is a widespread problem (Stacey & Steinle, 1998; Wearne & Hiebert, 1988). This research has established that there is a wide variety of misconceptions that interfere with new learning. Stacey and Steinle (1998) present a substantial listing of misconceptions and Stacey and Steinle (1999) demonstrate that for many children, misconceptions persist through years of schooling.

This paper reports on the development of two computer games designed to assist children with misconceptions about decimals. The games are intended to be used by children at home or in class to supplement normal instruction. Our unpublished work shows that the incidence of misconceptions about decimals can certainly be reduced by even a little targeted teaching. However, not all children get this and some of them do not get it when they need it. In this context, we decided that computer games may be useful addition to the pedagogical armoury.

This paper reports two exploratory studies to guide further development of two computer games. The principal aims were to find out what children learned from playing the games and to design appropriate teaching to be programmed into the games. In addition, the studies field-tested features of the games, such as the attractiveness of the "storyline", the clarity of instructions, technical issues such as the timing of display of feedback and so on. These results are not reported here, but they confirmed the general directions taken and assisted in fine-tuning the presentations of the games.

Computer Games as Teaching Tools

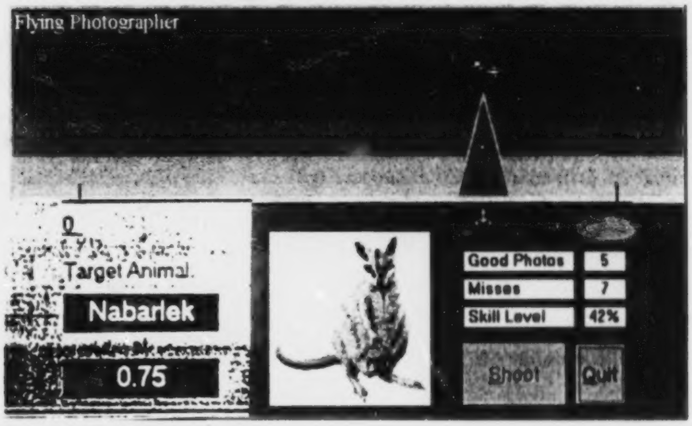
The impact of computers in education has been much discussed and many claims and recommendations have been made. Studies have shown that computer use in mathematics can heighten student interest and cater for a range of abilities, encourage students to pose and solve "What if...?" questions, allow for multiple representations of concepts and encourage the use of global strategies to solve a broad range of problems (Heid, 1997). Heid reminds us that it is important for games software designers to find out how different students use the games and suggests that the games' effectiveness is reliant upon how visible the mathematics is to the person using the game. Mathematics games software research stresses the need for games: to fill a gap in the current curriculum; to focus on the mathematics; to avoid making mathematics a hurdle requirement (i.e., having to perform a calculation before progressing to something exciting); and to lead the user to progressively deeper mathematical thinking. (Murray, Mokros, & Rubin, 1999).

In contrast to many "drill and practice" computer games, underlying the design of our games is a constructivist view of the teaching of mathematics. They have been designed within the literature on teaching that addresses misconceptions and challenges student's thinking. Bell's (1993) principles for the design of teaching have been useful. The games are designed around a variety of tasks which can highlight the errors that students make and present them with cognitive conflict to be resolved. Bell notes the importance of timely feedback, and this occurs within all our games. A degree of intensity of experience is provided by games - students can play as many times as they wish, with constant feedback, and the element of fun that games engender can make the experience more memorable. Bell notes the importance of adjusting the degree of challenge of the tasks that learners are faced with - in a computer games potentially this can be done in very sophisticated ways and a Bayesian network is being developed to do this (Dettman, Nicholson, Sonenberg, Stacey, & Steinle, 1999). Finally, Bell notes that the student should be exposed to a broad range of situations where the concept is illustrated in many ways and that this can be done by making changes of element, structure and context. This is achieved across the set of games, not in any one game.

In Bell's diagnostic teaching, cognitive conflict is resolved through children working together on tasks followed by teacher-led discussion, which also encourages reflection and review of learning. This cannot be guaranteed when children are playing a computer game, although students working in pairs can help. The computer game genre is often not one of thoughtful consideration and so it is a particular challenge to program useful teaching into the games. Useful literature on providing teaching within a computer game is scant. Most of the literature is concerned with on-line assistance for teaching the user to use large applications such as word processors (e.g., Duffy, Palmer & Mehlenbacher, 1992). The studies described below therefore wished to investigate whether the planned programmed teaching was indeed useful, whether children would ask for it when it was needed or whether it should be provided automatically when the student appeared to be making errors.

Study 1: Flying Photographer

The aim of this study was to observe what children learn from playing the game Flying Photographer and to investigate what teaching may enhance it. Flying Photographer (see Figure 1) was designed to develop student's appreciation of the size of decimal numbers as reflected in their positions on a number line. The student is in an aeroplane flying over a national park and has to photograph animals whose location is given as a decimal number on a number line. For example, locations in the (relevant part of) the national park may be given by numbers on the interval [1.2, 1.3] and the photographer may be seeking a potaroo at 1.254. The game is about estimation, not precision, so it moves fast and there is an (adjustable) tolerance interval. Flying Photographer gives immediate feedback, rewarding correct responses with a dialogue box that reads "Well Done!", a picture of the target animal and the correct location of the decimal on the number line is shown with a green triangle with a happy face on it. Flying Photographer indicates incorrect responses with a red triangle with a sad face in the photographed position. After three unsuccessful attempts to locate the number, the correct position is shown with a green triangle and a dialogue box directs the student to it with the comment "Wrong. Look at the green triangle."



Target Animal: Nabarlek

0.75

Good Photos: 5

Misses: 7

Skill Level: 42%

Note. The endpoints are 0 and 1. The player has successfully located a Nabarlek at 0.75 on a third attempt, indicated by the triangle with a happy face. This is the twelfth item in this game; the player has located five num-bers successfully and made seven incorrect responses, giving a skill level of 42%.

Figure 1. A screen shot from Flying Photographer.

Method

The Decimal Comparison Test (DCT), was administered to one year 5/6 class in an inner city primary school. This 10 minute paper and pencil test requires children to choose the larger of pairs of decimal number. The pairs of numbers are chosen so that misconceptions can be quite precisely identified, using methods outlined by Stacey and Steinle (1998). Four students who all have the same misconception about decimals were selected by the teacher to trial the game. All of these students see the decimal point as separating two whole numbers, rather like a time written in minutes and seconds. As a result, they believe longer decimals are larger. When confronted with a pair of numbers like 0.36 and 0.5 they choose 0.36 as the larger because thirty-six is bigger than five. Students with the same misconception were chosen so that the questions which they are most likely to get right and wrong were the same (and thus could be controlled by the experimenter). This particular misconception was selected because it is the most common misconception amongst year 5 and 6 children in Australian and international studies (Steinle & Stacey, 1998).

A task-based interview was used by a student teacher well known to the students. The children played two games of Flying Photographer to familiarise themselves with it with the

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researcher's help. After the initial two games, he also provided the teaching, either when requested by the child or when the child made three errors in a row. We distinguish these as solicited and unsolicited teaching. In both cases, the teaching consisted of placing a clear plastic overlay over the number-line on the screen, dividing it into finer gradations. If, for example, the endpoints of the number line were 1.2 and 1.3 , then the gradations marked 1.21, 1.22, 1.23, and so on. If the endpoints were 0 and 1, then the overlay marked 0.1, 0.2, 0.3, and so on. The overlay was in place for only one item then removed. The researcher observed and took extensive notes, observing any difficulties students had or comments they made. For analysis, these were incorporated with the computer file showing all the moves made by the child. On occasion, the researcher asked the child questions for clarification.

Results

Table 1 shows that the students all followed a general trend toward improvement in their ability to place decimal numbers on the number line as they played more games with the same endpoints. When the game presented new endpoints, students had to learn again. For example, Table 1 shows Student A was able (after at most three trials) to locate 55% of the specified numbers on the interval [0,1] in the first game. This improved to 100% by the fourth game. (The dashed line under *skill level* for game three shows that the player has prematurely quit that game, possibly accidentally.) When the interval was changed to [1.2, 1.3], the skill level dropped as new tasks were presented.

The shaded boxes indicate where Student A was shown the teaching screen. The results in Table 1 indicate that, when it was provided, the teaching screen was effective. Student C did not know what a number line was, so this was explained by the researcher and whole-number endpoints were used first. Student C had the most to learn, was given the most teaching and made good progress.

Table 1.
Results from Games of Flying Photographer

	Student A		Student B		Student C		Student D	
Game	Endpoints	skill level %	Endpoints	skill level %	Endpoints	skill level %	Endpoints	skill level %
1	0,1	55	0,1	70	**Whole**	45	0,1	85
2	0,1	83	0,1	87	**Whole**	75	0,1	85
3	**0,1**	--	0,1	88	**0,1**	32	0,1	94
4	**0,1**	100	0,1	100	**0,1**	88	1.2,1.3	54
5	**1.2,1.3**	--	1.2,1.3	37	0,1	93	1.2,1.3	54
6	1.2,1.3	89	1.2,1.3	17				
7	0,1	88	1.2,1.3	54				
8	0,1	100	1.2,1.3	-				
9	1.2-1.3	89	1.2,1.3	89				
10	Mixed	60*	1.2,1.3	100				

Note. * Average of three games

A DCT taken two months later showed that students B and D retained the same misconception. Student A however showed some progress, still thinking of the decimal part as like a whole number, but adding the understanding that 0 in the tenths column meant that the number was quite small and "close to zero". Student C made sporadic errors, indicating movement away from whole number thinking, but not yet solid understanding.

Strategies

As they played Flying Photographer, children developed four strategies, which do not show explicit understanding of place value, but rather show associated knowledge.

Benchmarks. Sometimes students successfully used known positions on the number-line to assist in placement of the number. For example, 0.25 was "a quarter of the way along" for student B. However, it was not helpful when he wanted to place 1.25 between endpoints of 1.2 and 1.3. He was expecting 1.25 to be quarter of the way along from 1.2 and was surprised to see the green arrow at the midpoint of the number-line and commented, "In the middle. I don't know. This is wrong, man."

Sections theory. When the teaching screen was used, student A saw the tenths divisions as naming sections of the number line. In a game with endpoints of 0 and 1, when presented with 0.95, he commented "that's in the 9 section". Similarly he observed that 0.66 is "in the 6 section and 0.098 would be "in the zero section". Student C also developed this strategy. He had placed 0.25 between 0.8 and 0.9, but upon seeing the green triangle commented "you go to the 2 and then five more little ones".

Zeroes in the tenths column. Student B observed that "zeroes in the tenths column make the number smaller". Student D also noted "I noticed there is a zero in it" after correctly placing 0.09. This is a significant step for students who regard decimal numbers as whole numbers would ordinarily regard 0.9 and 0.09 as the same number.

Ignoring initial digits that remain constant. When they moved away from the interval [0,1] three of the four students made comments that indicated that knew they could ignore the start of the number because it remained constant. Student A said, " if you pretend 1.2 is a zero then it is like the other game". Student B observed " 1 and 2 is at the start ... you don't count that". Student C made the comment, "you have to look at the first number and match zero with the number line". These students are beginning to reason in a way similar to putting words in alphabetical order. For example, once we have located *cats* after *catalogue* in a dictionary, we no longer look at the prefix *cat* to place *catnip*; we look only to the fourth letter of the word. Interestingly this strategy was not useful to student A in a game with mixed endpoints as can be noted from Table 1.

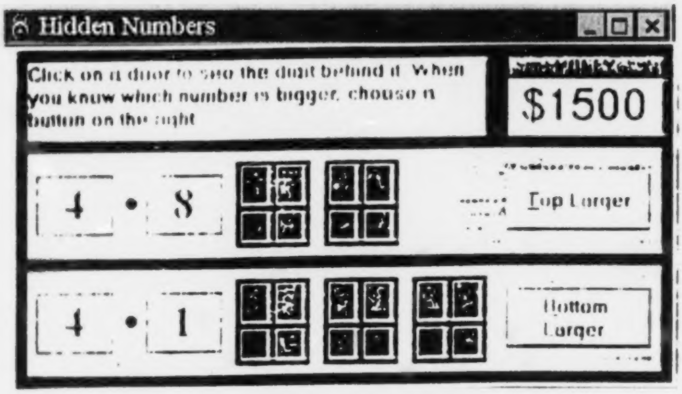
Was the Teaching Effective?

The programmed feedback of red and green triangles was extremely effective and used by all students. It aroused cognitive conflict in all students. One example of this occurred when student D was attempting to place 0.008 on the interval [0,1]. He expressed surprise at seeing the number near the beginning of the number-line. In a later game this same student had changed understanding and said, "I noticed that there's a zero in it" when placing 0.09 correctly near the beginning of the [0,1] interval. The researcher observed that less teaching was offered to the students than had been expected and that this was probably due to the success of the programmed feedback.

Only student A asked for teaching assistance and after game 5, he requested that the teaching screen not be shown, so he could work out the answers for himself. One other student was given the teaching screens. The teaching screen was helpful to those students and was probably the impetus for the development of the *sections* theory.

Study 2: Hidden Numbers

This study has the same aims as the first study, but has investigated a second game, Hidden Numbers. This is designed to combat misconceptions which judge the size of the decimal part of a number solely by its length, both longer-is-larger or smaller-is-larger. It also encourages the use of an expert *left-to-right* method of comparison of the digits in the columns in a decimal number. The common strategy of adding zeroes to the shorter decimal to equalise length is not a useful one for this game. This game requires the player to select which of two decimal numbers is the larger. Each digit of the decimal is behind a door, and students need to pay to open the doors, one by one. They can guess which number is the larger at any time. Hidden Numbers gives right/wrong *feedback* immediately a choice is made. The bank balance on the screen (see Figure 2) measures success.



Click on a door to see the digit behind it. When you know which number is bigger, choose a button on the right

Figure 2. The Hidden Numbers screen.

Note. The player has uncovered the whole numbers and tenths. This player has a bank balance of $1500. A whole number thinker would incorrectly choose the bottom number as larger even after observing the greater number of tenths in the top number.

Method

The DCT was administered to each child in a year 5 class from a Government primary school in the eastern suburbs of Melbourne. Fourteen of the 26 students in the class had the whole number misconception as discussed above, and 5 of these were selected by the classroom teacher to participate. The researcher was a student teacher, well known to the students. The game was programmed to present pairs of random decimal numbers, challenging to these children. An individual task-based interview was conducted as before. The results were logged in the output file and observations were recorded in real time.

Two forms of teaching were used. These had not been programmed but were provided on paper or simulated by the researcher on request or when a student had made a series of errors.

LAB demonstration. Linear arithmetic blocks (LAB - see Archer and Condon, 1999) were used to give the student a visual, linear model of the uncovered digits to date. So, for example if the student had uncovered a 2 in the tenths column of one number and 8 in the hundredths of another, the relative lengths of these represented in LAB were shown.

Unsolicited teaching - the pop-up screen. This was a simulated pop-up screen that read "Save money, open less doors" which was shown by the researcher when the player was regularly opening more doors than was necessary.

Results

Because the children had not seen LAB before, a short teaching session was conducted by the researcher before the game was played. Unfortunately for the research, this session may have been highly effective, because all students displayed a high degree of proficiency with the game, answering between 85% and 100% of the questions correctly. It is unclear if these results are attributable to the game or the pre-game LAB session.

None of the students requested teaching. Two of them received the pop-up screen and the LAB demonstrations. When they were asked after the game, all students reported that the LAB pre-game session was helpful and thought that a screen version of LAB teaching would be helpful.

Strategies

The students seemed to learn two strategies whilst playing this game. As with the students in the study above, students playing Hidden Numbers developed a strategy that is linked to understanding the effect of a zero in the tenths column. Students used this strategy to take what the researcher called an "expert punt". If one number had a zero in the tenths column, then there was a good chance that the second number was bigger. Additionally, all students showed some refining of the left-to-right strategy.

Discussion

Both studies found that these games were useful, although students developed knowledge associated with decimal numbers rather than direct place value knowledge. Interestingly, both games helped students to develop knowledge about zeroes in the tenths column to make judgements about the size of a decimal. Sackur-Grisvard and Leonard (1985) suggest that this adding of an extra separate knowledge of decimals comes next in a developmental sequence after whole number thinking. The *sections* theory was an unexpected outcome, which is part of the complex web of relationships that understanding the decimal concept entails. The experiments showed that teaching assistance should be programmed to automatically appear - students did not request it

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Scaffolding: A Suitable Teaching Characteristic in one-to-one Teaching in Maths Recovery

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This paper examines Bruner's notion of scaffolding in one-to-one teaching of Maths Recovery. The interactions of two teachers working with students were videotaped and then transcribed. The sessions are described and discussed. The results of this study showed that scaffolding is an important characteristic deemed to be suitable in one-to-one teaching of Maths Recovery. However, more study of one-to-one teaching is needed to further understand the types of characteristics that would enhance the teaching and learning process in one-to-one teaching.

In recent years one-to-one teaching programs have come to the fore with a renewed focus to see that all students have a reasonable chance of success in school, and to assist students who are at risk of school failure. But what teaching characteristics in these programs make them effective? This paper focuses on scaffolding, a teaching characteristic identified in a larger study into the effectiveness of teaching characteristics in the Maths Recovery Program, an early intervention program for students who are 6 to 7 years of age and in their second year of schooling (Wright, 1994). This program was developed by Wright over a three year period (1992-1995) and draws extensively on the work of Steffe and colleagues and related work by Wright (1994).

Maths Recovery

The Maths Recovery teacher administers an interview-based assessment to students identified by their class teacher as low attaining. The results of this interview are then used by the Maths Recovery teacher to develop instructional activities which are just beyond the cutting edge of the student's current arithmetical knowledge (Wright, 1995). The teacher of Maths Recovery undertakes a year-long professional development programme to learn how to develop a bank of instructional activities; to understand the purpose of the activities; and to present the activities in individualised teaching sessions. The instructional activities are used by the teacher to develop an individualised teaching framework. The framework is evaluated using videotaped records of teaching sessions, written notes made throughout the teaching sessions by teachers and the teacher's on-going reflections and evaluations of their teaching. This is to ensure that the instructional activities are those most likely to advance the student's arithmetical thinking (Wright, 1994).

Wright (1994) asserts that videotaping is crucial to learning the instructional techniques used in Maths Recovery. The teacher is videotaped during individualised assessment and teaching sessions. These videotaped records are used to evaluate the instructional activities, the teaching, and the assessment of children's arithmetical knowledge and the progress over time. Wright, Stanger, Cowper and Stewart (1995) identified nine underlying principles of Maths Recovery Teaching. Following are five of these principles which focus on teacher behaviour and interactions between a teacher and student and seem particularly relevant to an analysis of one-to-one teaching.

1. Teachers use their professional judgement in selecting from a bank of instructional settings and tasks, and varying this selection on the basis of on-going observations.
2. The teacher understands children's arithmetical strategies and deliberately engenders the development of more sophisticated strategies.

3. Teaching involves intensive, on-going observation by the teacher and continual micro adjusting or fine-tuning of teaching on the basis of his or her observation.
4. Teaching supports and builds on the child's intuitive, verbally-based strategies and these are used as a basis for the development of written forms of arithmetic which accord with the child's verbally-based strategies.
5. The teacher provides the child with sufficient time to solve a given problem. Consequently the child is frequently engaged in episodes which involve sustained thinking, reflection on his or her thinking and reflecting on the results of his of her thinking.

Maths Recovery is organisationally similar to Reading Recovery (Wright, 1994). Both focus on the second year of schooling; feature teachers observing, analysing and recording students' behaviours that inform their instruction; the student and teacher are both active participants in the learning environment; and both programs are intensive and individualised. The success of each of these programs hinges upon the teacher's ability to make and execute powerful decisions throughout each lesson so that, "instruction leads development rather than waiting for it" (Gaffney & Anderson, 1991, pp. 3-4). In both these programs Bruner's notion of scaffolding can be identified as an appropriate teaching characteristic used to bring about successful teaching and learning in one-to-one teaching.

Scaffolding

Bruner (1985) used the term *scaffold* as a label for the gradual withdrawal of adult control and support as a function of children's increasing mastery of a given task. This is similar to Vygotsky's assertion that every child, with assistance, can do more that he or she can by him or herself but only within the limits set by the state of his or her development. Imitation and instruction are crucial in a child's development. Vygotsky asserts that they bring out the human qualities of the mind and lead the child to new developmental levels. He states that what a child can do in cooperation today he or she can do alone tomorrow. According to Vygotsky "the only good kind of instruction is that which marches ahead of development and leads it; it must be aimed not so much at the ripe as at the ripening functions" (p. 104). Vygotsky states that the lowest threshold at which instruction can begin needs to be determined as well, but the upper threshold must be determined also; instruction must be oriented toward the future, not the past (p. 104).

Significant to Bruner's notion and Vygotsky's assertion is Diaz, Neal and Amaya-Williams' (1990) explanation that a teacher who scaffolds, keeps a task at a proper level of difficulty, always encouraging independent learning and avoiding unnecessary frustration. This links with von Glasersfeld's (1990) constructivist theory and Vygotsky's (1934/1962) zone of proximal development and sociocultural theory discussed in the larger study. According to von Glasersfeld (1990) constructivist theory of knowing, "knowledge is not an iconic representation of an external environment, but a mapping of ways of acting and thinking that are viable to the acting subject in attaining experiential goals" (p. 37). Von Glasersfeld explains that knowledge is the result of, "an individual subject's constructive activity, not a commodity that somehow resides outside the knower and can be conveyed by linguistic communication" (p. 37). He asserts that, "language is not a means of transporting conceptual structures from teacher to student, but rather a means of interacting that allows the teacher to constrain and guide the cognitive construction of the student" (p. 37). Vygotsky claims that a child's mental functions need to be fostered and assessed through collaboration with a teacher. He proposed that through collaboration the teacher is able to determine the distance between actual development and the potential development of the child, that is, a child's zone of proximal development (Hedegaard, 1990, pp. 349-350). A child's zone is not static, but instructionally sensitive. It must be recalibrated constantly to accommodate a child's new learning. The teacher, therefore, is supporting the child at the cutting edge of his or her competencies. For the child to achieve at the cutting edge the teacher provides minimal support and is constantly adjusting the amount of scaffold, to take account of the new learning's of the child. The abilities of the child are developed and strengthened through quality social interaction between the child and the teacher. Not only does this advance a child's development of higher mental function, as well, the organisational features of the social context are internalised and reflected in the child's performance (Gaffney & Anderson, 1991, p. 4).

Clay and Cazden (1990) assert that both the teacher and child share knowledge and responsibility for the task. However, the scaffolds are adjustable and temporarily used to help extend the range of work and accomplish tasks not otherwise possible. Similar to Clay and Cazden, Gaffney and Anderson (1991) assert that teachers must find tasks that engage the learner in performing at the upper level of his or her potential. This, Bruner (1985) asserts, can only be done:

> If a child is enabled to advance by being under the tutelage of an adult or a more competent peer, then the tutor or the aiding peer serves the learner as a vicarious form of consciousness until such a time as the learner is able to master his own action through his own consciousness and control. When a child achieves that conscious control over a new function, it is then that he is able to use it as a tool (p. 24)

According to Diaz, Neal and Amaya-Williams (1990), the work of Wood (1975-1976) has shown that "successful scaffolders focus children's attention on the task and keep them motivated and working throughout the session" (p. 140). Wood divides the learning task into accessible components and directs the child's attention to the essential and relevant features. Diaz et al. (1990) assert that "the teacher who scaffolds, demonstrates and models successful performance while keeping the task at a proper level of difficulty, avoiding unnecessary frustration and encouraging children's independent learning" (p. 40). This supports Clay and Cazden (1990) who state that within the zone of proximal development the child is not a passive recipient of the adult's teaching, nor is the adult simply a model of expert, successful behaviour. Rather the adult and child engage in joint problem-solving activities, where both share knowledge and responsibility for the task (p. 218). The Maths Recovery teacher performs the crucial function of scaffolding the task to make it possible for the child to reflect on the strategies and thinking involved and gain confidence in their own solutions, reducing the need to continually refer to the teacher for approval. Determining the optimal amount of scaffolding requires a high degree of craftsmanship and regulation on the part of the teacher. Scaffolds are adjustable and temporarily used to help extend the range of work and accomplish tasks not otherwise possible. The Maths Recovery teacher is constantly adjusting the amount of scaffold, to take account of new learning's of the student and anticipating and supporting the student's next step.

In Wright, Stanger, Cowper and Stewart's underlying principles of Maths Recovery teaching (1995), the teacher is involved in ongoing observation and continual micro adjusting of teaching based on his or her observations. Micro adjusting is informed by the student's performance on a task. When a student is not succeeding, the teacher adjusts the task to one closely related to the original task. A student's program must be adjusted from lesson to lesson and from moment to moment within lessons. That support keeps a student at the cutting edge of his her competencies, in his or her continually changing zone of proximal development.

The literature indicates that scaffolding is a useful characteristic to support children as they learn mathematics, and in particular those children who are seen to be at risk of failing. In the context of a one-to-one intervention program, namely Maths Recovery, this project seeks to investigate the ways in which teachers effectively scaffold children's learning.

Method

Scaffolding was identified from a larger study of one-to-one teaching, as an appropriate characteristic to analyse in Maths Recovery because it seemed highly consistent with Wright, Stanger, Cowper and Stewart's (1995) underlying principles of Maths Recovery teaching. Videotaped transcripts of teachers teaching in the Maths Recovery Program were used in the research. These were made available from Bob Wright, founder of the Maths Recovery Program. Initially, four teachers of Maths Recovery were observed for their teaching characteristics. From these observations two teachers were selected because they embodied a good number of positive (ie. appropriate) characteristics appropriate for Maths Recovery teaching and were particularly suitable in terms of the characteristic scaffolding.

Analysis

To illustrate the analysis being adopted in the present study, videotaped excerpts of interviews and teaching sessions were used. A video camera was set up to record the teaching environment, that is, the teacher, student, and tasks used during the teaching sessions. According to Wright (1994) the videotapes inform "research into children's learning; research into teaching; assessment of children's arithmetical knowledge; documentations of children's progress over time; and evaluation of instructional activities". For this study videotaped recordings of teachers teaching Maths Recovery to students were analysed. Transcripts were selected based on how well they informed the research literature on scaffolding. These transcripts were then written in the form of protocols. Two protocols using numeral identification, counting and subtraction are used to provide findings for this paper, however the project was considerably larger.

A necessary feature of scaffolding is that it can be applicable across settings and tasks. During a Maths Recovery teaching session on numeral identification and counting tasks, scaffolding from the teacher was identified. The student placed the numeral cards from "12" to "16" in order on the table. The teacher asked the student to identify the numerals on the table.

1. T: Right, count these ones for me.
2. C: (Uses finger to point at numeral cards) twelve, eleven.
3. T: Twelve. What comes after twelve?
4. C: (Pauses for four seconds) eleven.
5. T: Twelve. You count up to twenty for me.
6. C: One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, sixteen, sixteen.
7. T: Start again and I will point to the numbers when you get to them.
8. C: One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve (stops after "12") twelve.
9. T: (Pauses for seven seconds points to numeral cards "12" and "13") ten, eleven, (indicates with an upward inflection).
10. C: Thirteen, fourteen, fifteen, sixteen.
11. T: Let's try that again. We'll start with number ten, eleven (points to numeral card "12").
12. C: Twelve (pauses for six seconds) twelve, eleven.
13. T: Ten.
14. C: (Immediately responds) ten.
15. T: Eleven, twelve.

16. C: (Counts with teacher) eleven, twelve, thirteen, fourteen, fifteen, sixteen (as teacher points to numeral cards).
17. T: Right (with an upward inflection in the voice to indicate the child's success).

The Maths Recovery teacher performed the crucial function of scaffolding in this task to make it possible for the student to reflect on his thinking. The teacher asked the student to count the numeral cards that were placed on the table by the student. In Line 3 the teacher prompted the student by asking him what comes after twelve. At this time the teacher provided sufficient time for the student to reflect and think about the task. Gradually, the scaffolding was increased as the teacher realised that the student was not able to identify and count on from a given numeral. This is evident in Line 5. From Line 7 to 10 scaffolding had been adjusted to instructionally support the student so that he could accomplish the task. This was evident when the teacher invited the student to count with him. The teacher broke the task into simple components, paused and allowed time for the student to think about the task and supported the student's next step. The teacher was successful with keeping the student focused and working through the task to the point where the student successfully counted with the teacher. This was evident in Lines 15 and 16. This embodies Diaz, Neal and Amaya-Williams' (1990) explanation that a teacher who scaffolds, keeps the task at the proper level of difficulty, encouraging independent learning and avoiding unnecessary frustration.

In the next protocol the student was asked to solve a subtraction problem "55-11" using bundling sticks. When the student was asked the question, "How did you know that?" She was able to describe confidently the strategy she used. Had the teacher not supported her and been willing to be involved in meaningful dialogue she may not have explained her strategy so clearly.

1. T: Fifty-five take away eleven (places five bundles of "10" sticks and five sticks under cover, removes one bundle of "10" and one single stick and covers these). I'll leave it uncovered (removes cover from eleven). I took away eleven. Fifty-five take away eleven.
2. C: (Reflects for ten seconds appears to be thinking and points to covered card) was that fifty-five?
3. T: Fifty five was under there (points to covered card) and then I took eleven away.
4. C: (Immediately) forty-four.
5. T: Very good girl. How did you know that?
6. C: Because you took one of each away. (Removes cover from "55") there was five in here (places single stick with group of four, points and counts) one, two, three, four, (checks again) one, two, three, four, five. And then there was five in here (places single bundle with group of four bundles) ten, twenty, thirty, forty, fifty. Then you took away one which was four (removes one stick) then you took away ten (removes one bundle) which would make four.

Here the teacher and student, both shared their knowledge and responsibility for the task. In Line 2 the student reflected for ten seconds, and sought clarification for part of the problem. The teacher repeated the problem, keeping the task at the same level of difficulty. She did not micro-adjust but allowed the learner to master her actions through her own consciousness and control. This is a good example of what Bruner (1985) asserts, "an adult serves the learner as a form of consciousness until the leaner has master consciousness and control" (p. 24). This was evident in Line 4 and Line 6. The student was asked how she solved the problem and responded in a manner that indicated that she was motivated and working throughout the task. When the teacher and student interacted meaningfully like this, opportunities were there for the teacher to understand and negotiate meaning with the student.

Discussion

Both teachers in this study were similar with scaffolding when the students seemed to be experiencing difficulty or seeking clarification of the task. This served to minimise frustration on the part of the students and kept the them focused and working on the task that was nevertheless at a reasonable level of difficulty. Both teachers micro-adjusted their teaching depending on the student's success on the task. Both teachers provided instructional support based on continual observations of students. This kept the students at the cutting edge of his or her competencies. The notion of scaffolding affirms Vygotsky's (1934/1962) theory of a zone of proximal development and sociocultural theory "every child can do more with assistance, than they can by themselves but only within the limits of their development" (p. 103). Significant in the teaching of both teachers was their confident approach to one-to-one teaching, which, in turn, seemed to engender motivation and confidence in their students. Evident in the teaching of both teachers was the underlying principles of Maths Recovery discussed earlier and particularly the forth principle, the teachers built on each of the student's intuitive, verbally-based strategies which will serve as a base for the development of written forms of arithmetic.

Conclusion

The conclusion of this study indicates that the research literature about scaffolding is appropriate for developing an understanding of one-to-one teaching in Maths Recovery. However, this research highlighted the need for more study of one-to-one teaching to establish what teaching characteristics bring about successful teaching and learning in a one-to-one setting. There was minimal research literature available which specifically focussed on one-to-one teaching. From this data, it would appear there is a need to research teaching characteristics such as scaffolding, further, to establish just how successful teachers of one-to-one teaching are and to develop a bank of characteristics deemed suitable to use in teacher education and training.

Recently there have been many debates about how teachers teach, and how effective they are at using their skills and abilities to meet the demands of the curriculum and students. In the past the assumption was that teachers were considered effective if their students sat quietly and listened to the teacher, completed all the necessary work, responded correctly to teachers' questions and passed tests. It was considered important that the teacher is in control and the students were required to adapt to the teacher's environment (Lyons, Pinnell & DeFord, 1993), While some aspects of this are important they are a small part of teaching. The notion of scaffolding appears to have an impact on the teacher and student. A teacher who consistently exhibits this characteristic is more likely to enhance learning and be a more effective teacher. A teacher's observational skills and reflective practice enables them to make important decisions at crucial moments in a student's learning. They create a learning environment that supports good teacher and student interaction, where students are not passive recipients but active, motivated participants who are supported throughout their learning.

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Low Achieving Mathematics Students' Attitudinal and Achievement Changes as a Result of Using an Integrated Learning System

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This paper reports on a study to measure the effectiveness of an integrated learning system in improving mathematics achievement for low achieving Years 5 to 9 students. The study found that statistically significant gains on the ILS were not supported by scores on standardised mathematics-achievement tests. It also found that although student attitudes to computers decreased (significantly for some items), the students still liked the ILS and felt that it had helped them to learn.

One of the by-products of the growth of information technology in education has been the development of computer-based integrated learning systems (ILSs). These systems have three essential components, namely, *substantial course content*, *aggregated learner record system*, and a *management system*. They track learners' task responses and progress, and provides performance feedback to the learner and teacher (Underwood, Cavendish, Dowling, Fogelman, & Lawson, 1996, p. 33). This paper reports on a study to investigate how effective an ILS was on improving mathematics achievement for low-achieving mathematics students. The study compared achievement gains in mathematics learning as measured by standardised mathematics achievement tests with gains as measured by the ILS's management system. It also analysed changes in students' attitudes to computers in the classroom, and measured students' beliefs about the ILS.

According to the manufacturer, the core mathematics course of the ILS used by the students was developed to foster the development of foundation skills and concepts and to promote the use of higher-order thinking skills. It was designed to work with individual students in a closed manner, that is, the curriculum content and the learning sequences were not designed to be changed or added to by either the tutor or the learner (Underwood et al., 1996). Thus, the ILS marginalised the teacher's role and removed students' initiative and autonomy in the system's learning process (Bottino & Furinghetti, 1996). However, it should be noted that the manufacturers endorsed the ILS only as a tool for teachers to use to consolidate already introduced material and to diagnose student difficulties. They argued that it is the teacher's role to introduce the material to be practised on the ILS, and to remediate the difficulties identified by the ILS.

The core mathematics course was based on USA syllabi but correlated reasonably well with Australian syllabus requirements. It was divided into a range of *topics* (e.g., numeration, addition, multiplication, fractions, space) which were then sub-divided into collections of tasks that were sequenced in terms of performance at different *levels* (given in Years, e.g., 4.34). The difference between levels was constructed so that high mastery at one level (approximately 85%) is the same as mastery (above 60%) at the next level. The individual tasks were developed and placed in levels as a result of large-scale trials in the US.

For their initial *placement* on the ILS, students were given a large number of tasks at different levels until the system found the level at which they have reasonable mastery (about 65-75%). Students *progressed* when they achieved high mastery at one level; the system automatically raised them to the next level. To maximise the chance that task performance correctly represented level, the tasks within a level were presented randomly. The distributors argued that any reduction of randomness would affect the accuracy of placement and, therefore, the potential for students to achieve mastery. Without mastery, students may not experience the continual success, and therefore the motivation for achievement, that lay at the theoretical heart of the ILS. The ILS management system provided reports summarising students' levels of performance overall and for each topic area and giving the number of hours the students had spent on the ILS.

The mathematics tasks were in the form of electronic worksheets that were generally attractive in their presentation and sometimes creative in the way they probed understanding. They attempted to encourage the construction of knowledge by providing 2-D representations of appropriate teaching materials in mathematics (e.g., Multi-base Arithmetic Blocks, place value charts, fraction and decimal diagrams). Built into the core mathematics course were online resources that enabled students to get special help during a session should the need arise. However, use of the Help and Tutorial icons automatically graded performance as incorrect. The Toolbox icon made available calculators, rulers, tape measures and protractors for student use and also provided complex tools (e.g., graphing and drawing) for advanced levels.

For some topics and levels, there appeared to be insufficient task variety to prevent repetition. Furthermore, some tasks had novel presentation formats that students found difficult to interpret (e.g., spring scales used to determine number size, not object mass). Other tasks required inflexible and/or novel solution formats that sometimes resulted in students' correct answers being marked incorrect (e.g., failing to type the units digit first in operations, the omission of zero in decimal numbers such as 0.63). There was a tendency for questions to be closed (i.e., "find the right number") and based on speed (although the teacher could vary the time limits on answers). Time delays (e.g., while an algorithm was completed with pen and paper) could lead to the ILS's defaulting to incorrect. For each level and topic area, there were worksheets that could be printed to provide students with extra practice and teachers with a guide to the types of activities that needed consolidating.

Previous evaluations of the core mathematics course of the ILS have been mixed. Becker (1992) reviewed 32 evaluations and concluded that no independent evaluation had found significant gains from the use of the ILS. However, Underwood et al. (1996) found substantial positive gains from use of the ILS for mathematics performance in computation (although student numbers were small). Baturo, Cooper, McRobbie and Kidman (1999) and Baturo, Cooper, Kidman and McRobbie (in press) found that many classroom teachers felt that the ILS was successful in supporting mathematics learning in their classrooms, while Baturo, Cooper and McRobbie (1999) showed that it was possible to progress in the ILS without gaining any mathematics knowledge.

Method

The study reported here was essentially a pretest-posttest design without a control group (Campbell & Stanley, 1963). Instruments measuring achievement in mathematics, attitudes to computers, and beliefs about the ILS were administered, and progress ratings collected from the ILS's management program.

The study was part of a larger multimethod (Brewer & Hunter, 1989) project that also involved interviews with students and teachers and observations of classrooms. It ran for the last two terms of the year. Approximately 1000 Years 1-10 students from 23 primary and secondary schools (predominantly low socioeconomic status) throughout Queensland were involved in the larger study.

Sample

The subjects of the pretest-posttest design study were the 805 Years 5-9 cohort from the larger study. However, for a variety of reasons (e.g., time pressures, student nonattendance), the numbers of students varied for each instrument. For example, approximately 500 students completed the pretest, 135 students completed the pre-and post computer attitude survey, 350 completed the posttest, while 374 students responded to the questionnaire concerning students' perceptions of the ILS. Approximately 400 ILS reports were collected.

Data Sources

There were four data sources – *mathematics standardised achievement tests, computer attitude survey, student evaluation questionnaire,* and *ILS progress reports.* The three tests, Year 5/6, Year 7/8, and Year 9/10, had been developed by Education Queensland to monitor mathematics achievement in all Queensland state schools. Each test comprised two booklets with the first booklet containing items that were considered to be less difficult than those in the second booklet. These tests were used for both the pretest and the posttest. The 12-item survey measured students' attitudes to using computers at school and had been developed and used by Education Queensland in an earlier study of the ILS. Items were responded to on a six-point Likert Scale ranging from "very strongly disagree" (assigned value = 1) to "strongly agree" (assigned value = 6). The open-ended questionnaire was developed by the research team and sought students' perceptions of their experience with the ILS. Items of interest in this analysis included, "Did you like working on the ILS?" and "Did the ILS help you to learn?". Students were requested to answer the items "yes/no" and to give reasons for their response. The ILS progress reports were constructed by the ILS management system and provided information on the students in the study with respect to the levels they reached, their gains since placement, and the aggregated time they had spent on the ILS.

Procedure

The *tests* were administered at the beginning and end of the study by the classroom teachers. Because of the nature of the students, their low achievement levels, and the focus of most schools on remediation, many teachers chose to give a pretest that was below their class year level. Therefore, most students (primary and secondary) completed the Year 5/6 test (some Booklet 1, some Booklet 2, some both booklets). The *survey* was attached to the mathematics achievement tests and was therefore to be completed at the same time as the test (i.e., twice – at the beginning and the end of the study). The *questionnaire* was administered by the class teacher at the end of the trial. The ILS *reports* were collected by the class teacher at the end of the trial.

The classroom teachers organised the ILS sessions with their students to suit their schools' timetables and positions of computers. However, typically, students engaged with the ILS for about one-half to three-quarters of an hour each week over the 16 week period.

Analysis

The test, survey and ILS reports were analysed using analysis of variance and Pearson correlation. The questionnaire was analysed using Log Linear Analysis. Due to the number and types of responses, the results were analysed by instrument, rather than by school year level. As well, the mathematics results were analysed separately for each test booklet (Book 1 and Book 2) at each test level. Also, only students with test responses were included in the students whose progress reports were analysed.

Results

Mathematics Achievement

Table 1 summarises the statistical analysis of the pre- and posttest scores for the tests and the ILS. Analysis of variance of the Year 5/6 and Year 7/8 results showed statistically significant differences for ILS progress levels for each group and between the pre- and posttest mean scores for Book 1 of the Year 7/8 test. Of interest is the decline in mean scores pre- to posttest for Book 2 of the Year 7/8 test.

Further analysis of subgroups of students was undertaken according to the number of hours (less than 6 hours, 6-10 hours, more than 10 hours) they were involved with the ILS. The Year 5/6 test results were not statistically significant except for the 6-10 hours subgroup where there was a statistically significant gain in scores on Book 2 ($p < 0.05$). The Year 7/8 test results were not statistically significant for Book 2, but were for Book 1 for the less than 6 hours and 6-10 subgroups (the same as they were for the whole group). The scores for the more than 10 hours subgroup were not statistically significant ($p = 0.09$). The ILS progress reports were statistically significant for all year levels and subgroups and this more fine grained analysis also suggested a relatively constant rate of gain independent of number of hours the students were involved with the ILS.

Table 1

Pre- and Posttest Mean Scores and Analysis of Variance of Differences in Mean Scores for Mathematics Achievement and ILS Progress for Years 5/6 and 7/8

Test completed	Number of students	Pre test Mean (SD)	Posttest Mean (SD)	Difference Means	*t* statistic
All students Year 5/6					
Test Book 1	80	10.99 (3.49)	11.46 (3.03)	0.47	1.38
Test Book 2	69	09.75 (3.01)	10.32 (2.87)	0.57	1.58
ILS progress (year levels)	100	03.50 (0.74)	03.86 (0.76)	0.36	14.97**
ILS rate of gain = 0.047 yrs/h					
All students Year 7/8					
Test Book 1	46	14.26 (5.00)	17.11 (5.14)	2.85	5.28**
Test Book 2	35	12.23 (5.12)	11.17 (6.38)	-1.06	-1.03
ILS progress (year levels)	47	04.38 (1.11)	04.82 (1.19)	0.44	9.37**
ILS rate of gain = 0.062 yrs/h					

Note. *$p \leq 0.05$; **$p \leq 0.01$.

With respect to Years 5/6, the Pearson correlation between gain scores for ILS progress and Book 1 was -0.11 and between ILS progress and Book 2 was -0.20, which were not statistically significant. For the Year 7/8 data, the corresponding correlations were -0.03, and -0.08, which were also not statistically significant. This suggests that the tests and ILS may well have measured relatively unrelated aspects of mathematics. Yet analysis of the task objectives of the core mathematics course and the mathematics achievement test tasks showed there was considerable overlap. For example, both the Year 5/6 test and the computation strand of the ILS contained addition, multiplication, subtraction and decimals examples within the range of ability of the particular year levels. A similar degree of overlap was found between the ILS and the Year 7/8 test.

Computer Attitude Survey

Students completed a survey about their attitude to using computers at school pre- and post- their experience with the ILS. Table 2 reports the items of the survey, the mean pretest and posttest scores (standard deviations), difference in mean scores and analysis of variance of difference in mean scores (t statistic).

For all items in which the difference in mean scores (Pretest to Posttest) was statistically significant, the posttest mean score is lower than the pretest mean score. The results show statistically significant decreases in mean scores for Item 2 (*Using computers at school is interesting and enjoyable*), Item 5 (*Using computers at school helps me to think and understand better*), Item 8 (*Using computers at school is time well spent*), and Item 12 (*Using computers at school is important for getting a job*). As well, all items showed a decrease in mean scores from pretest to posttest reflecting a general decrease in attitude towards the use of computers in school. The number of statistically significant t-tests suggests the results are more than would be expected by chance alone. Nevertheless, because of the possibility of Type 1 errors, caution needs to be exercised in interpretation, in particular those mean differences statistically significant at $p < 0.05$. Item 2 is particularly interesting as it shows a large decrease in finding computers at school as "interesting and enjoyable," pre to post the ILS experience. This is also a relatively large effect of about 0.7 SD which is an educationally significant effect size.

Table 2

Mean Pretest and Posttest Scores, Items and Analysis of Variance of Difference in Mean Scores for Computer Attitude Survey (N= 135)

Item	Pre test Mean (SD)	Posttest Mean (SD)	Difference Means	t-statistic
Using computers at school:				
helps me do my classwork better.	4.38 (1.52)	4.33 (1.56)	-0.05	1.98
is interesting and enjoyable	5.23 (1.26)	4.19 (1.65)	-1.04	-16.13**
makes me feel more confident about doing my schoolwork	4.32 (1.60)	4.06 (1.56)	-0.26	-1.85
helps me work at my own speed	4.60 (1.58)	4.20 (1.66)	-0.40	-3.58
helps me to think and understand better	4.57 (1.55)	4.12 (1.51)	-0.45	-4.03*
encourages me to learn	4.57 (1.59)	4.07 (1.63)	-0.50	-6.20*
helps me to do my homework better	4.05 (1.85)	3.58 (1.74)	-0.47	-1.11
is time well spent	4.82 (1.50)	4.18 (1.68)	-0.64	-6.61*
makes me feel happier about doing my schoolwork	4.45 (1.60)	4.04 (1.73)	-0.41	-0.31
helps me to make friends.	2.89 (1.78)	2.65 (1.78)	-0.24	-3.24
helps me learn difficult work.	4.57 (1.59)	4.18 (1.73)	-0.39	-2.35
is important for getting a job.	5.26 (1.28)	4.74 (1.61)	-0.52	-4.66*

Note. $^{*}p < 0.05$; $^{**}p < 0.01$.

Attitude to the ILS

Items 1 and 2 of the Student Evaluation Questionnaire respectively asked students to say whether they liked working with the ILS and whether they thought the ILS helped them learn. Each item required an explanation to support their response. Students' responses were categorised as follows in Table 3.

Table 3

Responses to Student Evaluation Questionnaire, Items 1-2 in Terms of Attitude and Learning Belief with Respect to the ILS and School Type (Primary School and Secondary School)

Attitude	Like		Dislike	
Belief	Primary(n=172)	Secondary (n=98)	Primary (n=53)	Secondary (n=51)
Learn	163	81	27	21
Not learn	9	17	26	30

The Log Linear Analysis found two statistically significant interactions between attitude and beliefs about learning (G = 89.64, df = 1, $p \leq 0.001$)and between school type, attitude and belief about learning (G = 6.24, df = 3, $p \leq 0.05$). Irrespective of school type, a significant majority of students liked the ILS and believed that it helped them learn. Furthermore, almost half of the students who disliked the ILS believed that it would help them learn.

Discussion and Conclusions

The crucial question under investigation in this study was: *Does the ILS improve students' mathematics learning outcomes*? To assess the ILS's impact on student learning, data were gathered via the ILS progress reports, pre and post achievement tests, pre and post attitude surveys, and diagnostic interviews. However, rather than clarifying the question under investigation, the data gathered on mathematics achievement, attitude and beliefs, and ILS progress revealed ambivalent findings and raised further questions.

Mathematics Achievement

The ILS progress reports showed that nearly every student made significant gains for all populations in mathematics. However, these statistically significant gains were not matched by gains made in the achievement tests (as for Becker, 1992), although there was one statistically significant result for Book 1 of the Years 7/8 test (as for Underwood et al., 1996). This ambivalence in findings from the achievement tests could be due to the small number of students who completed test, survey and ILS reports before and after the study. It does not seem to be due to differences between items in the tests and items in the ILS worksheets.

However, the following questions do arise with respect to why the ILS in each instance reported a significant gain:

- Does the ILS initially place students at a level below their true ability level and thus "inflating" gains?
- What actually is the ILS measuring when it determines correctness or progress?

The statistically-significant ILS gains were measured in terms of school years. In this investigation, most students gained less than half a year across the two terms of the trial (half a year). Therefore, most students did not progress enough to keep up with time. However, most students in the trial were predominantly low achieving, and many had ILS levels that were half their grade level. Hence, any reasonable gain for these students could be considered educationally significant.

Although the independent achievement tests did not support the ILS progress reports, teachers generally endorsed the ILS as helpful to their students (Baturo et al., 1999; Baturo et al., submitted). However, they were unable to say whether the ILS improvements were transferred to classroom performance. Thus, the following questions emerge with respect to teachers' perceptions of the ILS and with its actual effects:

- Is there any transfer of the ILS success to mathematics performance in the classroom?
- If there is no such transfer, then what is it about the ILS that impresses teachers?

Attitudes and Beliefs

1. One of the major reasons that teachers gave for endorsing the ILS was its perceived effect on affective traits. Teachers believed that it made students more confident, a behaviour which they argued was evidenced through increased participation in classroom discussion, promoted more engagement in the learning process, and improved general classroom behaviour. This belief of the teachers was supported by the questionnaire where 244 students out of 374 gave responses to the questionnaire indicating they liked the ILS and believed it was helping them learn. However, it was not supported by the survey that measured attitudes to computers before and after the trial. This showed few significant gains in affect across the trial but some significant losses, particularly in attitude towards computers. This ambivalence in the findings raises the following question:

- What in the ILS experience makes students like the ILS and believe they are learning?

Teachers tended to endorse the ILS if they felt that their students had "got better" on it. However, "getting better" had a lot of meanings. Differences were discovered at this point. For some teachers, getting better was equated with improved behaviours such as attending to tasks, participating in class discussions, and generally being less disruptive. As one teacher said, "He did more work on the computer in twenty minutes than he has done all day for me!" For other teachers, getting better was improved procedural performance, for example, being able to "do sums" correctly and quickly. For others, it was equated with improved cognitive performance although, as said earlier, all except one teacher were unable to provide specific examples of this. Thus, endorsement was based on a variety of perceived improvements in either the affective or cognitive domains of learning.

Progress Versus Achievement

Most students worked on the ILS for 45 minutes or less per week (8 hours or less over 16 weeks). This is a small time for the ILS to affect attitude and achievement, particularly when compared against the total weekly time spent on mathematics (usually 200-225 minutes in the classrooms studied). However, the ILS did report significant progress, which leads to the following question:

- Is there something in working with the ILS that enables progression to occur through experience with the program rather than through improved knowledge?

The ILS is reminiscent of the Individually Prescribed Instructional (IPI) packages that proliferated in the US in the 1970s with the ILS activities presented in electronic, rather than paper, form. Both systems have a management system which marks students' responses, directs unsuccessful students to other similar activities until "learning" takes place, and directs successful students to another higher level, and the process is repeated continuously. The only real difference between the two systems is that, in any ILS session, activities cover a variety of mathematical topics whereas in any IPI session, activities are presented in finely detailed sequences within the one topic. The pedagogical flaws in IPI systems were exposed by Erlwanger (1975). In a similar analysis of the ILS, Baturo, Cooper and McRobbie (1999) found that it was possible to progress on the ILS in fractions without improving fraction knowledge; a finding that echoed Erlwanger's earlier finding.

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Teaching for Abstraction: Reconstructing Constructivism

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Mathematics is abstract in the sense that mathematical concepts are formed by a process of abstraction—the recognition and reification of a similarity between a set of previously unrelated objects, events, or ideas. It follows that effective mathematics teaching should promote the abstraction process. Traditional methods seldom do so. Constructivist methods theoretically encourage abstraction, but this is rarely emphasised and the result may be an inefficient teaching procedure. Three examples are given of how explicit attention to the abstraction process could lead to radically different approaches to teaching. Some principles of what we call *Teaching for Abstraction* are abstracted from these examples.

Many students have difficulty relating abstract mathematical ideas to the everyday world. Students who do well in mathematics examinations often cannot give examples of the concepts they have learnt or suggest situations where the procedures they have carried out correctly might be useful. Students who do not do so well make little sense of mathematics and are frequently reduced to rote learning.

We believe that a major reason for this state of affairs is a misunderstanding of the nature of abstraction in mathematics. The aim of this paper is to clarify the nature of abstraction and to derive a framework for teaching mathematics intended to lead to meaningful and purposeful learning.

Mathematics is Abstract

Mathematics deals with concepts, procedures, theorems, definitions, and proofs. The *concepts* are abstract objects (e.g., points, lines) and abstract relations between such objects (e.g., parallel, intersect). *Procedures* and *theorems* are universal statements about these concepts: rules for obtaining one abstract object or relation from others (e.g., how to construct parallel lines) and general assertions about these concepts (e.g., properties of parallel lines), respectively. Finally, *definitions* are statements relating new concepts to concepts already met, and *proofs* are logical arguments linking rules or theorems deductively.

Mathematics is abstract because the constituent concepts are abstract. But what does this mean? What is an abstract concept?

Abstraction as a Process

A common theme in the work of Dienes, Piaget, and Skemp earlier this century is that concepts are formed when experiences are connected to one another on the basis of their similarities (Mitchelmore & White, 1995). Skemp (1986, p. 21) summarised the theory concisely as follows:

> *Abstracting* is an activity by which we become aware of similarities ... among our experiences. *Classifying* means collecting together our experiences on the basis of these similarities. An *abstraction* is some kind of lasting change, the result of abstracting, which enables us to recognise new experiences as having the similarities of an already formed class. ... To distinguish between abstracting as an activity and abstraction as its end-product, we shall ... call the latter a *concept*.

In the case of everyday concepts (such as "spoon"), the similarities between examples of the concept frequently relate to the objects' general appearance and purpose. Piaget referred to the *empirical abstraction* of everyday concepts.

Elementary mathematical concepts (such as "three") are formed through the same process of classification and abstraction, but with one important difference: The objects which are classified are not concrete objects nor even mental objects (everyday concepts), but objects or concepts which are related to one another in a consistent manner. For example, the abstract concept "three" arises from the realisation that counting particular sets of objects (one, two, three) always ends in the same number—no matter how the objects are rearranged. Piaget called the process involved in the formation of elementary logico-mathematical concepts *reflective abstraction* and emphasised its active nature: "The [mathematical] abstraction is drawn not from the object that is acted upon, but from the action itself" (Piaget, 1970, p. 16). In common with many other writers, Piaget also emphasised the fact that abstraction is constructive. Dubinsky (1991, p. 99) even claims that "Piaget seemed to feel that this constructive aspect of reflective abstraction is more important than the abstraction (or extraction) aspect". Sfard (1991) uses the term *reification* for the final stage of the abstraction process where the learner constructs a new mental object to embody a perceived similarity.

Generalisation also plays an important part in the abstraction process. After the learner has already abstracted a concept from a given set of situations, he or she may realise that the concept—possibly with some modification—can be extended to a wider set of situations. Generalisation thus refers to the process of rendering a known concept *more* general. The term is also used to mean an invariant relation between concepts—in mathematics, a procedure or a theorem.

Abstraction (together with the associated generalisation) is therefore fundamentally important both to mathematics and to the learning of mathematics (White & Mitchelmore, 1999). We would therefore expect it to be one of the main aims of mathematics teaching. Is it?

Do Current Methods of Teaching Mathematics Teach Abstraction?

We claim that none of the usual approaches to teaching mathematics explicitly teaches the abstraction process. However, some methods do promote abstraction.

The ABC Method

Traditional mathematics teaching the world over follows what we have called the ABC Method (Mitchelmore, 1999): **A**bstract concepts and procedures are taught **B**efore **C**oncrete examples and applications. The theory is that "knowledge acquired in 'context-free' circumstances is supposed to be available for general application in all contexts" (Lave, 1988, p. 9), but in practice this does not happen. Because the knowledge is context-free, it is *abstract-apart* (Mitchelmore & White, 1995) and cannot be easily related to familiar situations. Concepts can only be readily applied if they are *abstract-general*, that is, if they represent that which is general to a variety of situations. The ABC method proceeds in opposite direction to the abstraction process.

Constructivist Approaches to Mathematics Teaching

Constructivists, following Piaget, emphasise the need for children to construct their own understanding but rarely (unlike Piaget) talk about abstraction. Exceptions are to be found in the works of Dubinsky (e.g., 1991) and von Glasersfeld (e.g., 1995).

However, many elements of constructivist teaching could be expected to promote abstraction: The emphasis on discussion of existing knowledge or experience; the challenge of problem solving; the admission of contrasting methods and the reconciliation of conflicting solutions; the use of small group cooperative learning—all promote reflection which could lead to the recognition of communalities and hence either the abstraction of new concepts or the generalisation of existing ones.

For example, consider the Dutch "Realistic Mathematics Education" (RME) movement. (Treffers, 1991, p. 26) describes the five basic principles of RME:

1. Mathematics learning is constructive
2. Learning proceeds over several levels over a long time period
3. Reflection plays an important role in learning
4. Learning is interactive
5. Mathematical ideas are interconnected

The general constructivist characteristics are clearly visible.

Typically, the RME approach to teaching a topic consists of three stages:

1. Develop rules of operation in several specific, familiar, everyday contexts
2. Demonstrate that the same structure is present in several such contexts
3. Formulate, symbolise and study the common structure

Treffers (1991, p. 32) calls the first step *horizontal mathematising*: "The modeling of problem situations [so] that these can be approached with mathematical means." The second step consists of the recognition of structural similarities and the third step the construction of a new mental object. Treffers calls these steps *vertical mathematising*, which is "directed at the perceived building and expansion of knowledge within the subject system, the world of symbols." We can recognise the three steps as together constituting the abstraction process previously described.

Dienes and Multiple Embodiment

Dienes (1963) is the only example we know of an attempt to teach abstract structures by leading students to identify similarities between isomorphic structures. For example, place value concepts were abstracted from commonalities between his Multibase Arithmetic Blocks and tree diagrams. One of Dienes' basic principles was that of *perceptual variability*: "To abstract a mathematical structure effectively, one must meet it in a number of different situations to perceive its purely structural properties" (p. 158). Dienes aimed to create a teaching environment in which children learned by reflecting on their experiences in a variety of different situations.

Dienes was not happy with the outcomes of his experiments. "We assumed ... that abstraction would arise from a multiple embodiment of the concepts to be abstracted. By this I mean that situations physically equivalent to the concept-structure to be learned would, if handled according to specific instructions leading towards the structure, result in abstracting the common structure from all the physical situations. ... But as we observed children going through the 'abstraction exercises', it soon became clear that the picture was far more complex than we had assumed" (1963, p. 68). For example, requests to do "something like this" in a different embodiment or to say "how they are alike" initially brought out common features instead of a common structure—but only the first time through, as if children were just not used to looking for deeper similarities. Nevertheless, he claims that "artificial exercises in forming isomorphisms could act as teaching devices to help [children] recognise similarities when they see them" (p. 85).

Our view of Dienes' experiments is that the various "concrete materials" he used to embody mathematical structures were only concrete in the sense of being constructed out of physical materials. They were not familiar objects in children's experience and were in fact already abstractions from that experience—abstractions made by the researcher and not by the children. This view is borne out by findings that children often have difficulty relating Dienes' blocks to arithmetical procedures (Boulton-Lewis, 1992).

Teaching for Abstraction

We suggest that it would be valuable to design mathematics teaching in such a way as to explicitly promote the abstraction of crucial mathematics concepts. To be consistent with our approach to abstraction, we shall first give some examples and then draw out the similarities which we shall call *Teaching for Abstraction.*

Mathematics has a highly developed specialist vocabulary. For example:

- Point, line, plane, angle, parallel, symmetry, congruence, theorem, proof, ...
- Length, area, volume, time, measurement, …
- Whole numbers, fractions, integers, addition, multiplication, powers, ...
- Variable, function, equation, graph, domain, range, ...
- Linear, quadratic, exponential, polynomial, logarithm, trigonometric, ...

Some of these terms (such as "trigonometric") are only classifications, but most represent concepts—objects or relations. As such, they are candidates for teaching for abstraction. We have selected three, one each from the major curriculum areas of measurement, space, and number.

Example 1: Area

Like many crucial mathematical concepts, it is not possible to define area. Any definition (such as "amount of space") is quickly seen to be circular. (Just try to define "space" without using the word "area"!) But we all know an area when we see one, that is, we have learned to identify a certain similarity between a variety of what we now recognise as area situations. Teaching area for abstraction means helping children to learn this similarity and hence to abstract the concept of "area".

The area similarity is based on two factors: (1) It is a property of a region or a set of regions. (2) The regions are assumed to be uniform in some way, an assumption which is basic to the principle of conservation of area. So children need experience of several situations involving uniform regions if they are to abstract the concept of area. Area cannot be learned simply from cutting up paper, drawing on paper, or substituting in formulae because these experiences may not be connected to children's physical experience; paper and formulae are already abstract.

Some suitable area situations would include wall painting, book covering, floor tiling, seed planting, and land sales. All these involve operations on regions, and the uniformity assumption is either reasonable (e.g., all parts of a paddock are equally grassy) or obvious (e.g., the same tiles are used all over the floor). Children could investigate such situations in their own right before even mentioning the word "area".

Eventually, the teacher could challenge children to identify what is the same across several such situations. The similarity that is identified would then be called "area", and children could look for some other areas situations which share the same similarity. Reflection about properties which are common to all these situations would lead to some powerful, general ideas—for example, that area measurement units may not overlap or leave gaps. Abstract relations (e.g., the formula for the area of a rectangle) could then be developed which are recognised as applicable to all area situations—and limited in their applicability by the uniformity assumption. Children would then have acquired an abstract-general concept of area.

Example 2: Angle

Research (summarised by Clements & Battista, 1992) has shown that children readily form concepts of corner, slope, and turn but have difficulty integrating them into a single concept of angle. Our research (Mitchelmore, 1997, 1998; Mitchelmore & White, 1998, in press) shows that, whereas most Year 2 children see the corners in furniture, walls, and street intersections as similar in an angular sense, scissors are not readily recognised as having corners until Year 4 and leaning signposts not until Year 6. Also, about one-third of Year 8 students cannot demonstrate the angles which show how corners are similar to sloping and turning objects and a similar fraction see no similarity between a turning wheel and an opening door.

The concepts of corner, slope, opening, and turn are based on superficial similarities: Corners have two (more or less) visible angle arms, slope seems to be a property of a single static line, opening is a movement of a single line about its end-point, and turning is a global movement which need not involve any obvious lines or points. To form a concept of angle, children need to recognise a deeper similarity between all these situations—namely, that they can all be represented by angles. (The astute reader will notice that, like area, it is not possible to define angle without falling into circular definitions.)

Teaching angle for abstraction would fall into two stages. In the first stage, the teacher would help children to form separate abstract-general concepts of corners, slopes, turns, and openings by examining many examples of each concept and looking for superficial similarities. Carefully planned teaching could, we believe, lead to children forming such concepts in Year 2 or 3. Once each concept is secure, it could be developed to the point where children could compare and even measure corners, slopes, turns, and openings separately. The word "angle" need not be mentioned.

In the second stage, the teacher would lead children to link the concepts of corners, turns, and openings. Children would search for similarities both between the geometrical configurations in the three cases and between the ways that corners, slopes, and turns are measured. (For example, the right angle plays a significant but different role in each context. But four right-angled corners, turns, or openings all fill the space around a point.) The similarities would be expressed in the standard angle diagram, and the word "angle" used to name the emerging concept. Properties of the concept would then be studied (e.g., the standard method of measuring angles in degrees). Because the angle concept would have been abstracted from three superficially different contexts, it would be an abstract-general concept.

The next stage would be to generalise the angle concept, first to slope and then to other contexts such as direction and rebounds. These contexts are more complex than corners, turns, and openings and need to be studied in their own right first. For example, slope involves the physical concept of horizontal so its inclusion in the angle concept must wait until the corresponding physics is understood.

In a study funded by an ACU Large Grant, individual teaching experiments are currently under way to investigate how easy it is teach children in Years 2 and 4 to recognise angular similarities between corners, slopes, turns, and openings.

Example 3: Fractions

Hunting (1995) has shown conclusively that children often form a concept of "one-half" before primary school. He writes: "To understand 'one-half' in a deeper sense, a child must recognise something which several different mathematical structures have in common" (p. 122). For example, they must realise that, whenever they meet halves, two of them will always be equal and together make up the whole. They can only do this by experiencing many halving situations, reflecting on their similarities, and ignoring their differences (such as when one half of an apple has a blemish and the other is clear). It would appear that students abstract the concept of one-quarter in a similar fashion a little later.

The next step seems to be the formation of concepts of one-eighth (and possibly one-sixteenth) and three-quarters (and possibly three-eighths, etc.) by operating on one-half and one-quarter in various contexts and again noticing similarities among the relations between the various fractions in the different contexts. For example, three-quarters may be abstracted from experiences of three-quarters of an hour, three-quarters of a cup, centre three-quarter at football, and so on, linked to the knowledge that three-quarters in each case comes half way between half and full. How rapidly this abstraction is made probably depends on the activities the child regularly engages in; cooking and music seem to be particularly rich contexts for the use of fractions.

Generalisation, firstly to fractions of the form $\frac{1}{n}$ and then $\frac{m}{n}$, probably occurs as a result of reflection on the similarities underlying the binary fractions already abstracted. In informal research studies, my students have frequently reported cases of 10-year old children who seem to have a solid concept of one-quarter but only as a half of a half; at least, this aspect appears more important to them than the fact that there are four quarters in a whole. Similarly, many children of the same age seem to have a concept of three-quarters but not to realise that three-quarters is three times as much as one-quarter. It is a conscious awareness of such relations, we suggest, which enables children to conceive of dividing anything into any number of equal parts and then combining them to form general fractions. We do not believe that the general fraction concept is formed by abstracting the similarities between general fractions in different contexts: Fractions like $\frac{3}{7}$ of an hour, $\frac{3}{7}$ back at football, and $\frac{3}{7}$ time in music simply do not occur outside of textbooks.

Understanding of fractions therefore seems to develop rather differently from understanding of angle. Children do not form developed fraction concepts in different contexts and then abstract a more general concept later. Teaching fractions for abstraction would thus proceed rather differently than teaching angles for abstraction.

The first stage would be to assist children to abstract concepts of $\frac{1}{2}$ and $\frac{1}{4}$ from familiar examples of these fractions—perhaps as early as Kindergarten. The next stage would be to extend children's fraction concepts to include $\frac{3}{4}$ and $\frac{1}{8}$ (and possibly $\frac{1}{16}$, $\frac{3}{8}$, etc.) by finding these fractions in familiar contexts, exploring their relations to other fractions in those contexts, and then identifying the similarities between the contexts. Their fraction concepts would be abstract-general if they are seen to apply to all fractions contexts in a similar way.

Relations such as $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ would be seen to be expressions of familiar, general relations rather than the results of formal calculations.

The third stage of teaching fractions for abstraction would consist of generalising the structure inherent in the familiar fractions, still in familiar contexts but with an immediate recognition that a general structure is being constructed. For example, one would ask what "one fifth" or "two thirds" could mean and then look for examples of them in familiar contexts. The next stage would be to investigate how to combine unfamiliar fractions—but still in familiar contexts. This might include, for example, finding $\frac{2}{3}$ of $\frac{3}{4}$ of a cup of milk in order to reduce a recipe for 3 people to one for 2 people—first practically and then through reasoned pictorial arguments. The final stage would be to search for means of operating on fractions symbolically, and would lead to the standard "four rules" for fractions.

Principles of Teaching for Abstraction

The above examples illustrate the three essential principles of Teaching for Abstraction: familiarity, similarity, and reification.

Familiarity. Students become familiar with several examples of the concept (i.e., several contexts from which the concept will be abstracted) before teaching the concept itself. These may be objects (e.g., tiles, furniture, road maps), operations (e.g., cutting, painting, filling, ...), or abstract ideas (e.g., half, turn, rectangle, ...). The examples are discussed using the natural language peculiar to each context (e.g., corner, slope, turn, opening), not that of the concept to be abstracted (e.g., angle). However, the teacher will anticipate the abstraction to be made later (e.g., by including examples of slopes and turns in which the two arms of the abstract angle are clearly visible).

Similarity. The concept is taught by finding and making explicit the similarities underlying familiar examples of that concept. The similarities may be superficial (e.g., between the appearance of corners and scissors) or structural (e.g., between the way familiar fractions are related in different contexts). Whichever it is, students' attention is directed to the critical attributes which define these similarities and which are embodied in the concept to be abstracted (e.g., the uniformity assumption underlying the area concept). The teacher then introduces the specialist language associated with the concept and uses this vocabulary to "define" the concept (in the sense of making it more definite) by showing how it relates to the similarities on which it is based

Reification. As students explore the concept in more detail, it becomes increasingly a mental object in its own right, detached from any specific context. Almost any use of the concept is likely to assist its reification, providing the relation between the abstract concept and familiar examples of the concept is maintained. Some possibilities:

- Find how to use the concept in practice (e.g., by estimating the area of the school on a map).
- Investigate how to operate on the abstract concept, but always relate the results to some familiar context (e.g., calculating $\frac{2}{3}$ of $\frac{3}{4}$ and then checking it using a diagram).
- Define and work with special cases (e.g., percentages as special fractions).
- Look for generalisations involving the concept (e.g., area formulae).

Teaching for Abstraction clearly has much in common with other constructivist approaches, and many of their principles apply equally well. One difference is that Teaching for Abstraction has no problem with the fact that much of the content of school mathematics is pre-determined. Our belief is that, instead of merely hoping that abstract mathematical ideas will develop as a result of cooperative learning, reflection on experience, and so on, a more deliberate attempt to foster the abstraction of crucial mathematical concepts would pay handsome dividends in terms of student learning and understanding.

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Graphical Representations of Statistical Associations by Upper Primary Students

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Students' graphical representations of statistical associations are explored through analyses of responses from 97 students in grades 4, 5, and 6. The task involved representing (a) the association "People grow taller as they get older," (b) this association with an age modifier, and (c) this association with sex differences. Three levels of representing bivariate associations and of representing multivariate associations are described: unsuccessful, partial, and complete. Issues for developing student representations are discussed.

Graphs are commonly used to convey at a glance large amounts of statistical data, particularly data involving bivariate and multivariate associations, which are otherwise difficult to convey without reporting formal statistics such as t-tests or regression lines. Graphs produced by students may convey not only statistical data, but also how students understand statistical data, and how they understand bivariate- and multivariate-associations. Although many studies have explored students' interpretations of graphs (for a review, see Leinhardt, Zaslavsky, & Stein, 1990), it is only recently that students' representations have begun to receive attention by mathematics education researchers (e.g., Mevarech & Kramarsky, 1997; Moritz, 1999; Moritz & Watson, 2000; Watson, 2000).

Mevarech and Kramarsky (1997) asked 92 grade 8 students to represent the following four claims of association between time studying and achievement grades in school:

1. [increasing function] the more she studies, the better her grades;
2. [constant function] no matter how long she studies, she always gets the same grade;
3. [curvilinear function] up to three hours, the longer she studies the better her grades, but beyond three hours, she becomes tired and her grades become lower; and
4. [decreasing function] when she studies more, her grades decrease.

Approximately 55% of students appropriately represented claims 1, 2, and 4 using a labelled two-axis graph, whereas the curvilinear function of claim 3 was correctly represented by only 38% of students. About 10% of students represented generalised ideas of graphs or represented relations between the variables in an idiosyncratic way. Three common alternative conceptions were evident in responses that represented (a) only a single point in a graph, (b) only one factor in each of a series of graphs, and (c) an increasing function irrespective of task requirements. These conceptions were evident in approximately 20%, 20%, and 10% of students' responses respectively (students could exhibit more than one conception). The first two conceptions involve eliminating variable data to reduce complexity of the representation. The last conception indicates that constant, curvilinear, and decreasing functions may be more difficult for students, for example involving appreciating that as one measure changes, the other measure may remain constant, and thus representing a repeated value on that measure.

A number of studies (Moritz, 1999; Moritz & Watson, 2000; Watson, 2000) have reported on responses from students in grades 6 to 11 to represent in a graph "an almost perfect relationship between the increase in heart deaths and the increase in the use of motor vehicles", as claimed in a newspaper extract and presented in a written survey task. Moritz and Watson (2000) described a framework of four levels according to degree of success in representing the association. At each level, different categories of response were evident. At the level of *No Association*, some responses were pictures and others were basic graphs that displayed no trend and no labelling of variables. At the level of *Unsuccessful Association*, some responses were labelled graphs that did not show an association. Also at this level, many responses were single comparison graphs, often bar graphs comparing a single value of heart deaths with a single value of car use; the reduction of each variable to a single value appeared to be a means of reducing the complexity of the graph. At the level of *Partial Association*, some responses displayed a trend but did not specify the appropriate variables, and others represented a double comparison involving two values for each of heart deaths and car use. At the level of *Complete Association*, some responses showed bivariate association of heart deaths versus car use, and others were a series comparison of heart deaths and of car use displaying similar time-series trends.

The current study aimed (a) to extend to upper primary school students the findings of Mevarech and Kramarsky (1997) concerning students' representations of increasing and curvilinear functions, and (b) to extend to different tasks the findings of Moritz and Watson (2000) concerning levels of conceptual development leading to the ability to use a series comparison representation to compare two bivariate trends.

Method

A survey item, shown in Figure 1, included two parts (B1 and B2, together forming part B) involving bivariate association based on items by Mevarech and Kramarsky (1997) and a third part (M) involving multivariate association to extend the work of Moritz and Watson (2000). The item was administered during November 1998 to whole classes as a written survey along with three other items. Each of the four items was presented on a single A4 sheet of paper. One item asked students to tell a story about an unlabelled, unscaled line graph involving a non-monotonic negative trend, and two other items were on unrelated topics. The two graphing items were always the second and fourth items in students' surveys, with approximately half in each order.

One day at school, the class began to talk about how tall people are. The teacher measured everyone's height. Then they began to talk about their brothers and sisters. Some were taller, and some were shorter.

(B1) David said, "People grow taller as they get older".
Draw a graph to show what David is saying. Label the graph.

(B2) Mary agreed, "People do grow taller. But when you are 20 years old, you stop growing."
Draw a graph to show what Mary is saying. Label the graph.

(M) The teacher said, "For 10 year olds, girls and boys are about the same height.
But men usually grow to be taller than women."
Draw a graph to show what the teacher is saying. Label the graph.

Figure 1. Survey item to graph statistical associations ["*(B1)*", "*(B2)*", and "*(M)*" not on original survey].

Responses were gathered from 23 fourth-grade, 25 fifth-grade, and 26 sixth-grade students at a private school for males, and 18 fourth-grade, and 13 fifth-grade students at a private school for females. When requested by students, the researcher or classroom teacher offered assistance in reading the questions and encouraging students to do their best when they were uncertain about their responses. Of these 105 surveys, 93 students' complete responses were included in the analysis, and another 4 responses to part (B) were deemed complete and also included; 8 surveys were not analysed because of non-response to parts (B2) or (M), possibly due to time constraints and other class interventions.

Each student's responses to part (B) were assigned to one of the three levels, as were responses to part (M). The three levels were comparable for responses to parts (B) and to part (M). At the unsuccessful association level, responses failed to show an association in the data, often omitting one of the relevant variables. At the partial association level, responses indicated an association between some variables but not all critical features were displayed. Responses at the level of complete association represented all critical features. For part (B2), critical features included a curvilinear trend that showed increasing height with age and also constant height with changing age by age 20. For part (M), three critical features included (a) at age 10, females same height as males, (b) association of height with age (for females, males, or both), and (c) for some age greater than 10, males taller than females. The coding schemes were devised to assess at higher levels those responses indicating the student understood the numerical association of the variables and had some coherent system for representing this, and not to penalise the student for violating a graphing convention, such as interchanging axes or reversing the order of an axis scale. Despite the task requirement for labelling, it was acknowledged that some students might have assumed height and age were obvious variables since they were written on the page just above where their graph was to be drawn. Hence "indicative labelling" was credited: students were given the benefit of doubt if they provided some features of the graph or the data that indicated a distinction between height, age, and sex. Examples include a pictograph of people implying the vertical axis indicated height, or the number 20 indicating a significant age in response to part (B2).

An independent coder assigned responses to levels.[1] There was agreement for 80% of responses to part (B) and 70% to part (M). Many of the discrepancies were due to the coder misunderstanding the criteria. Following joint discussion, there were few remaining cases of disagreement: one for part (B) and four for part (M). For these cases, students had represented features of the associations in idiosyncratic unconventional ways that appeared to the author to represent relevant features, and the author's categorisation was preferred despite the coder remaining unconvinced after discussion. Compared to the author's initial coding, the final coded data had no differences for part (B), and one difference for part (M).

There were no significant sex differences, nor item order effects, except at grade 5 for part (B): of students for whom this was the 2nd or 4th item of the 4 in the survey, 69% ($n = 26$) and 25% ($n = 12$) respectively represented the curvilinear trend at the level of Complete Bivariate Association. This difference may be due to chance, to less time devoted by students when the last survey item, or to the other graphing item influencing responses.

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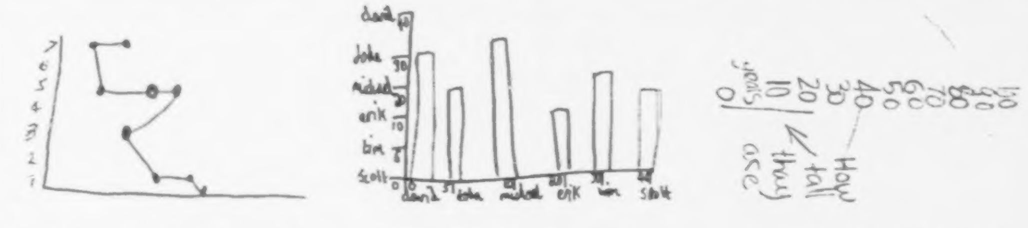
[1] It is acknowledged that within each level, various categories of response might be observed, such as by Moritz and Watson (2000), however analyses of responses into categories are beyond the scope of this paper. The coder was in fact asked to assign responses to levels and also to categories, with two categories at the levels of Partial Bivariate Association and Partial Multivariate Association, and three categories at the level of Complete Multivariate Association. The coding scheme of levels and categories was presented with wording of criteria and stylized illustrative graphs. Inter-rater reliability for coding of categories was 69% for part (B) and 58% for part (M). Many of the discrepancies were due to the coder misunderstanding the criteria, such as using the illustrative graphs intended to assist coding; for example believing that form of graph (bar or line) was one of the criteria.

Student Responses to Represent Bivariate Association

Level 1. Unsuccessful Bivariate Association

Responses at this level indicated that students had difficulty coordinating the representation of two variables to show a bivariate association. Some students super-imposed the two variables on a single axis. If Cartesian axes were used, the data indicated either that there was only one unique measure, or there was no association between the two measures. Three examples are shown in Figure 2. In the first example, a grade 4 male represented a generic graph with no indication of the variables or of an association. The bar graph by a grade 5 male (second example) displayed six male names presented in reversed order on each axis, but there was no indication of the variables or of an association. The third example, by a grade 4 female, was labelled to indicate numbers denoted age, and position (length of the line) denoted height; position, however, was also used to denote age with the numbers on a linear scale, thus age and height were superimposed on a single axis.



Part (B1), Grade 4 male *Part (B1), Grade 5 male* *Part (B2), Grade 4 female*

Figure 2. Three responses at the level of Unsuccessful Bivariate Association.

Level 2. Partial Bivariate Association

At the level of Partial Bivariate Association, responses to part (B) displayed some but not all of the features relevant to the task. Students represented the increasing trend of part (B1), but often had difficulties representing the curvilinear association for part (B2). These difficulties included errors or omissions of labels or scales, that resulted in failure to represent significant features of the association, such as failure to show constant height for changing age, or failure to show that this occurred by age 20. Three examples are shown in Figure 3. In the first example, a grade 4 female represented an increasing trend to part (B1) with height implied by the pictograph form and age scaled on the horizontal axis; for part (B2) she drew a single person at age 20. The second example is a pictograph that displayed a constant function for part (B2), but not the growth that occurred at younger ages, as had been represented for part (B1). In the third example, a grade 6 male reversed the axes, which posed no problems for representation until age 20, beyond which he represented the constant function and showed no change in vertical axis value. This error may indicate the student's confusion that height would naturally be on the vertical axis. His unsatisfactory solution, however, was to replicate constant height values on the horizontal axis.

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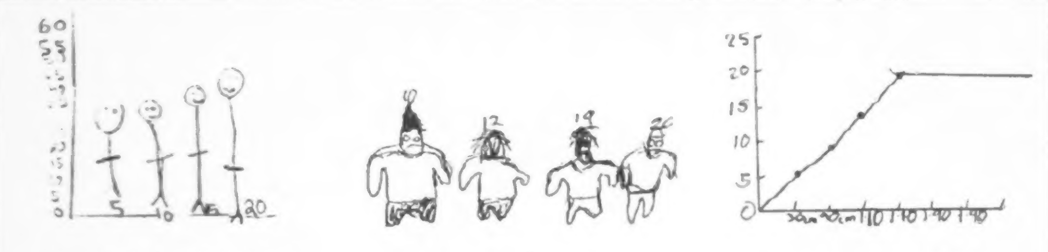
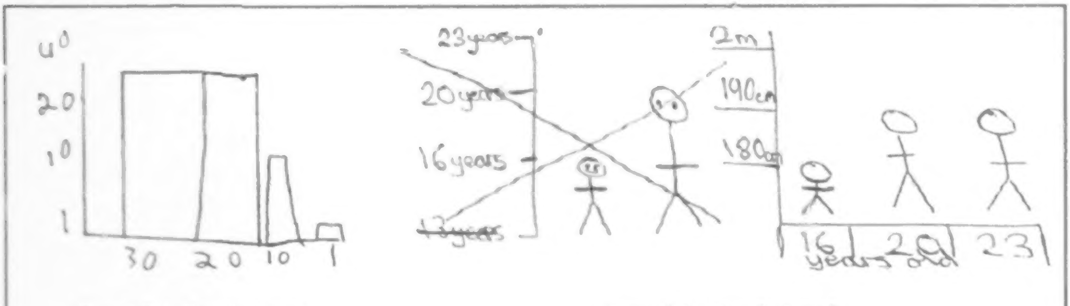


Part (B1), Grade 4 female *Part (B2), Grade 4 male* *Part (B2), Grade 6 male*

Figure 3. Three responses at the level of Partial Bivariate Association.

Level 3. Complete Bivariate Association

At this level, responses represented all relevant features of the task: a curvilinear trend that showed increasing height with age and showed constant height with changing age by age 20. Two examples are shown in Figure 4. In the first example, a grade 6 male reversed the horizontal scale and did not label the graph, but features of the graph data indicate for values 20 and 30 on the horizontal axis (age), there is a constant vertical value (height). In the second example, a grade 5 male learned in part (B2) about the need for independent axes to display repeated values of the same height for different ages. In response to part (B1), he drew a pictograph showing an increasing trend, with height implied in the pictures and age labelled with a vertical scale also. He then began drawing a similar graph arrangement to part (B2), but found difficulty drawing a 23 year old, since his system could not represent age differing when height remained constant, and he then developed an appropriate two-axis system.



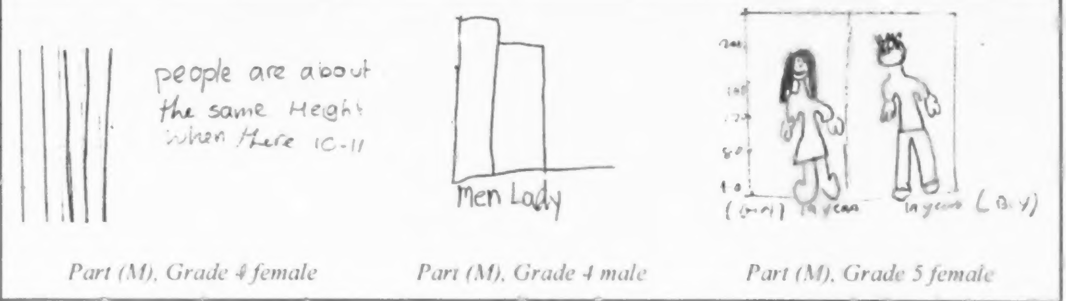
Part (B1), Grade 6 male *Part (B2), Grade 5 male*

Figure 4. Two responses at the level of Complete Bivariate Association.

Student Responses to Represent Multivariate Association

Level 1. Unsuccessful Multivariate Association

Responses that were unsuccessful at representing the multivariate association omitted one variable or super-imposed two variables to represent bivariate data. Three examples are shown in Figure 5. In the first example, a grade 4 female represented a constant function for part (M) just as she had for part (B2), however for part (B2) constant height was implied to be across varying ages, whereas for part (M), she annotated that the graph represented the same height across varying people (and possibly sexes) at the given age 10. The second and third examples in Figure 5 illustrate a method of omitting the variable of age to reduce complexity of the graph to a single comparison of the heights of a single female and male.

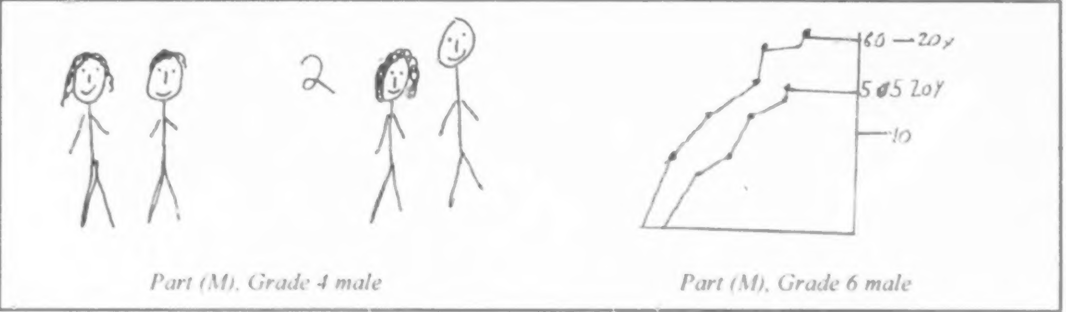


Part (M), Grade 4 female *Part (M), Grade 4 male* *Part (M), Grade 5 female*

Figure 5. Three responses at the level of Unsuccessful Multivariate Association.

Level 2. Partial Multivariate Association

At this level, responses indicated all three variables—height, age, and sex—and displayed some but not all of the features relevant to the task. Two examples are shown in Figure 6. In the first example, a grade 4 male represented a double comparison of female and male heights, indicating the same height at age "1", and males taller at age "2", but there was no indication of the critical feature of age 10. In the second example, a grade 6 male displayed an increasing trend for both females and males (unlabelled), but did not clearly represent the critical feature of same height at age 10.

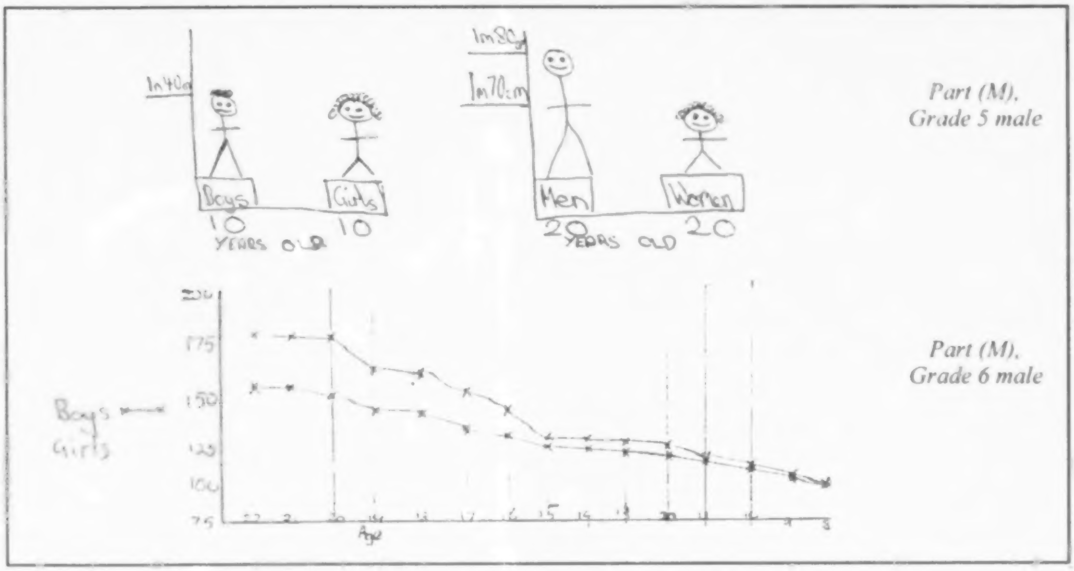


Part (M), Grade 4 male *Part (M), Grade 6 male*

Figure 6. Two responses at the level of Partial Multivariate Association.

Level 3. Complete Multivariate Association

At this level, responses represented all relevant features of the task: (a) at age 10, females same height as males; (b) association of height with age (for females, males, or both); and (c) for some age greater than 10, males taller than females. Two examples are shown in Figure 7. The first example is from the same grade 5 male who drew the pictograph in Figure 4 for part (B2). He adapted his two-axis system from Figure 4 to produce a double comparison, the horizontal axis denoting sex as a variable, and two graphs denoting two ages. In the second example, a grade 6 scaled the horizontal axis increasing right-to-left rather than left-to-right, but represented all relevant features of the task.



Part (M), Grade 5 male

Part (M), Grade 6 male

Figure 7. Two responses at the level of Complete Multivariate Association.

Numerical Analysis of Student Responses

Table 1 shows the percentages of responses levels to part (B) and to part (M) for each sex and grade. In response to part (B), more than 90% of students represented partial or complete association. In response to part (M), about a third of grade 4 students were unsuccessful in representing the multivariate association, but by grade 6 most students represented complete association.

There was an association of response level to part (M) according to response level to part (B) ($\chi^2_2 = 27.31, p < 0.001$)[1]. Four students' responses were level 1 both to part (B) and part (M). Of 38 students who responded at level 2 to part (B), 12, 19, and 7 responded to part (M) at levels 1, 2, and 3 respectively. Of 51 students who responded at level 3 to part (B), 3, 15, and 33 responded to part (M) at levels 1, 2, and 3 respectively.

Table 1

Percentage Responses of Level by Sex and Grade

	Bivariate Association (N=97)					Multivariate Association (N=93)				
	Female Grade		Male Grade			Female Grade		Male Grade		
Level	4	5	4	5	6	4	5	4	5	6
1. Unsuccessful	12	0	5	4	0	33	8	37	16	10
2. Partial	53	31	32	48	35	33	23	37	48	33
3. Complete	35	69	63	48	65	33	69	26	36	57
n	17	13	19	25	22	15	13	19	25	21

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[1] To ensure χ^2 test reliability, for parts (a, b), Levels 1 and 2 were merged to avoid low expected cell counts.

Discussion

Three levels of representing bivariate associations and of representing multivariate associations were described, corresponding to levels observed by Moritz and Watson (2000). Over 90% of students from grade 4 represented an increasing trend at the level of Partial Bivariate Association, and at least 35% represented a curvilinear trend at the level of Complete Bivariate Association. These were high success rates in comparison to the grade 8 students in the study by Mevarech and Kramarsky (1997), of whom 55% and 38% respectively represented an increasing function and a curvilinear function. Their criteria, however, included labelling of axes. Indicative labelling was considered a strength of the current coding scheme, in acknowledging student understanding implicit in representations, for example, the height label being implicit in a pictograph of a person.

The task to represent a curvilinear association (part B2) posed difficulties of representation for over 30% students, who responded at the level of Partial Bivariate Association. Some of these students did not have a consolidated sense of a Cartesian coordinate system, which is perhaps not surprising at these grade levels. This did not mean that the task was inappropriate for these students; on the contrary, for some students, the difficulties posed in part (B2) provided an important learning exercise to revise their representational system to display two independent axes, one each for height and age.

The task to represent multiple variables (part M) involving comparison of bivariate associations posed difficulties for many students. The solution for over 30% grade 4 students was to represent a comparison of heights of one female and one male, thus eliminating age as a variable. This strategy to reduce representational complexity has been observed by other researchers as "one-point" graphs (Mevarech & Kramarsky, 1997) and "single comparison" graphs (Moritz & Watson, 2000). Responses at the levels of Partial and Complete Multivariate Association were either in the form of double comparisons (first examples in Figures 6 and 7), or in the form of series comparison graphs (second examples in Figures 6 and 7). These two forms are similar to those observed by Moritz and Watson (2000) in their use of a comparison schema, though for that task, the two forms were at the Partial and Complete levels of success, respectively. Given students' success in representing bivariate associations, sequences of graphing tasks that are carefully designed by teachers and researchers for students in upper primary years may prove fruitful in exploring and promoting these students' learning in representing statistical associations.[1]

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Reasoning and Expressing Probability in Students' Judgements of Coin Tossing

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A survey item based on a newspaper article about coin tosses was administered to 1256 students from grades 6 to 11. Few students determined the probability of four successive tails. Most students considered that heads and tails were equally likely for a subsequent fifth toss, often describing the probability as "50-50". Students in higher grades were more likely to respond appropriately. Results are discussed with reference to equiprobability, independence, the gambler's fallacy, and the outcome approach to probability.

> Australian cricketers lost in darkest Africa ... Border wanted to bat, but he has now lost eight of nine tosses in international matches on tour ... When he retires soon, his new sporting life will not, presumably, be as a two-up player. ("Lights, cam and a fair bit of inaction", *The Mercury*, 6 April, 1994, p. 52)

Newspapers extracts such as this illustrate that understanding probability is an advantage in various social contexts including sport, gambling, and the daily news. In these contexts, events such as a sequence of losing coin tosses often provoke intuitive comments of "that's not fair" or "how unlucky". "The language of chance is widely used in a colloquial way... students should be helped to refine and extend their use of this language so that they are more able to make sense of their everyday experiences" (Australian Education Council [AEC], 1991, p. 163). Possible activities for upper primary school students (Band B) include "analyse simple experiments (e.g. those involving single coins, dice and simple spinners, and equally likely events); make a systematic list of possible outcomes and assign simple numerical probabilities based on reasoning about symmetry" (AEC, 1991, p. 170). Also related to coins, for higher grade levels, the concept of independence appears in the curriculum for secondary students (Level 7), evident when a student "estimates probabilities... and assigns probabilities using complementarity and independence", such as to "calculate the probability of getting at least one head in five tosses of a coin using 1 – [minus] 'the probability of getting no heads in five tosses'" (AEC, 1994, p. 124).

The current study investigated school students' probability judgements concerning coin tossing sequences. Of particular interest were (a) the expressions of probability that students used indicating whether they used intuitive estimates or calculations to determine probabilities, (b) the incidence of responses indicating reasoning about equally likely outcomes and consistency with the concept of independence for coin tossing.

Reasoning and Expressing Probability

Much research of probabilistic reasoning has concerned errors people make based on intuitive misconceptions or heuristics (Fischbein & Schnarch, 1997). The *representative heuristic* is a belief that even small samples of data should reflect the parent population (Kahneman & Tversky, 1972). This heuristic explains the *gambler's fallacy*, a negative recency expectation; for example after a sequence of heads in coin tosses, tails is expected to even up the numbers of heads and tails. Students may exhibit inconsistencies in reasoning as they switch between reasoning based on intuitions or based on mathematical concepts (Konold, Pollatsek, Well, Lohmeier, & Lipson, 1993).

Truran and Truran (1997) suggest that students' responses to many probability questions should be interpreted with reference to the particular random generator involved and to the wording of the question, as students' intuitions and reasoning depend on their interpretation of the question wording. Typical questions focus on which outcome "is more likely", "will occur more often", or "is easier to get". Some students, however, appear to interpret most probability questions as what "you think will happen", that is prediction of an individual outcome (Konold, 1989; Konold et al., 1993). Known as the *outcome approach*, students interpreting probability questions in this way take one of three stances in their prediction: the outcome will happen, the outcome will not happen, or the outcome cannot be predicted. Metz (1997) further observed that some students believe that chance situations involve patterns-without-uncertainty (over confidence in predicting yes or no), and others believe in uncertainty-without-patterns (no basis for prediction), whereas the educational goal is an appreciation of quantifiable patterns of uncertainty.

Students' expressions of probability may indicate how they interpret a question. Results from other studies of development of chance measurement for students of grades 3 to 11 (Watson, Collis, & Moritz, 1997; Watson & Moritz, 1998) have indicated that younger students often use words or non-normative numerical expressions for measuring chance. Even an expression that appears to indicate a numerical value, such as "50-50", may be a colloquial expression indicating basic uncertainty but not measurement of chance (Konold et al., 1993). Thus any research evidence concerning students' reasoning with regard to equally likely outcomes or independence should be considered in the light of the problem situation, the wording of questions, and the expressions students offer.

Equally Likely Outcomes and Independence

The belief in equally likely outcomes for coins and dice is not universal. Kerslake (1974) found that 68% of grade 1 students and 59% of grade 4 students believe some numbers on dice are easier to get. Green (1983a) also found that only 45% of 11-12 year olds and 74% of 15-16 year olds believe there is no difference in whether a red or green face of a counter is more likely when tossed. When asked whether a 1 or a 6 on a die was easier to throw, Watson et al. (1997) found that 55% of grade 3 students and 87% of grade 9 students chose "equally easy", but fewer could provide a simple justification.

On the other hand it is possible to exhibit an *equiprobability bias* (Batanero, Serrano, & Garfield, 1996): an over-generalisation of equally likely outcomes to inappropriate situations. Lidster, Watson, Collis, and Pereira-Mendoza (1996) found that school students tended to assume dice were fair even after gathering data that suggested a heavy bias. Green (1983b) asked students about the outcomes of 100 drawing pins landing up or down, and found that around 60% of secondary students at any grade level chose the option "all these results have the same chance" although other multiple choice options more clearly reflected the given data of a previous trial that 68 pins landed point up and 32 landed point down.

Turning to independence, events A and B are *independent* if and only if $P(A|B) = P(A)$; that is, if one assumes tosses of a coin are independent, one expects that the likelihood of getting heads or tails is not affected by knowledge of previous tosses of the coin. Results of research studies, summarised in Table 1, have indicated that most secondary school students consider heads and tails equally likely for the next toss of a normal coin following a sequence of successive outcomes, though a significant number of students indicate a preference for a negative recency outcome in accord with the gambler's fallacy. Konold et al. (1993) described apparent inconsistencies in student reasoning, and distinguished between reasoning from representativeness that a *sequence* of five outcomes should display a balance of heads and tails, and reasoning about *the fifth outcome* that has two equally likely outcomes. As Truran and Truran (1997) point out, "given that most naive students do not have the combinatorial skills to assess these probabilities [of several five-toss sequences], such a response [based on representativeness] is not surprising" (pp. 94-95).

Table 1

Results of Four Studies of Students' Preferred Outcome in Coin Tossing Situations

Research Study	Previous event	Student response (H, T, or =), Grade/Age, and Proportion of response
Green (1982)[a]	5 Heads	(=) 11-12yo, 67%; 15-16yo, 80%
Brown, Carpenter, Kouba, Lindquist, Silver, and Swafford (1988) [b]	4 Tails	(=) grade 7, 47%; grade 11, 56% (H) grade 7, 38%; grade 11, 33%
Konold (1989), Konold et al. (1993)	4 Heads	(=) high school, 79%; uni graduates, 96%
Fischbein and Schnarch (1997)	3 Heads	(=) grade 5, 7, 9, 11: 40%, 55%, 70%, 90% (T) grade 5, 7, 9, 11: 35%, 35%, 20%, 10%

[a] *Note.* [a]Cited by Borovcnik & Bentz, 1991; [b]cited by Konold et al., 1993.

Research Questions

The current study of school students' responses to coin tossing outcomes investigated students' estimates and expressions of probabilities. When asked about a sequence of four tails, would students (a) calculate the probability of the sequence, (b) estimate the chance as less than 0.5 to indicate at least an intuitive sense for the measurement of chance of the sequence, or (c) use an outcome approach of basic uncertainty such as "50/50"? When asked about a subsequent toss, would students consider heads and tails equally likely, consistent with the concept of independence, and would they assign probabilities of 0.5 for each outcome? It was of interest whether appropriate responses would be more likely from students in higher grades or from those students who appropriately measured chance on traditional probability items concerning simple events.

Method

Survey responses ($N = 1581$) were gathered from 1256 students in grades 6 to 11 at 20 government primary schools, secondary schools, and secondary colleges distributed around Tasmania. Curricula in these schools were individually adapted in general adherence to the Australia-wide curriculum (AEC, 1991). During 1995, 206 grade 6 students, 301 grade 8 students, 294 grade 9 students, and 72 grade 11 students were surveyed; during 1997, 80, 206, and 39 of these students, then in grades 8, 10, and 11 respectively, were surveyed again. Responses were also obtained from additional students during 1997: 166 in grade 6, 74 in grade 8, 90 in grade 9, 46 in grade 10, and 7 in grade 11. Approximately equal numbers of females and males were surveyed at each grade level in each year.

A survey item, shown in Figure 1, was developed involving four parts closely related to the interview protocol of Konold et al. (1993). This item was last on a 10-item survey involving newspaper extracts about chance and data concepts (Watson, 1994). The survey was administered to whole class groups during 45 minutes of class time. Students who did not respond to the item, due to time or inclination, were excluded from analysis.

During the recent Australian cricket tour of South Africa, the Hobart Mercury (6/4/1994, p. 52) reported that Allan Border had lost 8 out of 9 tosses in his previous 9 matches as captain. Imagine his situation at this point in time.

(a) Suppose Border decides to choose heads from now on. For the next 4 tosses of the coin, what is the chance of the coin coming up tails (and him losing the tosses) 4 times out of 4?

(b1) Suppose tails came up 4 times out of 4. For the 5th toss, should Border choose

☐ Heads ☐ Tails ☐ Doesn't matter

(b2) What is the probability of getting heads on this next toss?

(b3) What is the probability of getting tails on this next toss?

Figure 1. Survey item (italics denote labels not shown in original survey item).

Responses to part (a) were coded into four categories: value < 0.5, value $= 0.5$, other value (value > 0.5 or indeterminate), and no response. Common response expressions were also identified, in particular distinguishing various expressions of the probability value 0.5. Responses to part (b1) were coded as "H", "T", or "=" according to multiple choice selections "Heads", "Tails", and "Doesn't Matter" respectively; in a few cases where no choice was made, the choice was inferred from parts (b2) and (b3). Responses to parts (b2) and (b3) were coded either as both values $= 0.5$, or as other values (including "chance" or "luck", indeterminate values, differing values, or the same values but $\neq 0.5$). For part (a) and for part (b)[1], analysis was conducted to examine if there were any significant differences between students in comparable grades who were (i) repeating and non-repeating students (grades 8, 10, and 11), (ii) females and males (grades 6, 8, 9, 10, and 11), and (iii) drawn from 1995 and 1997 samples (grades 6, 8, 9, and 11). Of 24 tests in total, only two were significant with $p < 0.01$: for grade 10 responses to part (a), appropriate responses with a value less than 0.5 were more common from males ($\chi^2_3 = 12.93$, $p = 0.0048$) and from non-repeating students ($\chi^2_3 = 12.93$, $p = 0.0048$). As these differences were of marginal significance for the number of tests, response data were combined across repeating and non-repeating students, females and males, and samples from 1995 and 1997.

Results

In response to part (a), few students at any grade level appropriately evaluated the chance of four successive tails as 1/16, as shown in Table 2. Many students, particularly from grade 9 or below, did not respond. Of those who did respond, most responded with a value of 0.5. Those responding with a value of 0.5 often expressed variants of "50%", "50-50", "50/50", "2/4", "2 out of 4", and "2 times out of 4". The first three expressions are consistent with a valuation of 0.5, but are also consistent with the outcome approach that "it is possible"; for example, a grade 8 male responded, "it's both half & half because you can't predict what's going to come up". The expression "2/4" may be a description of expecting 2 tails out of 4 tosses, or from various thinking about 1/2; for example, one grade 11 female responded: "P(half + half + half + half). The chances of Border losing the tosses 4 times out of 4 are 4/8 or 2/4. There is a 50-50 chance he will lose on each of the 4 tosses." Students in higher grades were more likely to respond with values less than 0.5; common expressions included "25%", "1/4", "12.5%", "1/8", and various chance words such as "unlikely". One grade 11 female responded "1/2 + 1/2 + 1/2 + 1/2 = 1/8 chance of tails four times out of four", and another responded, "1 in 5 chance {HHHH TTTT HHHT TTTH TTHH}". Some students gave values greater than 0.5 such as "3/4", "4/4", and "probable". Many responses were of indeterminate value, such as chance expressions that did not clearly indicate equality or inequality with 0.5, for example "anything is possible" and "doesn't matter, it's the luck of the toss". Other expressions included "no", "yes", "2/2" (unknown intended probability), and "there is equal chance of either coming up" (the student may have been considering a single toss rather than a sequence of four tails).

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[1] In χ^2 tests for part (b), "H" and "T" responses were combined to form four outcome categories: "=, each value = 0.5"; "=, other values"; "H/T, each value = 0.5"; and "H/T, other values".

Table 2

Percentage Responses to Part (a) by Grade (N = 1581)

Response category (Response expression)	Grade 6	 8	 9	 10	 11
Value < 0.5 (1/16[a])	0	0	3	5	16
Value < 0.5 (Other # or word)	12	9	11	12	13
Value = 0.5 (50%)	8	16	12	17	19
Value = 0.5 (50-50)	7	11	11	23	7
Value = 0.5 (2/4)	11	7	7	6	11
Value = 0.5 (Other # or word)	2	1	3	3	5
Value > 0.5 or Indeterminate	19	13	15	13	14
No response	41	42	38	21	15
n	372	455	384	252	118

Note. [a]Includes two responses of "1/2 x 1/2 x 1/2 x 1/2" where this expression was incorrectly evaluated.

In response to part (b1), most students selected "=", as shown in Table 3. Students in higher grades were less likely to respond "H" or "T", and more likely to respond "=" with values of 0.5 for parts (b2) and (b3). Those responding with values of 0.5 often expressed variants of "50%", "50-50", "50/50", or "1/2". Many students gave inconsistent responses, combining "H" or "T" in part (b1) with values of 0.5 in parts (b2) and (b3). Many responses of indeterminate value were due to non-response to parts (b2) or (b3).

Responses to part (b) differed according to responses to part (a). Of students' responses to part (a) with (i) a value less than 0.5 (n = 214), (ii) a value of 0.5 (n = 569), and (iii) other responses or no response (n = 798), the proportions of students responding "=, each value 0.5" to part (b) were 59%, 72%, and 38% respectively. Hence 8% (n = 1581) of the sample responded in the correct response category on both parts.

Table 3

Percentage Responses to Part (b) by Grade (N = 1581)

Response to part (b1), values for parts (b2) and (b3)	Grade 6	8	9	10	11
=, Each value = 0.5	32	55	52	73	79
=, Other values [a]	30	20	25	14	9
H, Each value = 0.5	5	6	5	3	3
H, Other values [a]	16	10	8	6	2
T, Each value = 0.5	6	4	5	3	3
T, Other values [a]	12	6	6	2	3
n	372	455	384	252	118

Note. [a] Responses include "chance" or "luck", indeterminate values, differing values, or the same values but ≠ 0.5.

A subset of responses (n = 1497) were matched to the students' chance measurement levels (Watson & Moritz, 1998). Levels were based on responses to three items: (1) chance of outcomes when a die is rolled, (2) chance of outcomes drawn from a bag, and (3) comparison of chances of drawing one colour of marble from two boxes with the same ratio of colours. Level 0 indicated responses not related to judgements of uncertainty. Levels 1, 2, and 3 indicated basic uncertainty, a qualitative judgement, and a quantitative judgement, respectively. Levels 4 to 6 indicated proportional reasoning in the third item, with higher levels for more explicit numerical justification. The distribution of responses to parts (a) and (b) of the coin tossing item differed according to chance measurement level, as shown in Table 4 (part (a), χ^2_{21} = 145.3, $p < 0.0001$; part (b), χ^2_{21} = 173.7, $p < 0.0001$). Higher levels were associated with more appropriate responses to parts (a) and (b).

Table 4

Percentage Responses by Chance Measurement Level (N = 1497)

Response category for part (a)	Chance Measurement Developmental Level 0	1	2	2.5	3	4	5	6
Value < 0.5	20	8	8	8	11	17	13	35
Value = 0.5	20	8	29	34	41	45	42	34
Value > 0.5 or Indeterminate	20	23	23	16	15	13	15	6
No response	40	61	41	42	33	25	30	24
Response to part (b1), values for parts (b2) and (b3)								
=, Each value = 0.5	10	24	33	44	59	68	58	79
=, Other values [a]	30	40	33	25	19	16	26	9
H/T, Each value = 0.5	0	8	10	12	10	7	4	5
H/T, Other values [a]	60	27	23	20	12	8	11	6
n	10	62	177	358	270	436	53	131

Note. [a] Responses include "chance" or "luck", indeterminate values, differing values, or the same values but ≠ 0.5.

Discussion

For the event of four successive tails, less than 30% of students, even by grade 11, responded that with a probability less than 0.5, and very few could calculate the probability to be 1/16. Many students in grades 6, 8, and 9 gave no response, and many students in all grades expressed a value of 0.5 such as "50%", "50-50", or "2/4". These results indicate not only that students are poor at *calculating* the probability of compound events by multiplying probabilities of independent events, but also that they are poor at *estimating* such a compound probability using intuition. Many students appeared not to be used to probabilistic reasoning about compound events, and they based responses on the outcome approach (Konold et al., 1993) in that they were unwilling to predict the outcome. It is not clear, however, whether these students considered the outcome to be four tosses or one. It had been hoped that four tails would be a long enough run length to prompt students' intuitions to acknowledge that the sequence was not only possible but also *rare*. Perhaps for many students a more strikingly long run length is required to trigger this thinking. The dilemma associated with run length and equiprobability is illustrated in the following dialogue from an interview with a grade 9 male.

> *I*: What is the chance of getting tails four times in a row?
> *S*: Pretty good chance, oh not really, not really a pretty good chance. Just the luck of the flip.
> *I*: Yeah ok. How likely, do you think would it be to get four tails in a row, what sort of number?
> *S*: Oh, 50 percent I reckon... They both have the same amount of chance of coming out. [...]
> *I*: What about 100 tails in a row, do you think that is very likely? [...] Would that still be 50 percent or would that be a different number, for probability of getting 100 tails in a row?
> *S*: Oh... Not... One percent I reckon.
> *I*: Right, it might be one percent. Do you think for four tails in a row, it is 50 percent, or...?
> *S*: I suppose it would have a good chance of coming up.
> *I*: Yeah. Would that mean it's 50 percent or a different number?
> *S*: 50 percent I still reckon.

Concerning the fifth coin toss following four successive heads, the high incidence of correct multiple choice responses closely mirrored that found in small samples by Konold (1989), though the trend of improvement with increasing grade was less pronounced than that found by Fischbein and Schnarch (1997). Two aspects would appear to demand attention in the classroom: expressing measurement of chance, and acknowledging independence to reduce the gambler's fallacy. These two aspects may be related. For example, another grade 9 boy in an interview, responded to part (b1), "I reckon he'd have a better chance of doing heads, cause I reckon the next one would come up heads", then suggested 50% for parts (b2) and (b3). When the interview commented about heads being more likely, and thus whether the chance would be 50%, the student responded as follows.

> *S*: I'm not sure... it would be fifty percent because you wouldn't know which is going to come up.
> *I*: So do you think it really doesn't matter which one he picks, or do you think he is generally better off picking heads?
> *S*: Oh it doesn't really matter, I suppose. I'll rub that one out... It's just the luck of the flip.

It is not clear whether the student's rejection of the gambler's fallacy is in favour of the outcome approach, that one cannot predict, or in favour of measuring the chance as 50%. A similar indeterminacy occurred in interpreting responses of about 10% of students, who were inconsistent by combining a response of "H" or "T" with values of 0.5 (see Table 3). Evidence of improved performance in relation to chance measurement level (see Table 4) gives some hope that understanding of traditional probability problems may assist students to reject both the gambler's fallacy and the outcome approach, in favour of measuring chance. Konold (1989) concluded by advocating use of asymmetric distributions to encourage chance measurement, along with the assertion that some outcomes are more likely than others, rather than symmetric distributions, which are consistent with the view that there is no "basis for prediction" as outcomes are equally likely. Introducing asymmetric binomial distributions, such as those resulting from dropping drawing pins and counting the two outcomes (point up or point down) (Green, 1983b), may be an important step to prompt students to measure chance numerically, whether in situations where empirical data may be required to provide chance measurement estimates, or in classical probability situations of equally likely outcomes. Further encouragement to measure chance numerically should reduce reliance on the outcome approach and on intuitions such as the gamblers' fallacy, and overall, tasks based on compound events should encourage students to combine reasoning about simple events with the concept of independence.

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The Practicum as Context: Two Snapshots

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Practicum sessions allow student teachers to learn about the profession through participation in it. This paper uses Lave's notion of situated cognition, and argues that academics need to be aware both of changes that are taking place in school contexts and of how such changes are shaping student teachers' lived experience. The paper reports results from two surveys of students' observations of primary mathematics education—the first implemented in 1990 and the second a decade later. Some changes are noted, and it is proposed that these are worthy not only of discussion in tertiary classrooms but also of further research.

Teacher education (not only in the formal sense) is set in a community of practice that includes teacher education courses (pre-and post-graduate), school-based interactions, curriculum advice from many sources, parental expectations, teachers' and children's personal interests and beliefs, staffroom discussions, and many more factors. The traditions of institution-based learning are powerful, and achieving and then maintaining an image of a successful teacher means gradually taking on accepted ways of acting and thinking. This is first done in apprenticeship situations, with participants gradually taking more central roles in planning, teaching and evaluation of their own practice.

The relevant discourses, however, suffer learning discontinuities. It is assumed that theories presented in arenas that are divorced from school settings can be applied during practicum experience and in later teaching—but in fact the communities of studentship, scholarship and schooling often differ in important ways. In tertiary education, knowledge is frequently abstract: statements about teaching and learning and its underlying philosophies are proposed as if they are objective, unproblematic, rational and universally applicable; and teaching is often presented as a value-free and context-free enterprise. There is an assumption that knowledge and skills developed in one setting can be transported to other contexts. For example, separating foundation studies from curriculum studies presumes that later integration will be possible. Further, it is expected that tertiary students will be able to link group university-based experiences with their personal experiences as participants in schools; and hence will be able to take on new roles—as for example, "reflective practitioners" and "agents of change"—but in institutional contexts that may not support either of these initiatives. However, such expectations rely on participants being able to stand outside of processes that constituted their professional being in the first place—so we need to be cognisant of the elements of those processes.

The practicum allows student teachers to learn about teaching as an apprentice, with learning being situated in the practice of teaching. This paper explores some differences exposed by two snapshots of practicum experience in primary schools—one taken in 1990 and one a decade later in 2000. It draws on the theory of situated cognition, which recognises the dialectical development of individual knowledge through interactions in social contexts and focuses attention on how learners come to assimilate and re-generate the knowledge of the community itself. Knowledge, here, for the teaching community and its apprentices, is expressed as much in professional action as verbally; and perhaps more so, because cognition and articulation about practice are situated in, and are responses to, specific but constantly evolving practical circumstances. Thus school experience can be thought of as "an integral part of generative social practice in the lived-in world" (Lave & Wenger, 1991, p. 35)

Situated Learning

Situated cognition pursues theorists such as Vico, Mead and Vygotsky and their rebellion against mind-body dualism. An influential author in this field in recent times is Jean Lave (and later Lave with Wenger), who stated that "Developing an identity as a member of a community and becoming knowledgeably skilful are part of the same process, with the former motivating, shaping and giving meaning to the latter, which it subsumes" (Lave, 1988, p. 65). The key idea in Lave's theory is *situated activity*, the term she uses for "a distributed form of cognition" (p.1). This is not a theory of learning, but a theory of social practice of which learning is only one dimension; a view that "generates interconnected views of perception, cognition, language, learning, agency, the social world, and their interrelations" (Lave, 1988, p. 66). In Lave's theory, (as in the Vygotskian tradition), development of knowledge, skills and roles results from participation in social activity, with participants being constituted by the socially constructed setting and the activity itself. Initially they participate on the periphery, but increasingly they take on roles and cooperate in ways that shape the setting and its activities. Thus reasoning and understanding are deeply bound in the activities through which learning takes place— "'understanding' appears to belong directly in the experienced world, in activity" (Lave, 1988, p. 170). The setting, with its institutional structures, norms, tools, ways of working and patterns of interaction, is paramount. Lave claims that "apprentices" contribute to, and hence re-construct, this setting; but do so in ways that are shaped by their perceptions of the context and its aims. As Adler (1998) noted, while shifting Lave's theory of learning mathematics through participation into school learning raises questions about what constitutes a community of practice, situated cognition is a powerfully theory for considering learning about teaching.

There is a range of theoretical positions that link the development of professional knowledge with social experience. In one genre of interpretations identified by Rommetveit (1987), individual cognitive processing is seen to be occurring in a social world, but cognitive activity itself is not reconceptualised in social terms. A second point of view dissolves the individual-social duality, with the world arising from agents' constructions. Here, situatedness "is not located with physical locatedness in the world. It is not possible to walk into a situation. Instead, language use and, thus, meaning are situated in interested, intersubjectively negotiated social interaction" (Lave, 1991, pp. 66–67). However, Lave noted that both of these views, in their own ways, imply that the social world can be separated from individuals. Neither holds that "subjects are fundamentally constituted *in* their relations with and activities in that world" (p. 67). This latter assumption is the proposition that underpins a third guise, the position that Lave took: *situated social practice* (Lave 1985, 1988, 1993, 1996). According to this view, cognition and communication are situated in "the historical development of ongoing activity" (Lave & Wenger, 1991, p. 67). In Mousley, Zeegers and Herbert (in press), we use this latter genre to explore how our student teachers' development of self-concept is shaped by the ways in which they are constructed by others as "student teachers", as well as by the university and school experiences. However, here we focus only on a few more structured aspects of the context of their learning. Our inquiry is into some of the observable patterns in the community of practice—or at least what our students' perceptions of these are.

The Study

Motivation for the Study

A decade ago, one of us was involved in research that collated and summarised some perceptions of eighty second-year Education student teachers' experiences during one particular practicum (Mousley & Clements, 1990). The student teachers had been asked to complete a questionnaire with 25 questions about the mathematics lessons they had observed (but not taught) during the previous two weeks of school experience in primary schools in the Geelong, Victoria, area. The results of that research were presented at the 1990 conference of the Mathematical Association of Victoria. One member of the audience at that time agreed with the presenters' point that the geographical area may have led to conservatism. However, he was offered a copy of the questionnaire to implement with his own students in Melbourne, and was quite surprised to find that his results were almost identical—his students' perceptions of practicum learning contexts were, in fact, remarkably similar.

Since 1990, there have been some key changes in mathematics education: the introduction of national and state curriculum and standards frameworks, initiatives focusing on numeracy, broad-based testing programs, growth in the availability of electronic technologies, and increasing claims about (and professional development in) group work, co-operative learning, constructivist approaches, and so on.

Teacher educators need to be aware of these changes; and of whether and how they are perceived by student teachers. Thus we were interested, a decade later, to find out whether school-based contexts for teacher education have changed significantly. The big questions for us were whether school experience now sits more comfortably with the theories and examples presented in our university program than it did a decade ago, and whether the content of the university-based components sufficiently reflects changes taking place on school resources practices, and expectations.

Methodology

Even though questionnaires were used, and many of the items leant themselves to quantitative analysis, this is essentially an interpretive project, where the term refers not so much to interpretations made by researchers (for these are a necessary component of all research) but to what is collected during the research process. We aimed to capture the students' perceptions of school-based contexts in which they learn about teaching mathematics, so many of the questions were open-ended in order to gain insight into these interpretations. We recognise that these perceptions are just snapshots: each person's understanding is a picture resulting from individual synthesis of reality but is not a photograph of reality. It reflects an internal network of ideas that interacts with reality in a mutual transformation (Labinowicz, 1985, p. 5). Further these snapshots sit within similarly limited pictures—the previous and current questionnaires made no claims to wider applicability of results as no attempt was made to construct representative samples.

As in the previous study, the 37 students were second-year Education students, undertaking their first unit of mathematics education, and their second 10-day school experience round. Because the school experience was taken in a block this year, the average number of days attended (9.8) was higher than in 1990 (4.95), but the lower number of students involved (37:80) meant that the total of days that had been spend in schools was similar (364:396). In 2000, nearly all observed at least five Maths lessons, and most saw more than seven, the mean being 7.8. It is pleasing to note that the upper grades in the classrooms observed this year nearly all have Maths every day. As an aside, the number of composite grades is now much higher (58% compared with 14% a decade ago) and these were clustered around the Years 2/3/4 level. In later discussions, the students questioned the wisdom of composite classes, because any straight grade already has a wide distribution of achievement levels in key learning areas and the composite classes seemed to exacerbate this, especially in cases where the grades were taught separately in key subject areas.

Both the 1990 and 2000 questionnaires were filled out during tutorial time so plenty of time was available, and replies were anonymous for both the students and the schools. Both questionnaires used a range of types of questions for different purposes. Forced choice or fill the gap style was used for collecting information where only a few possibilities were available (e.g., state/independent school, number of mathematics lessons observed, whether or not written work was usually corrected). Three-point scales were used to gather data on the frequency of use of texts, worksheets, calculators, and so on. Additionally, there were some relatively open questions, where, for example, students were asked to tick any terms that applied to the majority of Maths lessons observed and write a short description where applicable. Even more open questions were used to describe features of the classroom interaction, such as any general patterns that the lessons seemed to follow. Responses were collated and/or tallied under a number of headings, and trends noted. Percentages used in the results below have been rounded to the nearest whole number. No levels of significance were calculated or are claimed.

We had attended to some of the limitations of the first questionnaire by adding further questions and options as well as including more open questions. However, for the purposes of this paper we will restrict the reporting of results to only those questions that were asked in the first iteration. Again, a number of items and responses presented difficulties in interpretation, given the lack of common vocabularies and no opportunity for follow-up interviews.

Results

The majority of Maths lessons were held after morning recess. Only 10% of students reported that classes were held regularly before recess, although 21% students said Maths could be taught any time in the mornings. Only two students saw Maths classes in the afternoon; two others said Maths was taught at various times of the day; and all of these four students reported use of an integrated-curriculum approach. In 1990, most (85%) lessons were also held in the mornings, but many more (44% of the morning classes) were held before recess. From the students' written comments this year, it was clear that time allocated to "literacy" is a relevant factor here; and it was interesting to note frequent use of the term "block"—many respondents used this term. For example, the only student who reported seeing fewer than five Maths lessons wrote "The Numeracy block was interrupted by the Swimming block".

In 1990, it was reported that many teachers (38%) used textbooks as their main resource for planning lessons, 15% used school policies and year-level policies, and 12% used state guidelines. A disturbing outcome, in both 1990 and 2000, was the fact that more than 10% of the students did not know what resources their teachers used in planning the lessons.

However, 66% of those who answered this question said that teachers used "The CSF" (Victoria's Curriculum & Standards Framework), 35% used textbooks, and 33% worksheets from previous years. Again, there was a further variety of other resources mentioned and these were hard to categorise (e.g., "various books", and "the net"). We noted that the text most frequently mentioned advertises its congruence with the CSF overtly, and that in any case all of the texts and commercial worksheets mentioned are structured broadly on the CSF "learning outcomes".

In 35% of the current classrooms, textbooks were used extensively during seatwork (c.f. 31% in 1990). Of these, 5% said they were used in every lesson and 32% in some lessons (ranging fairly evenly from 25% to 75% of the time). The most frequent reason given by teachers for their use was that they "cover" all areas. We were interested to note a few students' comments about texts and worksheets being used only in Maths classes.

All current students reported that worksheets were used in Maths classes (c.f. 57% in 1990), and 59% said they were used in all lessons. The ratio of use in "all/most lessons" to "some lessons" was close to 1:1 at each grade level from Prep to Year 4, but 4:1 in Years 5 and 6. The majority of the worksheets (84% of those who reported their use) seemed to be commercially produced (54% in 1990). Some students commented that their teachers frequently download worksheets and other resources from the internet, and we classified these as "commercial".

Table 1
Modes of Communication

Mode of Communication	Teacher		Children		No Response	Year level differences
	1990	2000	1990	2000		
Explaining		75%		25%		Similar across all levels
Discussing		54%		46%		Diminishing in upper grades
Asking questions		65%		35%		Diminishing in upper grades
Responding to questions		42%		58%		Increasing in upper grades
(Total: "Teacher talk")*	80%	59%	8%	41%	12%	
Listening*	14%	31%	72%	54%	14%	Similar across all levels
Write		5%		95%		Diminishing in upper grades
Draw		54%		46%		Diminishing in upper grades
(Total: "Write and draw")*	8%	29%	82%	71%	10%	

*Only these questions were asked in 1990.

Patterns of communication seem to be much as they were in 1990 except for more use of children's talk. The results are difficult to compare because the data were collected differently; but nevertheless the results from each year (see Table 1) are of interest. By the way, we enjoyed and empathised with two responses where one student reported the teacher listening 25% of the time and the children 0%, and another claiming that the children did "lots of listening, but no active listening to the teacher".

One area where there is a noticeable difference in results is in reporting about group work and co-operative activities. Whereas in 1990, only 16% of the students had reported that children worked co-operatively with each other, 65% of this year's cohort saw this "regularly". In 1990, 35% of the students reported seeing group work, and this year the equivalent response was much higher, at 74%. In 2000 there was also a lot more "ability" grouping (42%, compared with 11% previously), but the increased number of composites may well have contributed to this result. There was less friendship grouping (18%—30%). This year's students reported that group work was most commonly used before Year 5.

In 2000, the most prevalent words used to describe the perceived "roles of the teacher" were categorised as giving support (30%—compared with 24% in 1990), giving instructions (22%—17%), and supervision (18%—13%). Of course how one sets up categories and allocates terms used to these affects the results and such details were no longer available from the earlier questionnaire, but the overall pattern of results was similar. Strangely, few students included the types of actions that we categorise as "teaching", such as introducing or explaining maths concepts; but perhaps students take such actions for granted and of course some of the "giving instructions" and "supervision" could have served these purposes. Teacher's planning and evaluation received little mention, but perhaps such a perception is understandable because these teacher actions are not so visible. It is worth noting that 27 of the 37 students thought that the mathematics lessons observed followed a definite pattern of teacher introduction/demonstration, then seatwork. In only some classes (7%) was this followed by sharing of solutions, discussion, etc., and this type of activity was mentioned rarely in higher levels. It seems still very common, and even more so, for lessons to start with "mental maths" sessions and quick games that emphasises speed, competition and rote learning (57% of current students mentioned this, and 38% in 1990).

The student teachers were asked to "Suggest up to ten words describing the atmosphere of the classroom". As for the previous questionnaire, the most frequently words were "busy", "quiet", "repetitive", and "boring". The 2000 questionnaire asked for the year level of each respondents' placement, so a grade-by-grade-level analysis was possible, and this yielded some interesting features. For example, it was noticeable that many more negative descriptors (such as frustration, uncomprehending, annoyed, despair) were used to describe classrooms from Years 2 to 4 than were evident in either early or senior grades. We believe that despite our small sample this phenomenon seemed so pronounced that whether it occurs (and possible reasons) would be worth researching further, through gathering data on children's and teachers' feeling, and using more focused observation techniques. The negative words used to describe the atmosphere upper grades were fewer in number, and also related less to affective matters and more to productivity (e.g., wasting time, passive, "busy but not productive"). Of course there were positive descriptors too, such as "supportive" and "active" but again the transition from emotional/affective descriptors to engagement descriptors was clear.

One obvious difference across the decade is in the amount of "use of children's ideas and experiences", reported by 32% of students in 2000 and 11% in 1990. This year, it is clear that more teachers encourage children to use their own methods, as 54% of the students mentioned this (up from 6% in 1990), and many wrote comments like "sharing of methods used is an important aspect of many lessons". There now seems to be more integration across key learning areas and/or use of themes (22% compared with 6%). This year's students reported more use of contextual maths (54%:1%), with the majority of this being in Years 1 to 3. More activities were completed outdoors (16%:3%), but of course the weather may have been more conducive. In 1990 Mousley and Sullivan noted that despite encouragement for teachers to allow pupils to work out answers to sums and problems in their own ways, 93% of the observers who wrote about this felt that methods were imposed by teachers or textbooks.

In other aspects of the learning environment there had been less change in perceptions. In 1990, a little more than half thought the curriculum seemed "rigid", and the recent result was similar (43% thought that there was "flexibility"). Emphasis on speedy completion in 2000 was 22%, and 1990 was 15%. In 1990, only one observer believed the pupils had had any influence in decisions about the content of a lesson, and in 2000 no students saw evidence of this.

It seems that forms of feedback on students' written work have changed little. Most teachers still use ticks and crosses as the major form of feedback, although in an increased proportion of classrooms children corrected their own work (21% in 1999 and 37% in 2000) or peer-correction was used (10%:21%). Oral comments were also given (23%:14%), but neither questionnaire asked students to distinguish between individual and class feedback here.

In 1990, it was reported that 5% of the students saw calculators being used. In 2000, 40% saw calculators used (5% in every or most lessons, and 35% in some lessons), and 32% commented that they were freely available for use at any time. There was no question about computer availability or use in the 1990 questionnaire. In 2000, 29% saw computers used for activities requiring or developing mathematical knowledge, but none reported more than occasional use. The most common type of uses were "games" (seen by 35% of students) and "drill and practice" (16%). Some students noted that they had been encouraged to use these technologies even in classrooms where they were not observed to be used!

In Summary

The 2000 questionnaire was a broader and richer instrument than the original survey. For example, we collected data on the uses of electronic technologies and on whether (and how) the teaching experiences met the student teachers' expectations. It yielded far more data than we can report here, and we have a further cohort of students in schools as we write.

Like most research, however, it has also furnished more questions than answers. For example, it would be interesting to use broader and more controlled studies to find out if the following observations can be supported Australia wide, or even State wide, and if so to examine each one's effects on teaching and learning.

The 2000 students reported increasing:

- organisation of days into learning area "blocks";
- use of State curriculum documents in forward planning, with continuing reliance on textbooks for lesson planning and class work;
- "children talk", with lessening "teacher talk" and "child listening" (with investigation of the nature of that talk and listening necessary);
- group work and co-operative activities;

- use of composite classes but also more "ability grouping";
- "mental maths" sessions and number-fact games;
- use of children's ideas and experiences;
- encouragement for children to use their own methods;
- integration across key learning areas, especially with use of themes;
- use of commercial worksheets;
- self correction as well as correction by peers; and
- use (more varied use) of electronic technologies.

From the data we have available, some tentative hypotheses could be developed. It seems that the national framework agenda is having an impact on planning as well as timetabling. Perhaps the "constructivist" movement is leading to greater use of group work, co-operative activities, use of children's ideas and methods and discussion about these—and one could ask whether the consequent variety of methods and solutions necessitates more self- and peer-correction. A greater focus on numeracy and testing could be leading to wider use of ability groups and emphasis on number facts and speed.

There is need for further research on the effects of such changes, not only on achievement of learning objectives and mathematical understandings but also on teacher's and children's attitudes and beliefs. For example, increased availability of calculators and computer programs is undeniable, but we know little of the effects of this or of which mathematical ideas and processes are being privileged in both the teaching and the learning that takes place.

In summary, we all know that the contexts in which our students first practise teaching are changing. From the data collected, it seems that some of the aspects of our own student teachers' school experience now fit better with the ideas presented in lectures, tutorials and workshops—but then some do not.

Conclusion

To take Lave's position, student teachers are apprentices in a professional community, but are also agents in the perpetuation and reconstruction of the field and could be increasingly influential in this respect. However, this influence is framed by their perceptions of the context and its aims. Because the students themselves, as well as their knowledge of mathematics education, have been constructed within and through schooling over a period that includes the past decade, it is difficult for them to see mathematics pedagogy as a set of changing and problematic practices. As teacher educators, we need to find ways of helping them stand outside of the situation. Thus the above list of changes will be a rich resource for group discussions, assignment tasks, and further inquiry. We also maintain that this list raises questions worthy of further research.

Most importantly, student teachers need to become aware of, as well as develop the skills to analyse and critique, the various structural components and interactive features of the contexts where their learning about teaching takes place. They will then be more able to consider how aspects of school settings have framed, and continue to shape, their own pedagogical assumptions and practices.

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A Comparative Analysis of the 1996-1999 Calculus TEE Papers

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In this paper we present a comparative analysis of the 1996-1999 Western Australian Calculus Tertiary Entrance Examination papers. A classification scheme from the literature for question characteristics proved to be a useful tool of analysis. We show that the introduction of graphics calculators led to an enhanced role for diagrams and changes in the structure of questions on graphing, with implications for teaching practice.

Introduction

Since 1998 the availability of graphics calculators has been assumed for the West Australian public examinations of Mathematics, Physics and Chemistry at tertiary entrance (Year 12) level. This paper is part of a longitudinal study evaluating the impact of the introduction of the technology on the Calculus examination. Here we consider the characteristics of questions in the 1996-1999 Calculus papers, look in detail at the style of function graphing questions for the four years and consider implications for teaching practice. The analysis is in part a critique of our own practices as examiners for the 1998 and 1999 examinations.

Test and Examination Questions with Graphics Calculators

Examination questions can be graphics calculator *active* where use of the tool is necessary or greatly simplifies a solution, graphics calculator *neutral* where graphics calculator usage and traditional methods are equally viable, or graphics calculator *inactive* where use of the tool is not possible (Harvey, 1992). When the technology was introduced for the Calculus Tertiary Entrance Examination (TEE), the policy was to include questions in all these categories.

The presence of graphics calculators affects the selection of questions for examinations. Graphics calculators can *impact* on traditional questions by enabling alternative methods, *have no impact* because they contribute no more than scientific calculators to a solution, or *trivialise* questions by allowing solutions that require little or no mathematical input from the user (Jones & McCrae, 1996). A solution can also be significantly *reduced in complexity* without the question being trivialised. For an example with complex numbers see Forster and Mueller (1999). Questions can also be specially designed for technology usage, taking a different form to traditional questions and may include functions that students would not be expected to be able to manipulate by hand (Anderson, Bloom, Mueller, & Pedler, 1997).

In an examination, graphics calculators can be used as the first option to generate answers or used to check non-graphics calculator methods (Jones & McCrae, 1996). Checking might involve replication of a hand method, or use of different representations, such as using a graph to verify algebraic working. Where a non-calculator approach is needed for the written answer, students might use the tool to get started (Lauten, Graham, & Ferrini-Mundy, 1994), to help with working and with the final answer. It is expected that procedural work is off-loaded to the technology, for example, for the evaluation of definite integrals (Jones, 1996). Upon the inclusion of technology, the complexity and effectiveness of traditional examination questions must be re-evaluated and changes made to modes of testing students' understanding of some concepts. This paper explores these issues.

Research Method

We restricted our analysis of the Calculus TEE to the 1996-1999 papers on the basis that in 1996 the format of the papers changed to having questions ordered according to their degree of difficulty. Previously, the papers contained two sections. One with routine questions without interdependent parts and the other with longer questions, typically more demanding with interdependent parts. Of the four papers we consider, the 1996 and 1997 papers were set before and 1998 and 1999 papers after the introduction of graphics calculators.

For the analysis we used a coding scheme that we modified from one by Senk, Beckmann and Thompson (1997). First, we independently coded question characteristics according to the original scheme of Senk et al., then modified the scheme to suit the Calculus TEE (see Table 1). Finally, we independently re-coded the questions and, where we varied, we negotiated agreement guided by the official worked solutions.

The role of a diagram depends to a large extent on the ease with which a diagram may be obtained. Graphs can be quickly generated on graphics calculators, whereas without them drawing a graph might be impractical. When coding for 'Role of diagram', we took into account the absence of graphics calculators for 1996 and 1997 and their presence for 1998 and 1999. For interest, we coded 'Active', 'Neutral' and 'Inactive' for 'Graphics Calculator', whether the technology was available to be used (1998 and 1999) or not (1996 and 1997). Some questions belonged to more than one curriculum component and were recorded as belonging to each group.

Results

A summary of our comparative analysis for all questions on the 1996-1999 papers is given in Table 2. We illustrate the coding scheme with the analyses of the questions concerned with graphing of rational functions from 1996 and 1999. Their coding is provided in Table 3 and the questions are provided in below.

1996. Question 10. Let f be the function defined by $f(x) = \dfrac{-4x+5}{(x+2)(5-x)}$.

(a) State the poles of the function.
(b) Evaluate $\lim\limits_{x \to +\infty} f(x)$.
(c) Evaluate $\lim\limits_{x \to -\infty} f(x)$.
(d) Evaluate $\lim\limits_{x \to 4} f(x)$.
(e) State the x and y intercepts.
(f) Show that there are no turning points and sketch the graph of $y = f(x)$, clearly labeling all the important features.

1999. Question 13. If $f(x) = \dfrac{x^2+3x-10}{x^2+x-6}$,

(a) state the domain of f,
(b) evaluate $\lim\limits_{x \to 2} f(x)$,
(c) sketch the graph of f showing the intercepts, asymptotes and any other distinguishing features.

Table 1

Categories for Analysing Examination Questions

Category	Description
Curriculum component	Functions and Limits Theory and Techniques of Calculus Applications of Calculus Vector Calculus Complex Numbers
Skill	
Yes.	Solution requires a well-known algorithm such as solving equations or inequalities or bisecting an angle. Item does not require translation between representations
No.	No algorithm is generally taught for answering such questions, or item requires translation across representations
Level	
Low	A typical student in that course would use no more than three steps to solve.
Other	A typical student in that course would use four or more steps to solve.
Reasoning	
Yes	Item requires justification, explanation or proof or it is necessary to interpret the question before being able to start the answer.
No	No justification, explanation or proof is required. (By itself, 'Show your work is not considered reasoning.)
Realistic context	
Yes	The item is set in a context outside of mathematics (e.g. art, fantasy, science, sports).
No	There is no context outside mathematics.
Role of diagram	
Interpret	A graph or diagram is given and must be interpreted to answer the question.
Make	From some non-graphical representation (data, equation, verbal description) the student must make a graph or diagram.
Assist	The use of a diagram or sketch would simplify a solution, but is not essential for obtaining the answer.
None	No graphical representation is given or needed or a graph or diagram is given but is superfluous to answering the question.
[a]Graphics Calculator	
Active	Use of the tool is necessary to obtain a solution or it greatly simplifies the work needed to get a solution.
Neutral	It is possible to use the tool to obtain part or all the solution, but the question could be answered reasonably without the tool
Inactive	Use of the tool is not possible or is inappropriate.

[a] Over and above scientific calculator capabilities.

Table 2

Percentage of Part-questions in the Calculus Tertiary Entrance Examinations for 1996-1999 per [a] Major Curriculum Component by Characteristic

		Skill-based	Level	Reasoning	Context	Role of Diagram				Technology		
		No	Other	Yes	Yes	Interpret	Make	Assist	None	Active	Neutral	Inactive
Function and limits	1996	21	43	29	0	7	7	0	86	14	57	29
	1997	13	20	27	13	0	7	13	80	7	87	7
	1998	20	40	70	0	0	10	60	30	10	50	40
	1999	15	8	31	0	15	7	46	38	15	54	31
Theory and techniques of calculus	1996	27	45	27	0	0	0	27	73	27	9	64
	1997	0	23	46	8	0	0	8	92	23	15	62
	1998	10	40	20	10	0	0	0	100	10	0	90
	1999	20	40	30	0	0	0	0	100	10	0	90
Applications of calculus	1996	38	77	69	15	8	8	15	69	31	31	38
	1997	25	75	83	33	0	8	33	58	25	17	58
	1998	44	61	67	56	33	11	11	45	17	6	78
	1999	40	40	45	60	14	14	19	52	30	10	60
Complex numbers	1996	43	43	36	0	14	29	0	57	7	36	57
	1997	63	38	38	0	38	0	0	63	13	38	50
	1998	56	56	44	0	0	44	22	33	22	0	78
	1999	80	60	60	0	0	50	20	30	0	30	70
[b]% of total part- questions	1996	30	46	37	13	7	11	9	72	17	33	50
	1997	20	41	45	14	6	4	14	76	18	39	43
	1998	33	53	55	27	12	14	20	53	14	12	73
	1999	35	35	40	28	10	14	21	55	16	23	61

[a] Calculus of trigonometric functions (6hours of coursework) and Vector calculus (10hours) are not included as few part questions fall only into these categories.

[b] number of part-questions were 54 for 1996, 51 for 1997, 49 for 1998, 57 for 1999.

Table 3

Coding for the 1996 and 1999 Calculus TEE Questions on Graphing Rational Functions

		Skill	Level	Reasoning required	[a]Role of diagram	[b]Graphics Calculator
1996						
	a	Yes	Low	No	None	Inactive
	b	Yes	Low	No	None	Neutral
	c	Yes	Low	No	None	Neutral
	d	Yes	Low	No	None	Neutral
	e	Yes	Low	No	None	Neutral
	f	No	Other	Yes	Make	Active
1999						
	a	Yes	Low	No	None	Neutral
	b	Yes	Low	No	Assist	Neutral
	c	No	Other	Yes	Make	Active

Note. [a] No technology available for 1996; [b] answered as though technology was available for 1996.

Changes in the nature of examinations can be attributed to a variety of factors but changes in 1998 and 1999 that seem to be connected with the introduction of graphics calculators relate to skills assessed and the role of diagram. The effects differed with the various curriculum components. These aspects are discussed below.

Skills

For all four examination papers the majority of the questions were skills-based (see Tables 2 and 3). This pattern has not changed to any large extent upon the introduction of graphics calculators, but scrutiny of the questions showed that the skills that are tested have changed. For example, the question, 'Evaluate $(1+i)^7+(1-i)^7$ using de Moivre's rule, from the 1996 examination has been made *redundant* by the presence of graphics calculators The use of de Moivre's rule used to be essential for obtaining the answer to expressions of this type quickly--the other alternative of using the binomial theorem would have been much more time-consuming. Thus the ability to apply de Moivre's rule correctly was an essential skill. With a graphics calculator the simplification requires only a single line entry of the character string. Here, requiring students to use de Moivre's rule is *inappropriate*, just like asking students to use a calculator to find logarithms in order to solve an exponential problem (Jones, 1996).

A further change in the papers is that in 1996 and 1997 testing of integration techniques was based on the evaluation of definite integrals while now indefinite integrals are used exclusively because of the numerical integration capabilities of graphics calculators. Definite integrals have been incorporated to a greater extent in application questions. Similarly, questions that require factorisation of polynomials as the sole task are now absent in view of them being *trivialised*.

Role of Diagram

Diagrams now play a greater role in problem solutions (see Tables 2 and 3). There are more part-questions that *require interpretation* of a diagram or where students are *required to make* a diagram, and more part-questions where a diagram *would assist* the solution. This enhanced role for diagram may be explained in part by the relative ease with which students can obtain graphs of functions, parametric and polar curves and in part by the ease with which markers can generate graphs to 'follow through' students' solutions. In 1999 a question asked students to derive and graph the velocity function from the position $s(t)=(t^2+1)/(t^4+1)$. The derivation is potentially problematic, as is graphing the velocity function with traditional methods. The task of marking the question would have been arduous without having the technology available to check answers that followed-on from the velocity functions students obtained--without the technology available the above position function would not have been used in a question of this type.

Effects on Curriculum Components

The effects described above have impacted on the various curriculum components in differing ways. The summaries in Tables 2 and 3 suggests that diagrams, usually graphics calculator generated graphs, could have assisted in answering questions from the component 'Functions and limits' more in 1999 than previously. This enhanced role of diagrams went hand-in hand with a reduction of the amount of guidance given for graphing, in particular for graphing rational functions. Scrutiny of the 1996 question (see the beginning of the 'Results' section), set prior to the introduction of graphics calculators, shows that it is very highly structured. Part questions, which relied on recognition of properties pertaining to the functions and involved algebraic manipulation, led students item by item through features of the graph and attracted part marks. The final step of drawing the graph required integration of several pieces of information and so was demanding but once the graph was drawn no further interpretation was required. In 1999, two part questions warned students about features of the graph, as did the statement to show 'other distinguishing features'. Choosing to use a graphics calculator meant students needed to interpret the graph on it in light of properties previously established, rather than using the properties to plot the graph. For the identification of all the critical features of the graph students needed to integrate (Boers & Jones, 1994) their reading of the calculator graph with given algebraic information and with mathematical properties established in the previous two part-questions: a highly demanding task that was a major source of error (Forster & Mueller, 2000a).

Another aspect of students being able to graph readily was the use of more complicated functions. For example, in the 1999 examination the function $f(t)=\begin{cases}(1-\cos(2t))/t & \text{for} \quad t\neq 0\\ 0 & \text{for} \quad t=0\end{cases}$ was used to test understanding of limits, continuity and other properties of functions. Without access to a graphics calculator, either the graph would have been supplied or graphing would have dominated the question. Otherwise the requirement for a graph would have been omitted in favour of algebraic methods, thereby making the question too abstract to be a suitable examination question at this level. In its present form,, the question allowed students the opportunity to demonstrate their mathematical insight with an unfamiliar function.

Another change that became evident through the comparative analysis was that questions for 'Theory and techniques of calculus' (integration and differentiation techniques) now largely preclude graphics calculator usage (see Table 2). For 'Applications of calculus', there is no pattern of increased opportunity for calculator usage (see Table 2), but in 1998 and 1999 it is the component for which there were opportunities for flexible problem solving (Gray & Tall, 1991) with both traditional and technology-assisted methods equally viable. For example, graphical or trigonometric methods could have been used in a 1999 question where tidal fluctuations were modeled as simple harmonic motion (Forster & Mueller, 2000b).

The component most affected by the introduction of the calculators (see Table 2) is 'Complex numbers'. Questions have become less skills-based, need a greater number of steps to reach an answer, and call on more reasoning. That is, in general in 1998 and 1999 the questions were harder than those for 1996 and 1997 in all the dimensions that measure difficulty. Diagrams played a greater role, but usually these were not graphics-calculator generated--there was actually reduced opportunity to use graphics calculators in questions on the topic.

Overall, opportunities to use graphics calculators, had they always been available, have not increased on the introduction of the technology for the TEE: see Table 2, which gives the breakdown in usage according to part-questions. On the basis of part-marks, in 1996 and 1997 approximately 53% of all marks could have been obtained through graphics calculator active or neutral part-questions, this percentage was 26% in 1998 and 39% in 1999. The lower mark allocation is largely caused by omission of skills-based questions of the type included in 1996 and 1997 that would be trivial in the presence of graphics calculators. Questions set in real-life contexts doubled from 1996-1997 to 1998-1999 (see Table 2). This is partly attributable to preferences of examiners but also reflects a move away from procedural towards more interpretative questions where graphics calculator usage is an option.

Concluding Discussion

The comparative analysis of the 1996-1999 Calculus TEE papers has made explicit changes in questions that have accompanied the introduction of graphics calculators. The availability of technology has impacted on the way concepts are tested and on what skills are tested. The use of de Moivre's rule to evaluate a complex number raised to a given power has become redundant, and polynomial factorisation questions have become trivialised. Definite integrals are no longer suitable for testing integration techniques. Yet, the concepts associated with de Moivre's rule, polynomials and definite integrals remain important aspects of calculus. There is a need to rethink how to test students' understanding of them, and in the Calculus TEE this involved a greater role for diagrams for questions on complex numbers. An increased role for visual methods is a notable change for the papers overall.

We ask, is it fair that the emphasis in the papers has changed so much in regard to visual methods? Dreyfus (1994) points to difficulties that students have with visualisation. Roth and Bowen (1998) paint dismal pictures of teaching practices associated with graphing and of the resulting confusion experienced by students. However, visualisation is considered helpful in supporting intuition and concept formation in mathematics learning, and the use of technology in particular has the potential of allowing flexible thinking (Dreyfus, 1994).

In 1998 and 1999, the need to integrate algebraic and graphical information when interpreting technology-generated graphs of rational functions called on the flexible thinking to which Gray & Tall (1991), as well as Dreyfus (1994), refer. Based on a sample of 20% of the candidates in the Calculus TEE, errors were widespread in graphing rational functions (Forster & Mueller, 2000a; Mueller & Forster, 1999). The implication for teaching is that interpreting graphs on a calculator should be a subject of instruction, particularly with regard to incorporating algebraic information. More encouraging was the outcome that approximately 45% of students in the sample appeared to adopt a graphical approach for answering the 1999 question on simple harmonic motion, indicating flexible use of the technology.

Current reform documents, for example, the Curriculum Framework for K-12 in Western Australia (Curriculum Council, 1998) promote the use of real-life contexts that are meaningful to students. A move in this direction is evident in the comparative analysis. The possibility of using functions with which students are not familiar when graphics calculators are available opens up opportunities for applications that model real-life situations more realistically than in the past.

In conclusion, implications of the analysis are that some skills have reduced importance with the introduction of graphics calculators, but this does not apply to the concepts to which they relate. Other skills associated with graphical interpretation and visual methods seem to warrant more emphasis when technology is allowed.

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Using Students' Perceptions to Modify Activities

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Teachers of Mathematics are encouraged to use a variety of methods and activities in their classroom practice but situational constraints lead to their preference for routines that are conservative and involve little risk. Regardless, some teachers do try new ideas with their students and improving the efficacy of these attempts may encourage further activity exploration. This paper presents a survey and graphing template that was developed by the first author while employed in Brunei Darussalam. They help teachers obtain indications of students' perceptions following any activity. Five characteristics are considered: Understanding Content, Communication, Relevance, Work Output, and Enjoyment. Practicality is central to success. Students require about ten minutes to complete the survey in the first instance and responses from 25 students take about twelve minutes to process.

When teachers attempt to improve student outcomes through the use of different activities they meet challenges that profoundly influence their enthusiasm for similar efforts. A natural question for a teacher to ask following the trial of any activity is *How effective was it?* This presentation describes a survey developed in Brunei Darussalam that has been found useful in helping teachers look at activities from the perspective of their students. Obtaining early indications of the strengths and weaknesses of an activity increases the likelihood of a few changes having a significant effect when repeating the activity with another class, or when attempting further activities with the same class.

Calls for Change

Teachers over the world face numerous and regular calls for reform (Fullan, 1993). Unfortunately, enacting reforms and changes is clearly a difficult matter and traditional practices show a tenacious resilience to change (Deal, 1990). These calls in Brunei Darussalam are manifest in several forms. Teachers are required to attend inservice training days and in 1999 all mathematics teachers in the country were obliged to attend a national conference aimed at sharing teaching ideas. The Department of Science and Mathematics Education, Universiti Brunei Darussalam, hosts an annual international conference at which selected teachers are directed to be present. A summary report following a school inspection in 1996 contained comments that strongly emphasised the need for a broader range of imaginative and challenging teaching and learning strategies in mathematics (H. Guan, personal communication, February 1, 1997). Teachers occupy the well recognised central role if educational changes are to be effected (Fullan & Hargreaves, 1992). Efforts and good intentions of organising committees will be futile and the inspector's report remain empty grandiloquence unless teachers try new approaches in a spirit of willingness.

Situational Constraints

Any attempt at educational change must include consideration of the situational constraints under which teachers and students operate (Fullan & Hargreaves, 1992). There is great security for teachers who emulate existing practices that are both accepted and expected by stakeholders in an educational system. Praxis changes do not come with guarantees of improved cognitive or affective student outcomes so teachers who engage their students in new activities require significant intrinsic motivation.

Teachers in Brunei face conflicting forces that both promote and discourage their attempts at new activities. Teachers are commonly allocated several classes at the same level and the punctuation of the year by common tests and examinations has these classes undertaking content concurrently. This provides an incentive for teachers to attempt new activities as the same preparation can be utilised for these classes within a short time period.

Individual teachers' attempts to introduce changes to teaching and learning in Brunei schools are required to dovetail into existing assessment schedules. Results from common tests are subject to close public scrutiny and tension for both students and teachers is greatly increased in years of external examinations. These constraints encourage teachers to conform to methods of teaching expected by students and administrators, as well as inciting any praxis exploration to be small in scale. As teachers retrench in their use of various teaching and learning techniques they are in danger of stagnating in their praxis, which in turn may influence their self-perception (Hargreaves, 1988). Attempts to counter this tendency must be a positive enterprise.

Obtaining appropriate and timely information that indicates merits and weaknesses of activities between successive implementations may assist teachers to improve the efficacy of those activities. Such personal adaptations would develop ownership of the activities, a factor considered crucial for teachers' adoption of change as well as being a means of increasing teacher satisfaction (Goldenburg & Gallimore, 1991). Teachers' enthusiasm towards exploring new ideas may then be enhanced and the incremental improvements accumulate to form significant changes.

Brunei has an expanding population and teachers from foreign countries are employed to fill vacancies. Capable local teachers are required to fill necessary school administrative positions, which increases the influence in classrooms of foreign teachers whose first language and culture differ from those of their students. Modes and perceptions of communication are culturally influenced (Levy, Wubbels, Brekelmans, & Morganfield, 1994) and cultural sensitivity is vital in any educational setting. Fisher and Fraser (1983) indicate that students' preference for an activity may be as important as the activity itself in determining students' attitudes, and hence influences their achievements. The success of an activity may be negatively influenced if the participants believe it transgresses their beliefs (Deal, 1990). Determining students' reactions to a new activity is important, but can be a difficult task, especially for foreign teachers.

Students' and Teachers' Perceptions

On occasions when teachers attempt new activities they rely on their own impressions if considering changes (Hargreaves, 1992). These are convenient and will always be strongly influential. However, differences between the perceptions of even experienced teachers and their students for the same situation are quite common (Wubbels, 1993). If students' perceptions are as influential as the literature suggests, an effort should be made to obtain timely data to assist in adaptations of activities that take into account students' perceptions.

Perception Instruments. The use of surveys to obtain perception information is established in educational research (Fraser, 1982). Surveys readily provide data for processing outside class contact time, permitting teachers to concentrate on teaching during time with students rather than gathering data. Problematic linguistic differences between students and teachers can be reduced through the provision of translations, and anonymity, which promotes the collection of valid data, is easily achieved.

Practicality. George Bernard Shaw has written that "Reformers have the idea that change can be achieved through brute sanity" (Fullan, 1991, p. 97). More than a good idea backed by theory and research is required. Practicality is vital if teachers are to engage any extra process willingly. Opinions of practising classroom teachers were sought throughout the development of the survey and graphing template of this paper.

Method

The *Students' Perceptions of an Activity Instrument and Display* (SPAID) consists of a survey supported by a graphing template that uses Excel. The survey asks students to compare their reactions to an activity with what they consider to be usual for that class. It takes about ten minutes of class time for students to complete it on the first occasion, decreasing with iteration for subsequent activities. This period of time has proved useful at the end of lessons for settling a class and tidying up equipment.

Five Characteristics. Teachers have their own ideas of what is important and it is impossible to please all. Mathematics and Science teachers experienced at teaching in Brunei were canvassed for areas of students' opinions that would be of interest to them following engaging a class in an activity. Suggestions from five teachers were received, being a mixture of local and foreign, male and female, and different racial backgrounds. Comments were compared with the external examination syllabi (University of Cambridge Local Examination Syndicate, 1999) and recent research into the concerns of teachers in Brunei (Wong, Nannestad, Palanisamy, & Lourdusamy, 1999). Five characteristics were identified and these are listed in Table 1 with a brief description of each.

Table 1

The Five Characteristics Measured by the Survey with Brief Descriptions

Characteristic	Brief description
Understanding Content	How well students consider they understood the content of the activity
Communication	How well students consider they understood the instructions for the activity
Relevancy	How relevant students considered the activity
Work Output	How hard students think they worked
Enjoyment	How enjoyable students found the activity

Scales from a range of existing educational environment instruments were considered for appositeness to the five characteristics and for use with Brunei students. All were found wanting. Individual items of SPAID were then formed based upon teachers' comments. Each characteristic has three items, giving a total of 15 for the survey. This small number of items abates the time required for students to complete the survey and for teachers to process responses, but considerably diminishes the value of statistical analysis in establishing validity. The validity of the survey is obtained through considering its viability in assisting teachers to effect changes cognisant of students' perceptions.

Responses. Students respond to items, such as *I enjoyed doing this work* (Item 5) by selecting one of four Likert-type responses; *A lot more true than usual* which scores 4, *A little* *more true than usual* scores 3, *A little less true than usual* scores 2, and *A lot less true than usual* scores 1. There are two negatively-worded items (Items 12 and 15) and these are scored in reverse order. An open-ended response at the end of the survey provides students with the opportunity to add comments. The anonymity of the survey has paved the way for comments that would not have been expected from Brunei students given their traditional acquiescence to authority. Linguistic challenges for foreign teachers have been noted when students respond to the open-ended item in their local language. However, obtaining translations through local colleagues does open opportunities for collaboration.

Three Versions. Two student versions of the survey were produced to reduce linguistic difficulties between foreign classroom teachers and their students in Brunei. The first is in a local language and the second version is in English (Appendix), the official language of mathematics instruction in Brunei secondary schools. Both versions are A4 in size and printing these on opposite sides of the same sheet of paper gives students a choice in response. Teachers are encouraged to use the Teacher Version to record their predictions prior to processing students' responses.

Computer Graphing. Manual computations are possible but there is a limit to the effort of even enthusiastic teachers. The Excel graphing template that accompanies the survey requires about twelve minutes to enter results from a class of 25 students and produces five pages of printouts. Responses from the students and the teacher are entered in the first page of the template and generate the graphs on the other pages. Page 2 gives two line graphs showing the average responses for each of the five categories; one graph compares students' responses with those of the teacher and the other contrasts male students' responses with female. The third page provides bar graphs of collated totals for each of the five categories. Page 4 provides the frequencies of responses for each of the 15 individual items and Page 5 breaks these responses into male/female.

Results

Teachers who have used the survey and computer displays have been positive about the information they received. Different information is useful for different situations and will always be balanced against the teacher's existing knowledge and professional expertise.

Example

Below is an example of how information from the survey helped improve the impact of one activity with two classes. These findings are neither startling nor dramatic. Such attempts to improve the efficacy of small individual events are grounded in what Goldenburg and Gallimore (1991, p. 69) describe as "the mundane but very real detail of teachers' daily work lives", considered to be a component of successful professional teacher development.

The students came from two lower ability Form 4 classes of sizes 19 and 21 within a boys-only school in Brunei Darussalam. The students and teacher completed the survey following a mathematics activity outdoors and Figure 1 gives the summary of responses for the first class. Students' average responses suggest that they considered the activity favourably compared with their perception of normal mathematics classes. Significant difference between the students' perceptions of the activity and the teacher's estimation of their perceptions is immediately apparent in the two categories of Understanding Content and Enjoyment. When combined with the close estimation of Communication, this implied to the teacher that students understood what they were to do but had trouble going about it. The gap between Enjoyment responses for students and the teacher prompted closer observation of students' enjoyment in future.

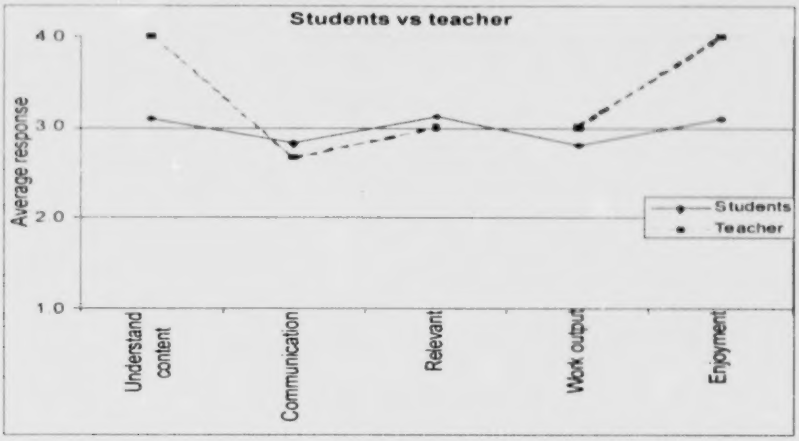

Figure 1. Summary of responses of the first class and teacher for the five categories.

The second class to engage in the activity was less academically able than the first and the teacher considered the students less likely to connect the relevance of the activity with the mathematics. Responses from the first class prompted more emphasis with the second class on showing how to go about the activity, breaking the steps into even smaller stages and demonstrating each in turn. These were simple to effect but would not have been considered without cognisance of students' perceptions.

Figure 2 shows the summary of responses for the second class. The average Understanding Content result of the second class was higher than the teacher estimated, suggesting that the students may have benefited from the changes made. The teacher still overestimated students' enjoyment of the activity but the biggest surprise was the underestimation of the relevance that students placed on the activity. This response provided further incentive to try other new activities with the class.

Possible Uses

Obtaining indications of students' perceptions following engagement in activities through a simple, quick and easy-to-use instrument may encourage teachers to temper their adaptations of those activities cognisant of students' perceptions. Information from SPAID can contribute to the cross-sectional development of a single activity with several classes within a short period of time, or can be used longitudinally to guide the development of several activities with a single class over a longer period.

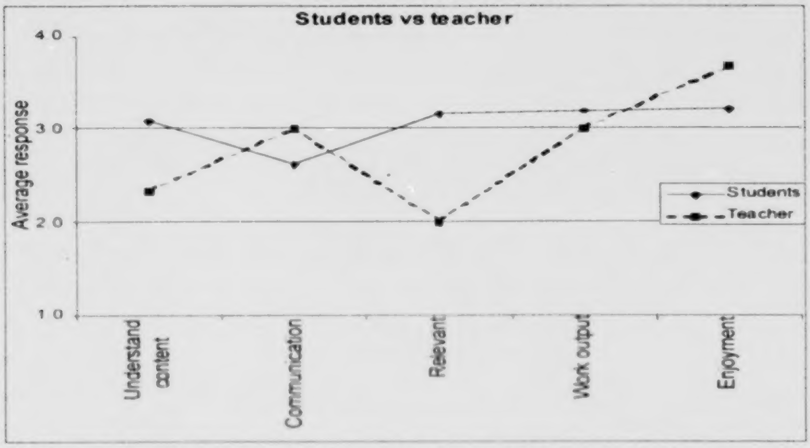

Figure 2. Summary of responses of the second class and teacher for the five categories.

Summary

The combined survey and graphing template that constitutes SPAID provides teachers in Brunei with an insight into their students' perceptions following the use of an activity. Responses require less than ten minutes of class time to obtain and twelve minutes to process using the computer template. Consideration of graphical displays and students' responses to the open-ended item can indicate adaptations to increase the efficacy of activities. These can be cross-sectional between iterations of the same activity with several classes, or longitudinal while introducing one class to several activities over a longer period of time.

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Appendix: English Version of the Survey

SURVEY

Class ....

Date ....

Activity ....

Male / Female

This is not a test. No names are required and there are no right or wrong answers.
Your teacher will use your responses to improve class activities so please answer honestly.

Your friends will give their own answers so do not copy what they put.

DIRECTIONS

Below are some statements. As you read them think about the activity you did. How do you feel about that activity compared with what you consider to be usual for this class?

If you think the statement is **a lot more** true than usual, circle **4**.
If you think the statement is **a little more** true than usual, circle **3**.
If you think the statement is **a little less** true than usual, circle **2**.
If you think the statement is **a lot less** true than usual, circle **1**.

If you change your mind about an answer, just cross it out and circle your new answer.

	Remember to circle how you feel about the activity that you have done.	A lot more true than usual	A little more true than usual	A little less true than usual	A lot less true than usual
1U	The activity helped me understand what I need to learn.	4	3	2	1
2C	I understood the instructions that the teacher gave.	4	3	2	1
3R	The activity that I did is useful.	4	3	2	1
4W	I tried my best to do the work.	4	3	2	1
5E	I enjoyed doing this work.	4	3	2	1
6U	This activity would help me explain the work to a friend.	4	3	2	1
7C	I understood how I had to do the activity.	4	3	2	1
8R	The activity I did will help me in the subject.	4	3	2	1
9W	I was happy to do the work because it was useful.	4	3	2	1
10E	I want to do more work like this.	4	3	2	1
11U	This will help me remember what I need to know for my tests.	4	3	2	1
12C	I was confused about what we were to do in the activity.	4	3	2	1
13R	What I did in the activity is useful for doing tests.	4	3	2	1
14W	I wanted to do the work for this activity.	4	3	2	1
15E	The activity was boring.	4	3	2	1
	I feel the activity we did would have been better if				

Thank you for completing this form and for your honesty.

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A Secondary Mathematics Teacher Explains his Non-Use of Computers in Teaching

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This study uses qualitative research methods to examine why a senior secondary teacher who was experienced in teaching mathematics and using computers would not use either generic software such as spreadsheets, or exploratory software (e.g. *Maths Helper*) in his teaching of mathematics. The teachers' beliefs about the nature of secondary mathematics, the nature of teaching, his own teaching practice and assessment issues underpinned his rejection of the use of computers in the classroom.

Internationally, there has been considerable support for the use of computers in mathematics teaching and learning (Gentile, Clements, & Battista, 1994; Kaput, & Roschelle, 1997). A number of authors have argued that teachers' beliefs about mathematics and learning are critical in making decisions as to the use of computers in their mathematics teaching (e.g., Henry & Clements, 1999; Sarama, Clements & Henry, 1998; Thompson & Thompson, 1996). Other researchers have linked teachers' preferred pedagogy with willingness to use computers (Ernest, 1992; Galbraith & Chant, 1990; Neyland, 1996; Sarama et al., 1998). The importance of assessment procedures in influencing teachers' practices has been recognised (Barnes, Clarke & Stephens, 1996; Becker, 1994). More specifically, assessment has been reported to have an effect upon teachers' use of computers and graphics calculators (Croft, 1998; Tobin, Routisky, & Jones, 1999). This paper explores the reasons why one senior secondary teacher does not use computers to teach secondary mathematics. It examines the importance of the teacher's beliefs in determining his responses to the use of computers, particularly exploratory software such as *Maths Helper* (Vaughan, 1997).

Theoretical Framework

Norton (1999) has described two orientations to the use of computers in mathematics teaching and learning. The first is *calculational* where computers are primarily seen as instruments to carry out procedures for deriving results (e.g., calculating numerical result or quickly graphing data). The second is *conceptual* where computers are used to build conceptual understandings by focusing students' thoughts upon the underlying mathematical ideas and relationships between these ideas.

Ernest (1996) provided a useful framework for describing beliefs about the nature of mathematics. His framework has two opposing poles, the poles being represented by *absolutist* and *fallibilist* beliefs. Ernest contended that teachers with absolutist beliefs were likely to view mathematics as objective, absolute, certain, incorrigible and based on deductive logic. Thus, such teachers would see mathematics as fixed, logical, inhuman and objective. By contrast, teachers with a fallibilist view of mathematics were likely to see mathematics as the outcome of a social process and, according to Ernest, see mathematics as open to revision. He contended that teachers with fallibilist beliefs frequently had a pedagogy associated with person-centred and humanistic approaches to teaching.

Kuhs and Ball (1986) identified four different teaching approaches. The first is *content-focused (performance)*, where the emphasis is on student performance (this has also been described as instrumental (Skemp, 1978; Thompson, Phillip, Thompson, & Boyd, 1994). The second category is *content-focused (understanding)*, which is similar to content-focused but with an emphasis on understanding the ideas and processes rather than performance. The third category is *classroom-focused*, where the main concern is that classroom activity be well structured and organised according to effective teacher behaviours. The final category is *learner-focused*, which is an approach consistent with constructivist theories of teaching and learning, namely where the learner is engaged in exploring and formulating ideas and actively constructing their own knowledge from the environment (Lucking, 1999). The latter approach also recognises the importance of social interactions (Cobb, Yackel, & Wood, 1992).

Method

The study was a qualitative educational case study (Stenhouse, 1990) involving a hermeneutic approach (Denzin & Lincoln, 1994). The design of the study and the analysis of the data heeded the criteria for the constructivist inquiry of Guba and Lincoln (1989). Techniques such as member checking, peer review, triangulation of multiple data sources, testing of developing assertions on emerging and previously collected data, and actively seeking negative instances, were used to enhance the trustworthiness and authenticity of the data.

Subject and Context

The teacher, (pseudonym Peter), was very experienced having been a teacher of mathematics for 23 years. He was highly computer literate and was responsible for overseeing the school administration computing resources, including programming and networking. Yet he did not use computers in his mathematics teaching.

Peter had extensive experience in software that has been used in mathematics education such as spreadsheets, LOGO, function plotters and statistics software. In addition he had used Structured Query Language to construct data base shells for the school, constructed the school web page and set up spreadsheet templates for analysing the school sports results. His formal qualifications included a Bachelor of Science degree, a Diploma of Education (a post-graduate preservice teaching award), a Graduate Diploma in Education (Computer Education) (an inservice award in computer education) and a Master of Information and Technology degree. He taught both computing and mathematics in his technology-rich suburban secondary school at the time of the study - information processing and technology (IPT) to Years 11 and 12 students and mathematics to Years 10 and 12. He stated that personal lack of expertise in using computers was not important at all in influencing his use of computers in his teaching. After acknowledging that the school was technology rich, he stated that, "if he wished to, he could use computers quite extensively in his teaching of mathematics".

Data Gathering Methods

Data was gathered using the following procedures and instruments:

- structured interviews, that explored Peter's experience, beliefs about the potential of computers in teaching mathematics, beliefs about secondary mathematics, and beliefs about teaching and learning and specific pedagogical practices
- lesson observations
- artefacts collection (students' workbooks, assessment items, Peter's web site publications on student learning.)
- member-checking interviews (Guba & Lincoln, 1989).

The first author volunteered to teach two of Peter's classes with the exploratory mathematics software *Maths Helper*. The first author introduced Peter to the software and showed him lesson plans that the first author had prepared to teach the algebra concepts that Peter's classes were about to study. *Maths Helper* contains mathematics tools that enable students to explore mathematics concepts and engage in mathematical modelling; for example, a line tool that can be placed on any curve to give the linear function of the tangent at that point, a maximum and minimum tool, an angle tool that gives readings in degrees and radians, a point tool, and an intersection tool. In addition, the software can calculate values and display them in tabular form, plot and calculate derivatives, plot integrals, normals, and lines of best fit for most functions that students encounter in secondary mathematics, and also determine correlation coefficients. The software can be used conceptually since it allows students to use the tools above to explore mathematical concepts, particularly algebra and calculus concepts.

Results

The results are presented as assertions. Excerpts of Peter's "voice" are presented as supporting evidence.

Assertion 1

Peter used computers rarely in his teaching of mathematics because he saw their potential as calculational and he believed that they did not facilitate student conceptualisation. Peter reported that using computers in mathematics teaching was "less effective than traditional methods in developing higher order thinking". However, he argued that they could be used to "do more complex situations where large numbers were involved" and "for crunching numbers". He stated, "I have not seen software that enhances students' mathematical thinking skills".

Assertion 2

Peter's beliefs about the nature of secondary mathematics were absolutist in nature. He believed that student use of the exploratory software such as *Maths Helper* was not necessary. He stated that school mathematics was essentially absolute since "virtually all (mathematics) done at school level is well developed… it would be a rare occurrence for a student to develop any new mathematics". Further, he had a low evaluation of students attempting to model in secondary school, "Mathematics modelling done in secondary school is fabricated, artificial and simplified … (because) students are nowhere near the level (mathematically), which is needed to look at real life situations." He stated that secondary mathematics courses were for preparing students for tertiary mathematics by developing the concepts and skills that could be built upon at a university level, so that students could begin to explore real models there. He added that the reason school mathematics should be treated with certainty "was because of the limited sophistication of thought of adolescence". Peter commented that using software such as *Maths Helper* to help students explore mathematical ideas "could waste time trying to rediscover things" that he "could explain quickly and besides they might get the wrong idea". In short such activities, "were not necessary for the development of basic skills and knowledge". It should be noted that most of Peter's post teacher training study was related to Information and Technology study rather than mathematics education.

Assertion 3

Peter believed that computers were not useful in the explaining mathematics concepts to students nor in illustrating examples of mathematics procedures. He claimed that teacher explanation and student practice of procedure was the most effective way to teach mathematics.

> I would be extremely explicit and I would be explaining things, the knowledge is coming from me so I believe that I am explaining it. I am a great believer in example, learning by example, learning skills and you actually learn skills from examples. Obviously it needs prerequisite knowledge and if you know the prerequisite knowledge you can just extend, extend slightly for people's area (of comfort) to be covering a new concept, then the students will be happy.

Peter's explanations gave insight into his image of teaching as well as his role as a teacher. He stated that students "should be able to perform basic skills and have the theoretical background to use numerical methods with understanding". The statements above support the conclusion that Peter had a content-focused (performance) orientation towards teaching (Kuhs & Ball, 1986). In aiming to achieve this goal he rejected learner-focused approaches since he believed that students would be like him and "much rather have an expert to help me quickly than spend hours trying to work out a solution for myself". He believed that software, including the exploratory software *Maths Helper*, could not aid in this process. As he explained, "I don't think it (computer technology) should be a substitute for theoretical approaches". Lesson observations showed that his practices supported his statements - he taught by example and explanation.

Peter compared the learning of mathematics to music learning. He said mathematics was "like a performance and, as in music, learning required that the student practise until the task could be done fast and faultlessly". He stated: "I believe it is important to drill, in that here is a need for doing the same thing so many times it becomes second nature". On his school web site, he noted:

> Teachers often convey information to the full student group orally, with perhaps the aid of visual cues. Teachers often give instructions and explanations using this technique because it can be a very efficient and economic way of communicating with a group of students. Listening should occur while performing no other tasks, apart perhaps, from taking notes on what is being communicated. It is imperative that all students listen to the teacher when required. Students need to practise their mathematics even after they have had initially had success at a task, just like a musician practises a tune after she as learnt it.

Assertion 4

Peter's beliefs about assessment supported his non-use of computers. Peter believed that assessment should be in the form of tests or examinations and this strongly influenced his teaching. For example, he stated: "Assessment is the determining factor of what we are on about and my teaching is actually geared towards assessment ... we have to get students to jump through those hoops". His view of assessment had implications for his use of computer technology:

> Well, like it or not, I believe that there is in Queensland, at the moment, a test called the QCS (Queensland Core Skills) (Computers are not part of the QCS assessment). So students have to become competent at pen and paper work, without a doubt that is the way to go if you are not going to disadvantage people. If I wanted I could actually quite easily spend a lot of time using computer technology. (However) I think I'd be doing the wrong thing by them ... I don't use computers much because of the syllabus, and the outcomes that are needed by students to succeed on a long-term basis in our society ... I don't think I have seen software which enhances the vast majority of students' knowledge and skills, that are required by out syllabus or assessed.

In short, Peter's interpretation of the syllabus assessment criteria supported his non-use of computers in his mathematics teaching. In actual fact, the syllabus recommended an "investigative" approach to teaching and learning and encouraged the use of computers in mathematics learning (Board of Senior Secondary School Studies, 1992). Peter would not use learner-focused and computer orientated approaches nor would he allow the first researcher to model this teaching with his classes because he not believe that such approaches would not be as effective as his own teaching in helping students pass assessment tasks.

Conclusions and Implications

Peter had an essentially calculational orientation in relation to the potential of computers in secondary mathematics learning. He did not believe that computer use (including spreadsheets and exploratory software) could compete with his existing teaching strategies in developing students' abilities to do and understand mathematics. His reasons for not using computers in his teaching of mathematics related to his beliefs about the nature of secondary mathematics, teaching, learning and assessment.

Peter's image of secondary mathematics was absolutist in nature in that it consisted of well-established facts and procedures that students needed to learn and practise. Thus, the potential of exploratory software (such as *Maths Helper*) which could empower students to construct mathematical knowledge, particularly by modelling mathematical concepts and phenomena, was largely irrelevant to Peter. In fact he stated that using such software could waste valuable time, which could be better used listening to the explanations of an expert and by practising demonstrated skills.

Peter's image of teaching and learning was primarily one of transmission where mathematical skills and knowledge are transmitted from the teacher to the learner (Atweh & Cooper, 1995; Perry, Howard, & Tracey, 1999). His understanding of constructivist learning theory appeared confused in that he appeared not to distinguish between student construction of new mathematical knowledge and student construction of their own understandings of mathematical knowledge that had been stipulated in the mathematics syllabus. His understandings of constructivist theory may have been one reason why he preferred a transmission approach to teaching. Peter's instrumental teaching orientation of content-focused (understanding) (Kuhs & Ball, 1986) was consistent with his beliefs about teaching and learning. His perceptions of the nature of teaching and learning mathematics may well have influenced his perceptions in relation to the potential of software to help students understand mathematics. Finally, Peter's interpretation of the syllabus assessment criteria served to support his existing beliefs and practices. In summary, Peter did not believe it was in the best interest of his students to require them to use software, particularly exploratory software, in their learning of mathematics.

This case study illustrates that beliefs about the nature of secondary mathematics, teaching and learning and interpretations of assessment requirements can act to reinforce each other to create strong resistance to the use of computers in mathematics teaching. The implications for the professional development of teachers are that attempts to encourage teachers like Peter to consider the potential of using software to teach mathematics needs to be multi-dimensional and consider teachers' existing beliefs and practices. That is, if reformers wish teachers like Peter to use computers in their teaching, it needs to be shown that the use of software such as *Maths Helper* could help students to conceptualise mathematical abstractions more effectively and efficiently than traditional practices.

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A Framework for Teaching Early Measurement

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This paper describes a framework for teaching length, area and volume in the early years of school. The framework is based on recent research findings that indicate the importance of students' knowledge of the unit iteration structure. The results of interviews with a sample of 16 Sydney teachers about their understanding of area measurement indicate that they place little importance on structuring in their teaching of measurement. Lessons from the framework were given in four schools and worksamples based on these are discussed.

Introduction

Measurement concepts are introduced in Kindergarten, then developed and extended throughout the primary years. However, students will not learn to estimate and measure successfully if they have inadequate understandings of measurement attributes and processes. They also need to be aware of key differences between measurement and number concepts, in particular, the notions of continuous quantities and variability. Measurement concepts have been found to develop later than number concepts because of the difficulty of dividing a continuous whole into equal sub-units as opposed to counting a set of discrete objects (Piaget, Inhelder, & Szeminska, 1960).

Length concepts are particularly important, as length is usually the first measurement process that students learn about. If students do not have a relational understanding of linear units they are unlikely to succeed in more complex areas of mathematics; for example, area and volume in which length units are extended to two and three dimensions respectively (Battista & Clements, 1996; Outhred & Mitchelmore, 2000).

The importance of understanding the development in the early years has been stressed by Hershkowitz (1990), who says that:

> It is surprising that most of the research that has been done in the last few years has concentrated on 4th to 10th grade (9 to 15 year-olds). It seems as though researchers try to learn about processes that start very early by investigating their traces at more advanced stages. (p. 93)

She listed the following stages through which meaningful learning of measurement develops: (a) conservation of the quantity being measured; (b) the meaning of the measurement unit and unit iteration (arbitrary units, standard units, correct use of measurement tools); and, (c) formulae for calculating the quantity being measured. However, some evidence suggests that the first two aspects might be mutually supportive and that it is not necessary to have a knowledge of conservation before beginning measurement activities (Nunes, Light, and Mason, 1993). Although Hershkowitz lists units and iteration, teachers seem to have paid little attention to the structure of the unit iteration.

The aim of this paper is threefold:

- to propose a framework for teaching measurement of length, area and volume that is based on theoretical research and that emphasises the structure of iterated units;
- to provide information from interviews with teachers about their teaching of area measurement;
- to provide some worksamples of students involved in initial classroom evaluations of the Framework.

Measurement Units and the Structure of the Iteration

Although length, area and volume measurement is taught in primary school there is a considerable body of evidence showing many secondary students do not have a thorough knowledge of these concepts (Carpenter, Lindquist, Brown, Kouba, Silver, & Swafford, 1988; Hart, 1993; Hart, Johnson, Brown, Dickson, & Clarkson, 1989; Schwartz, 1995). More recently, results of the international comparisons of mathematics (TIMMS) have indicated that Australian secondary students also have problems with linear measurement (Lokan, Ford, & Greenwood, 1996).

Commonly reported errors in these studies involve inadequate understanding of:

- the attribute being measured;
- what is counted when informal units are used;
- the use of measurement instruments.

Students first learn to compare quantities on the basis of perceptual judgements to obtain a sense of what a length, area, or volume is. Only gradually are perceptual judgements amended by cognitive intervention and fundamental to this shift would seem to be knowledge of measurement units. Concrete materials and practical activities have been widely recommended but their use has not always been found to be effective because students may not grasp the relationship between different forms of representation (Hart, 1987).

For example, research on students' area concepts suggests that the structure of the rectangular array is not intuitively obvious to students (Battista, Clements, Arnoff, Battista, & Borrow, 1998; Outhred, 1993) and therefore, they may not perceive the structure of the unit iteration. When students make arrays with concrete materials, such as cardboard tiles, the tiles structure the array whereas to draw an array requires knowledge of its structure. Thus, the step from making an array using concrete materials to representing it pictorially may be a very large one. Emphasising the structure of the unit iteration may assist students to link concrete, pictorial and symbolic representations of measurement concepts.

The concepts of a unit and unit iteration are basic to any understanding of measurement and students will not be able to partition a region into equal parts if they cannot anticipate what the partitioned region will look like (Hiebert, 1989). Students who can draw arrays seem to partition the rectangle by constructing composite units based on rows and columns. Therefore, although it is important to be able to visualise the partitioned region it is also important to have some knowledge of how such a region might be constructed. The results of studies by Outhred and Mitchelmore (2000) and Battista et al. (1998) suggest that the structure of a unit iteration of squares is not well understood by students.

Moreover, students who use multiplication to determine the number of elements in the arrays usually draw arrays using some lines, suggesting that the construction of composite units and the delineation of these units as single entities (by representing their boundaries by lines), are in some way related (Outhred, 1993). A strong relationship was found between measuring the sides of a rectangle and drawing an array using lines. A possible explanation is that drawing lines helps the students to connect the tessellation of unit squares that covers the rectangle with the lengths of its sides. Outhred suggests that students do not realise the significance of the side lengths unless they can visualise and draw arrays as an iteration of rows and columns.

Similar findings emphasising the unit iteration structure have been reported in research on students' understanding of volume (Battista & Clements, 1996; Hart et al., 1989). Volume concepts involve a higher level of visualisation than length or area as some units will be hidden by the layer structure. Campbell, Watson and Collis (1990) investigated the strategies that students in Years 2 to 6 used to find the number of cubes in pictured regular rectangular prisms. Nearly half the problems were solved by counting individual cubes and many students counted only visible cubes. Volume comparisons are more complex than area or length ones because of the difficulty in coordinating three dimensions. Children need time to learn how to decompose a three-dimensional array into layers, and to progress from repeated addition to multiplication as a method of enumerating the unit cubes.

Hart (1989) found that a number of factors contributed to the students' difficulties in understanding the formula for volume of a rectangular prism. Some students filled the containers with blocks then counted them. Although teachers referred to centimetres, students may not have realised that they were referring to the edges of the block; for example, some students measured length as cm^3 because they counted blocks.

Introducing measurement instruction with informal units has been suggested as a way of helping students to see the relationship between the 'continuous' nature of length and the 'parts' that may be counted (Wilson & Osborne, 1988). Australian curriculum documents (Australian Education Council, 1990) recommend the introduction of non-standard units of area before standard units and the measurement of irregular shapes before the measurement of regular geometric shapes.

However, the nature of these parts (or units) that are counted may not be obvious, and problems may arise from the use of informal units unless a fundamental aspect of area measurement is made clear—that the region should be covered without overlap or gaps. Moreover, the use of nonstandard units does not emphasise the structure of the iteration. When learning about area, for example, students should first be given tasks that emphasise the relationship between covering rectangular shapes with square units (Bell, Hughes, & Rogers, 1975). These authors found that early use of irregular shapes, with the associated complexity of fractional units, did not enhance students' concept of area.

The structure of the iteration of length, area, and volume units does not seem to be emphasised in textbooks and curriculum documents, yet it would seem to be central to understanding and linking these concepts. Simon and Blume (1994) found that preservice student teachers needed considerable experience before they began to understand the area of a rectangle as a quantification of surface and to see how the linear measures determined the shape and size of the array. If preservice students have difficulty understanding the structure of a tessellation, then they are unlikely to understand the structure of a three-dimensional packing of cubic units.

Methodology

The K-2 Framework for Learning Measurement has been developed for the Curriculum Support Directorate, NSW Department of Education and Training by the authors. It comprises a theoretical framework, activities, and example lesson plans. A draft document has been released to mathematics consultants for evaluation.

The second researcher interviewed teachers as part of a larger study of professional development models. Sixteen teachers (fifteen females and one male) were individually interviewed in the middle of the school year about their teaching of area concepts. Most of them (11) had more than five years teaching experience; three less than 10 years, four between 10 and 20 years, and four more than 20 years. Seven of the teachers were teaching Year 2, five Year 1, two Year 1/2, one K/1, and one Kindergarten. They were asked about lessons taught, the materials they used, what skills and understanding they were trying to develop, what they believed was the best way to teach area, and was knowledge they thought students would need in three years' time.

Lessons from the Framework were trialed in four schools in a low to medium socio-economic area of Sydney. Some examples of student worksamples were collected to show how students solved a selection of the tasks.

A Framework for Early Measurement Concepts

The Framework (see Figure 1) was designed to provide a sequence of similar conceptual levels for length, area, and volume, prior to the introduction of formal units. The three levels are: (1) identification of the attribute; (2) informal measurement; and, (3) the structure of the iterated unit. The third level does not seem to have been specifically included in measurement teaching programs. However, recent research (Outhred and Mitchelmore, 2000; Battista et al., 1998; Bragg & Outhred, 2000) suggest that this step is crucial because it is not understood by students. These levels provide a conceptual sequence for teaching the content of each strand—students are not expected to be at the same level in each strand.

Table 1

The Conceptual Framework for Learning Measurement

	Length	Area	Volume
Identification of attribute			
Early Level 1	Identify objects of equal length.	Identify congruent areas (possibly after cutting).	Identify volumes (fill or pack containers).
Later Level 1	Direct comparison of lengths.	Direct comparison of areas (e.g., by superimposing).	Direct comparison of volumes (e.g., by filling or packing).
Informal Measurement			
Early Level 2	Find how many identical units fit along a line.	Find how many identical units cover an area.	Find how many identical units fill/pack a container.
Later Level 2	Use numbers of units to compare lengths.	Use numbers of units to compare areas.	Use numbers of units to compare volumes.
Structure of repeated units			
Early Level 3	Replicate a given unit to fit along a line.	Replicate a square unit to cover rectangular areas.	Know how cubic units pack into rectangular containers (layers).
Later Level 3	Relate unit size to the number of units.	Relate unit size to the number of units.	Relate unit size to the number of units.

The Framework document includes a range of activities and sample lesson plans designed to help teachers to link their teaching of measurement concepts across length, area, and volume. It provides several different but related activities at each Level involving practical applications of estimation and measurement. Regular shapes are emphasised because we feel it is important that students develop an understanding of the structure of unit covering in area and unit packing in volume.

The design and presentation of the Framework make it easily accessible to classroom teachers of students in Kindergarten to Year 3. Each of the Levels includes a list of key strategies describing skills and understandings which students may demonstrate at that level. The learning experiences in each Level are referenced to the NSW Mathematics K-6 Syllabus, the Outcomes statements and the strategies to be demonstrated at that level.

The suggested experiences in each level include whole class demonstration and discussion lessons, as well as small group and individual activities. Lesson notes illustrate modelled mathematical language and teacher questioning within a structured lesson. The lessons also show how to introduce the attribute, key aspects of the measurement process, and recording and reporting back to the whole class.

Principles on which the Framework is Based

Students need to identify the attribute before they can directly compare quantities or measure it (Level 1). They also need to know that the quantity is unchanged if it is rearranged (conservation); that lengths, areas and volumes can be subdivided into identical parts (units); and that the number of units used gives a measurement of quantity (Level 2). A key understanding of the measurement process is the repetition (or iteration) of units. Unit iteration involves knowledge of repeatedly placing identical units so that there are no overlaps or gaps. For example, accurately aligning units along a length, constructing an array of units to measure the area of a rectangle, or packing or filling a container to find its volume. Another key understanding is that individual units must be equal in size and they can be combined into composite units, for example, a row of tiles, or a layer of blocks. Experience with composite units will help students to learn that fewer large units are needed to measure a quantity, but smaller units give more precision. Composite units can be used in calculations (and linked to early multiplication, eg. four rows of three squares).

Explicit links must be made so students see how the nature of the units changes from one dimension (length) to three dimensions (volume). For example, diagrams in textbooks use spaces and marks or lines to show length units but students may not know that the spaces on a scale represent unit lengths and that the marks delineate the end-points of the unit centimetres. Such knowledge is important if students are to use scales effectively. Similarly, students may not realise that lines on a grid show the edges of the tiles covering the region or be able to interpret two-dimensional diagrams of cubes.

Results and Discussion

The Interviews with Teachers Concerning Area Concepts

The interviews with teachers revealed their knowledge of area concepts they felt it was important to teach. By mid year the 16 teachers had taught a total of 19 lessons on area while two had not taught any area lessons. The content of the lessons they had taught is categorised in Table 2.

The descriptions teachers gave of the lessons they taught indicates that they conceive "area" to be a process of covering and counting, rather than of subdivision of a surface. Such an emphasis may be detrimental when students move away from concrete materials to more abstract representations that require knowledge of how to subdivide a region (area) or a three dimensional space (volume). No teacher mentioned any structural features of a covering (e.g., gaps or overlaps, congruent units, rows or columns). Even when teachers were specifically asked about skills and understandings these features were not mentioned. Most teachers thought students needed to learn that area is a covering (ten teachers)—only three used the term "surface". One teacher mentioned that the same units should be used for comparison and that an appropriate size unit should be considered.

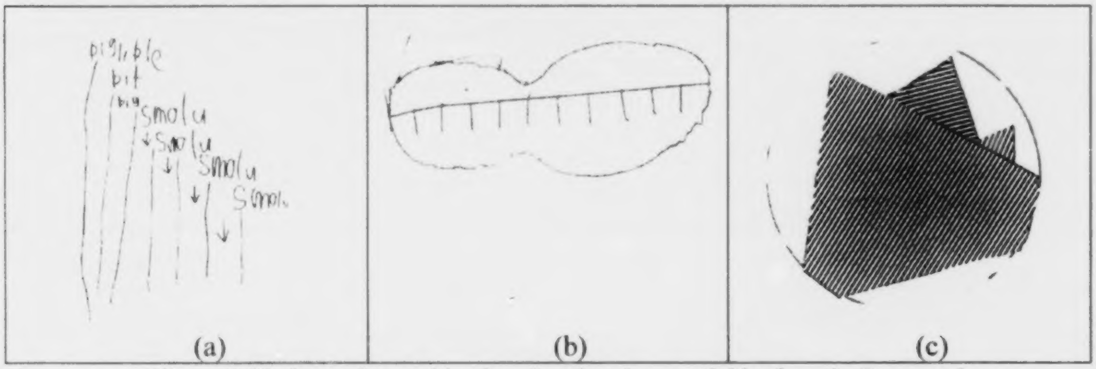
Table 2
Lessons Taught by the Teachers in the First Part of 1999

Open & closed shapes	2	Covering (with 3D objects)	4
Choose shapes and cover	2	Covering (with approximate 2D objects)	4
Covering (unspecified)	3	Other	4

Four teachers listed only two-dimensional units. Three–quarters of the group mentioned three-dimensional units—blocks, flats, centicubes, shoes, pencil tins, books, bodies, rods, unifix. One mentioned counters, a circular unit, while others used either a mix of 2D and 3D units or 3D materials only. The use of three-dimensional materials, especially blocks, may confuse students about the nature of the units and they may not know whether to focus on an edge, a face or the block itself. Even use of two-dimensional units did not seem to be associated with structural features of covering. For example, in one lesson students traced outlines of themselves onto paper, then covered the shape with envelopes.

The Implementation of the Framework in the Classroom

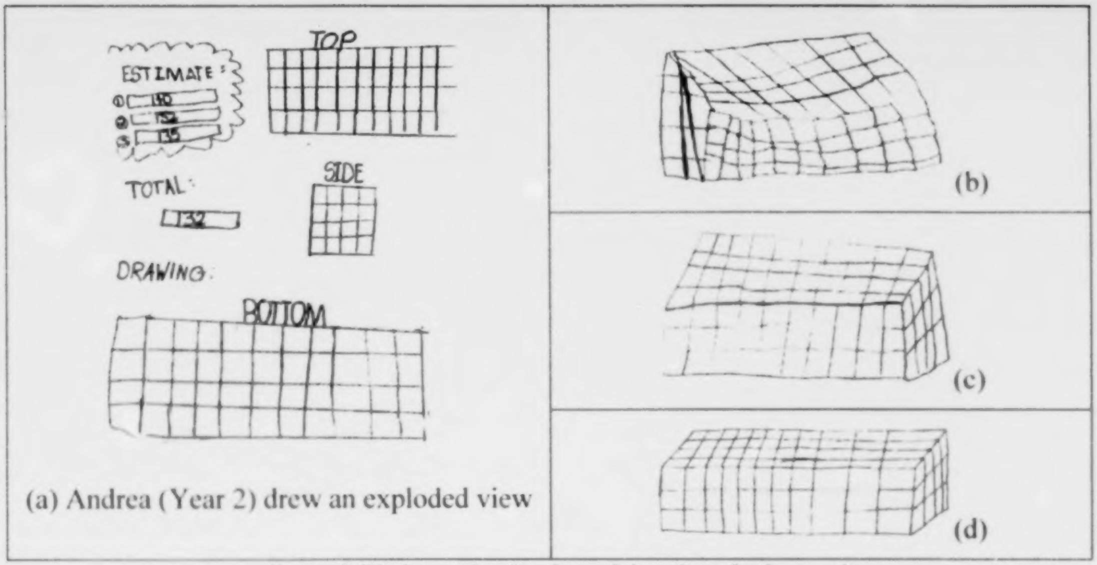
Work samples were obtained for only a limited number of lessons on length, area and volume. The worksamples have been included to illustrate how young students might engage with activities focussing on the attribute being measured and the structural features of the unit iteration. Figure 1(a) and 1(b) illustrate (a) ordering of length from "big" and getting "smolu" (Kindergarten), and (b) unit iteration of linear units (Year 1). The teacher emphasised in (a) that the lengths being compared must have the same base line; and in (b) that the length (a line) is being measured and how this length is subdivided into equal units. Figure 1(c) show s direct comparison of area (Year 2) by cutting and superimposing.



(a) (b) (c)

Figure 1. Work samples resulting from length and area activities from the Framework.

Figure 2 shows Year 1 and 2 students' drawings of the structure of the unit iteration for the volume of a rectangular prism. Students packed containers with blocks before making the drawings, which indicate that these young students had a good knowledge of the structural features of volume. Figures 2(a) and 2(d) were drawn by Year 2 students, 2(b) and 2(c) by Year 1 students. These students had previously been involved in a sequence of six lessons on the structural features of an array as part of a study undertaken by the second author giving them a basis on which to build volume concepts.



(a) Andrea (Year 2) drew an exploded view

Figure 2. Work samples showing unit iteration of volume units.

Conclusion

The Framework for Learning Measurement presented in this paper is based on recent theoretical research in measurement. The Framework includes a step that we believe to be crucial to learning measurement—the structure of the unit iteration. We believe that if students do not understand the measurement unit and the structure of the unit iteration they will not be able to progress to symbolic representations of measurement quantities.

The interviews with teachers suggest why students may have little understanding of fundamental properties of measurement units and their structuring in terms of aligning, covering or packing of length, area and volume respectively. Although the sample was a small one, only one of the 16 teachers mentioned features of the measurement unit ("appropriate size and the same units for comparison") and none mentioned the structure of the unit iteration—all seemed to think of area as "covering" something. The use of three-dimensional units may further confuse students about the nature of the attribute they are measuring. Results of a recent study (Bragg & Outhred, 2000) show that, although by Year 5 most students can measure a lines or draw a line of a given length, only high-ability students have a conceptual understanding of linear measurement, that is, the structure of linear units into a scale. Moreover, many students, also in Year 5, when asked what "area" is confused area and volume; for example, "area is the space inside something" or "the number of little cubes in the square" (Bragg, personal communication).

Students' misconceptions may simply reflect teachers' inadequate measurement concepts, combined with syllabus documents that give little direction in terms of key concepts. The worksamples shown in Figures 1 and 2 indicate that when structural features are stressed, students seem to be able to represent these in drawings. The role of such drawings is important as they link the activities with concrete materials to the abstract two-dimensional representations often used by teachers.

The connections between a linear scale, array structure and a three-dimensional packing are important to understanding the development of measurement concepts. These concepts are also closely linked to repeated addition and array multiplication. The difficulty of applying multiplication skills in a meaningful way has been documented for concepts of area and volume (Hart et al., 1989). They found many students continue until well into secondary school to determine both area and volume measures by counting, rather than by multiplying. Such methods do not generalise to fractional dimensions or to the formulae. Knowledge of the measurement unit, the spatial structure of the iteration and its links to multiplication need to be taught to students so teachers need to be made aware of the importance of these aspects of measurement.

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An Investigation of Literacy in the Teaching and Learning of Mathematics in Secondary Schools Serving Low Socio-economic Status Communities

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This paper reports on an investigation, commissioned by the Disadvantaged Schools Program (DSP) of the NSW Department of Education and Training, to identify the best ways to provide support for mathematics teachers in DSP secondary schools, so that they can assist their students to meet the literacy demands of mathematics. Case studies in four markedly different secondary schools were carried out by the authors. Through these, the role of literacy in mathematics learning, teaching and assessment, as perceived by the mathematics teachers and students, has been considered.

In recent years, literacy has been a significant issue in Australian education and has been taken very seriously by both State and Commonwealth governments. In New South Wales, a range of initiatives, under the banner of the State Literacy Strategy (e.g., NSW Department of Education and Training, 1999b), has been implemented in an attempt to increase the literacy levels of students generally, and to address the specific literacy needs of individual subjects and Key Learning Areas (KLAs) (e.g., NSW Department of Education and Training, 1999c). Individual schools have developed and implemented their own literacy plans in the light of these wider initiatives.

The Disadvantaged Schools Program (DSP) is a specific focus program within NSW Department of Education and Training schools which "supports schools serving students from communities with the highest concentrations of families from low socio-economic status (SES) backgrounds. It is an equity program focused on improving student literacy outcomes" (NSW Department of Education and Training, 1999a, p. 2). The DSP complements the implementation of the NSW State Literacy Strategy in its 478 schools through the provision of extra funding and support. The study which is the subject of this paper was commissioned by the DSP specifically to identify the best ways in which support could be provided to mathematics teachers in secondary schools serving low socio-economic communities.

Literature Review

As part of the commissioned project, an extensive literature review on the role of language in mathematics teaching and learning was conducted (Dawe, Dengate, Howard, & Perry , 1999). An overview is presented here.

The body of research into mathematics and language has expanded rapidly over the last ten years (e.g., Davis & Hunting, 1990; Ellerton & Clements, 1996; Mousley & Sullivan, 1996). A constant theme of this research is that language factors have a strong influence on the learning of mathematics and that there are strong links between the development of literacy capabilities and mathematics learning. At the secondary level, there has been a lack of awareness on the part of mathematics teachers to take seriously the literacy demands of their subject, and the literacy needs of their students (Dawe et al., 1999). Much of the research (e.g., Sullivan, 1999) has considered the levels and nature of classroom communication in mathematics classrooms and has led to equivocal conclusions about the value of such accepted practices as cooperative group learning (e.g., Gooding, 1994; Stacey, 1992). Other research has considered specifically the nature of 'good questions' or 'rich tasks' and their relationship to students' learning of mathematics (e.g., Sullivan & Clarke, 1991; Sullivan & Lilburn, 1997). In at least one Australian state, the notion of rich tasks has been developed as the centrepiece of systemic educational reform across the entire curriculum (Education Queensland, 2000).

Other issues of concern in the relationship between literacy and mathematics teaching and learning include the particular issues of socio-economic, cultural and language backgrounds. For example, Zevenbergen (1994) has argued that mathematics can, and often does, serve as a social filter and that 'middle class' and 'working class' schools were likely to imbue different discourses of interaction and success through the nature of their mathematics lessons. Similarly, cultural backgrounds can provide differences in the ways in which students view school mathematics and its importance in society (e.g., Barton, 1993; Stigler and Hiebert, 1999; Stigler & Stevenson, 1994). There are challenges raised by non-English speaking background students in mathematics classes but there are also opportunities provided by these students. No longer is bilingualism seen as a disadvantage to learning (e.g., Clarkson, 1991) but there are issues which affect students who are not fully competent in one or both of their languages.

The schools in which the case studies were conducted were selected because they represented secondary schools in the DSP. That is, they were schools serving low socio-economic status communities. Students in these schools do not perform as well as those in schools in middle or higher socio-economic status communities on statewide measures of literacy achievement nor on similar measures of mathematical achievement. The links between low socio-economic status and achievement in literacy and mathematics are well established (Hilton, 1998; Peterson, 1997). Moreover, it has been argued that the socio-economic status variable is so powerful that it may not be possible to change the nature and directions of these links without changing the nature of the socio-economic mix of the student body (Thrupp, 1998). The DSP in NSW is founded on the philosophy that it can make a difference. This study is part of the action arising from the philosophy.

Method

In early 1999, the four authors of this paper met with officers from the DSP to discuss the issues involved in linking literacy and mathematics. In particular, the DSP wanted to ascertain how it could help mathematics teachers in its secondary schools to address the needs of their students to meet the literacy demands of their mathematics learning. It was determined to use a case study methodology in four secondary schools in NSW. These schools were chosen to represent different DSP contexts.

- *School A.* This secondary school is located in a northwest NSW town. The school's student population is 250 and there are 31 teachers at the school, with three in the mathematics faculty.
- *School B.* This school is located in central west NSW. The school's student population is approximately 880 students. There are seven full-time mathematics teachers.

- *School C.* This school is located on the south coast of NSW in the suburbs of a major city. The school's student population is 640, with many students being the third generation of their family to attend the school. There are six full-time mathematics teachers.
- *School D.* This school is located in the western suburbs of Sydney. The school's student population is about 1150 and there are students from 62 different nationality backgrounds enrolled. There are nine full-time mathematics teachers.

The research questions for this study were devised by the researchers in conjunction with the relevant DSP officers. They were quite extensive. Those reported in this paper are as follows.

1. What are the expressed views of secondary mathematics teachers (in DSP schools) on:
   a) the role of literacy in their teaching of Years 7-12;
   b) the role of literacy in their students' learning in Years 7-12 and
   c) the literacy demands of external examinations and the impact on classroom instruction.
2. What are the expressed views of secondary mathematics students (in DSP schools) on:
   a) the role of literacy in their learning of mathematics in Years 7-12;
   b) the role of literacy in how they are taught mathematics in Years 7-12 and
   c) their ability to comprehend questions in either school-based or external assessment tasks.

In each of the schools, one of the authors met with the mathematics faculty, other teachers and students. Data were gathered using focus group interviews which were tape recorded and transcribed. Some mathematics lessons in each school were observed.

Results

Teachers Views on the Role of Literacy in their Teaching of Mathematics

In all of the case study schools, innovative and creative practices relevant to the role of literacy in mathematics teaching had been introduced. Activities had been developed including the students presenting a written account of certain curriculum topics, peer assessment of oral reports, the negotiation of meaning via the modification and reconstruction of students' own language, genuine reciprocal communication between teachers and students about the purpose, process and product of mathematical activities, underlining of key words in written explanations and definitions, packs of mathematical activities which take literacy issues into account, and close partnerships between mathematics teachers and support teachers. This indicated that the State Literacy Strategy has had an effect in these schools and that many of the teachers were aware of the importance of literacy issues in mathematics. As one teacher put it, "literacy in maths is critical. It is survival. If we, as teachers, don't do it we are going to have more children who see no purpose in school mathematics". Another teacher commented that "you have to teach the children to read. You have to be explicit".

It was a commonly expressed feeling that there were a lot more literacy issues to be dealt with in the lower level mathematics courses than in the higher level courses, in both the junior and senior years. One teacher suggested that:

> within my Mathematics in Practice[1] course there is a lot more literacy and discussion about the meaning of certain words and the context of maths as opposed to other classes. But if you are doing 2 unit calculus, there is not as much literacy.

While there was a recognition of the need to be cognisant of literacy issues in mathematics teaching, not all teachers were prepared to allow that their classrooms should include opportunities for all of the areas of reading, writing, speaking and listening. One teacher suggested:

> in my classroom, I try to minimise one of those - I'm a traditional maths teacher. ... I am perfectly happy to ask for specific feedback from kids by way of a statement that they might have made but I generally have not encouraged open discussion.

Teachers' Views on the Role of Literacy in their Students' Learning of Mathematics

The teachers interviewed stated that it was very important for the students to understand the mathematical terminology being introduced and to use it correctly. Students making the transition from 'conversational' to 'mathematical' language was a prime goal in the junior years of School D. In later years in this school, the correct use of terminology, logical connectives, prepositions and other difficult aspects of the mathematics register for second language learners were addressed through a whole-school literacy policy. The interviewed teachers suggested that these had a direct influence on students' 'deep' versus 'surface' learning.

In all the schools, the teachers were aware that many of the students in the lower level classes had poor literacy skills. They said that this fact was sometimes used as an excuse for the lower mathematical achievements of these students. In School A, some teachers saw that that this was an inevitable consequence of the students' backgrounds:

> the children just don't have literacy backgrounds to cope with the literacy needs of English and maths. Listening skills are not needed in the home as much as they were. Literacy in maths has not been a focus in the past. You look at the textbook and there is no emphasis.

Teachers' Views of the Literacy Demands of Examinations

Responses to this question varied between the schools. In School C, for example, there was no doubt in the minds of the mathematics staff that what is in past examinations affects what they teach and how they teach it. "Basically, the HSC determines what and how we teach, whether we like it or not". However, the teachers did not seem to think that these examinations necessarily presented literacy demands which were beyond or significantly different in nature to those literacy demands occurring in the everyday classroom. Hence, they did not see anything special, beyond the usual examination techniques and strategies which needed to be taught to their students.

In School D, the reading requirements of the external examination for Year 10 mathematics attracted a great deal of criticism for not providing an opportunity for students to really show what they could do. One teacher put it this way:

> Sometimes it just becomes who can answer ... who can fill in an exam paper better than others ... you might have the best maths ability but you don't know how to fill in the paper properly ... there are just too many different styles of questions ... I think they found that a bit of a shock.

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[1] The least academic of the mathematics courses in Years 11 and 12 in NSW in 1999.

The difficulties with examinations was reiterated by a teacher from School A who pointed out that changes in the mathematics curriculum had resulted in a greater teaching focus on problem solving and that this had resulted in different types of examination questions which demanded increased literacy abilities in the students. He suggested that "open ended exams have placed an increased emphasis on students' reading and literacy skills".

Students' Views on the Role of Literacy in their Learning of Mathematics

Some of the students interviewed identified stress, anxiety, frustration and annoyance as emotions that affected them in their learning of mathematics. Some felt sorry for those peers who could not do the work. While many students referred to the importance of listening and talking about mathematics in the classroom, they also saw reading, particularly reading the textbook, as a vital source of explanations and model answers. One student commented: "In textbooks, right, we actually learn it off more ... In class, if you don't get a concept, it is easier to go back to the textbook and look through the examples because it is much clearer". Writing out examples was seen as less important, although, in the students' eyes, it did help to consolidate the skills and routines. Writing in terms of explanation or as a reasoned argument (as in an essay) was not seen as a mathematical activity by these students.

A contrasting view was evident in School C where many of the students, while acknowledging that the textbook was an important source of information, highlighted a dilemma for them which arose when the teacher used different language and/or processes from those in the textbook to explain particular mathematical ideas. This has the potential to leave the learners in a confused state. Students in all the case study schools were adamant that they "generally find the way the teacher explains it easier to understand than that in the textbook ... they're actually talking. They're actually relating to you".

Students do not seem to talk to each other much, either in or beyond the classroom, about the mathematics that they are doing. Outside the classroom, "there are more exciting things" to talk about. Inside the classroom, "if you have a problem you might ask for help from someone sitting beside you". This seems to help because it "reassures you about what you are doing". When asked to whom they would go if they wanted help with a mathematics problem, students were adamant that they would go to a peer in the same class-"a reliable someone next to you".

Students' Views on the Role of Literacy in how they are Taught Mathematics

Students are aware of the huge amount of curriculum content through which the teacher must move. However, this does not stop them being critical of the pace at which they are expected to learn their mathematics, as shown by this comment from a School A student: "They will teach one topic and then move onto the next but I haven't understood the first one". The students know there is so much to cover that "some teachers just go on and on and on without giving us a chance to take a breath. Often when the teacher is talking too much it just goes in one ear and out the other".

In School B, extensive writing occurs, but most of it appears to be of a copying nature: "If she writes it, we write it, is our rule". In another class, an insightful view of oral interaction was given by one student: "The teacher asks a lot of questions when the work is really important". However, in this same school, senior students could not comprehend that the literacy demands of mathematics needed to be considered and suggested that language and mathematics were totally disparate: "English is in this box, maths in that box".

Another group of students felt that teachers were too harsh in quelling talking in class. "Teachers don't understand that you can talk and do work. Yes, you can talk and actually solve a problem." Not surprisingly, while teachers did not necessarily disagree with this sentiment, they were adamant that much of the talking in classrooms had little to do with the tasks which had been set for students to do. The observations of lessons undertaken by the researchers indicated that talk in classrooms certainly covers both of these aspects.

Students from another of the schools also had something to say about the talk that goes on in the classroom and the effect it had on their learning.

> Good teachers change the language and the ways that they explain it to the children. It is better if the teachers talk personally rather than 'up the front'. A good teacher helps you. They are strict but not grouchy, they let you have a bit of freedom and they let you talk. I would like to talk more in mathematics. Sometimes your friends can explain it better than the teacher-and that's when you are told to stop talking!

Some students, especially in School D, were keen to know more than the teachers seemed prepared to share. One commented: "Some teachers teach you exactly what you need to know in a syllabus, but they don't teach you anything more advanced, harder to do". However, this was not a universal comment: "For me, I couldn't keep up with the teacher. He was going too fast. He would explain once but not again, so you'd have to keep moving on. It was just too difficult for me".

Students' Views on their Ability to Comprehend Questions in either School-based or External Assessment Tasks

Many of the issues which have been identified above also have some pertinence for the literacy demands of assessment in mathematics. Key among these are the issues surrounding differences between the mode and density of language used when a teacher is explaining an idea and when it is used in a problem during an assessment exercise. One group of senior students was asked to consider a particular question from the 1996 HSC examination for the *Mathematics in Society* course. They wondered about the relevance of the question to them as students, even though it did make an attempt to situate the question in a real-life situation. The students also commented on the extraneous information through which they needed to wade before they came to what they were supposed to do. The use of what the students saw as 'difficult' language rather than more 'common' language was also criticised, especially when the latter could maintain the mathematical integrity of the question. In some cases, the students felt that too much information is given in assessment questions or the information is given in too many different forms, resulting in confusion. They cited the common occurrence of a drawing, a description and a diagrammatic representation of some trigonometric problems all being given and were adamant that: "Sometimes you are better off with just the triangle".

Discussion and Conclusions

The results shared above are not restricted to DSP schools (e.g., Ellerton & Clements, 1996; Miller et al., 1999). They highlight that both teachers and students in the four schools are aware of the importance of literacy issues in the learning of mathematics but do not necessarily know how to deal with them in the context of their mathematics lessons. However, they are trying, with many teachers using a wide range of teaching strategies to provide opportunities for students to deal with the literacy demands of mathematics. On the other hand, some teachers have yet to take seriously their role in the helping students meet these demands.

Direct instruction of content, followed by discussion, consolidation and practice is commonplace. Much less common are strategies such as practical work, problem solving, investigation using open-ended questions and cooperative group work. The role of writing in learning mathematics does not feature prominently in the work of teachers or syllabus and textbook authors. However, the expectations of the new Stage 5 and 6 mathematics syllabuses[1] - that the students will be able to express themselves in both speech and writing - may hasten the addressing of this.

Teachers need to engage with their students and provide structure through associated prompts and questioning. They need to use explicitly the students' mental imagery and their experiences to represent mathematical information in a wide variety of ways. Issues which arise from the different social and cultural backgrounds of the students need to be addressed. These issues do affect learning preferences and aspirations (Zevenbergen, 1994) and must be addressed in the context of mathematics learning. It is difficult to see how much progress can be made if, as one teacher from School C commented: "Another problem here is that 'it is seen to be cool not to be doing well'".

The impact of the DSP initiatives in the area of literacy in mathematics teaching and learning are beginning to bear fruit. Mathematics teachers have been seen as the most resistant group in secondary schools when it comes to consideration of the literacy demands of their subject. However, progress is obviously being made. The day-to-day routines of mathematics teachers serving in DSP schools are demanding and difficult. The issue of time is a major concern to many of these teachers, so the integration of literacy into content is critical - it cannot be seen as an 'add-on', or it will simply not be done. The leadership and enthusiasm of Head Teachers, Mathematics is critical. They can lead their colleagues through the development of specific teaching resources and approaches and can help them gain ownership of the strategies, with the result that literacy can be integrated into the content in such a way that the literacy demands of mathematics learning and teaching are met naturally. This is certainly the aim of the schools involved in this study and of the DSP as everyone involved moves towards the continual improvement of the mathematics education of all students.

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A Model for the Promotion of Statistical Thinking?

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This paper presents a model for statistical thinking in empirical enquiry, based on practitioner behaviour. The conjectured types of thinking that are fundamental to the statistics discipline are then used to analyse an assessment item and teaching situation. It is concluded that such an analysis is possible and that more discussion and research is needed on defining, clarifying, and articulating the statistical thinking that is embedded and inherent in statistical practice.

Over the last thirty years introductory statistics teaching has been moving from mathematical statistics towards empirical statistics. At the same time significant shifts have been occurring in statistical practice with the increasing availability and power of computer technology and with many applied statisticians broadening their domain of practice to all parts of the problem solving empirical enquiry cycle. These shifts in practice are refocusing the emphasis in teaching from *how to do*, for example, a graph, mean or confidence interval, to *how to think about* them. However, the development of students' statistical thinking can only be promoted if we know the type of thinking that is characteristic of the statistics discipline. Many articles discussing and characterising the core elements of statistical thinking have been written (e.g., Moore, 1990; Biehler, 1994).

The Wild and Pfannkuch (1999) four–dimensional model (Fig. 1) was an attempt to characterise practitioner thinking in the statistics discipline. The model was developed as a result of: interviewing statisticians and tertiary students about statistical projects they had been involved in; interviewing tertiary students as they performed statistical tasks; and analysing the literature. The focus of the statistical thinking was at the broad level of the statistical enquiry cycle, ranging from problem formulation to the communication of conclusions. The multidimensional nature of statistical thinking is captured in the model. Thus a thinker operates in all four dimensions at once. For example the thinker could be categorised as being currently in the analysis stage of the Investigative Cycle (Dimension 1), dealing with some aspect of variation in Dimension 2 (Types of Thinking) by criticising a proposed analysis against contextual knowledge of the situation in Dimension 3 (Interrogative Cycle) driven by scepticism in Dimension 4 (Dispositions). This paper focuses on the thinking that we believe is fundamental to statistics and how we might use these defined characteristics to analyse assessment items and teaching situations for the promotion of statistical thinking.

Fundamental Statistical Thinking

From our research five types of thinking that are inherently statistical emerged (see Fig.1(b)). We believe that these are the fundamental elements of statistical thinking.

Recognition of the need for data. The foundations of statistical enquiry rest on the assumption that many real situations cannot be judged without the gathering and analysis of properly collected data. Anecdotal evidence or one's own experience may be unreliable and misleading for judgements and decision-making. Therefore data are considered a prime requirement for judgements about real situations.

Transnumeration. In order to make a judgement in a real situation, data that 'measure' or capture the qualities or characteristics of that situation must be found. For this type of thinking we coined the word transnumeration which means "numeracy transformation to facilitate understanding". Once the data have been collected transnumeration thinking operates again as raw data are transformed into multiple graphical representations, statistical summaries, and so forth, in a search to obtain meaning from the data. At the final stage transnumeration thinking occurs when the meaning from the data, the judgement, has to be communicated in a form that can be understood in terms of the original situation.

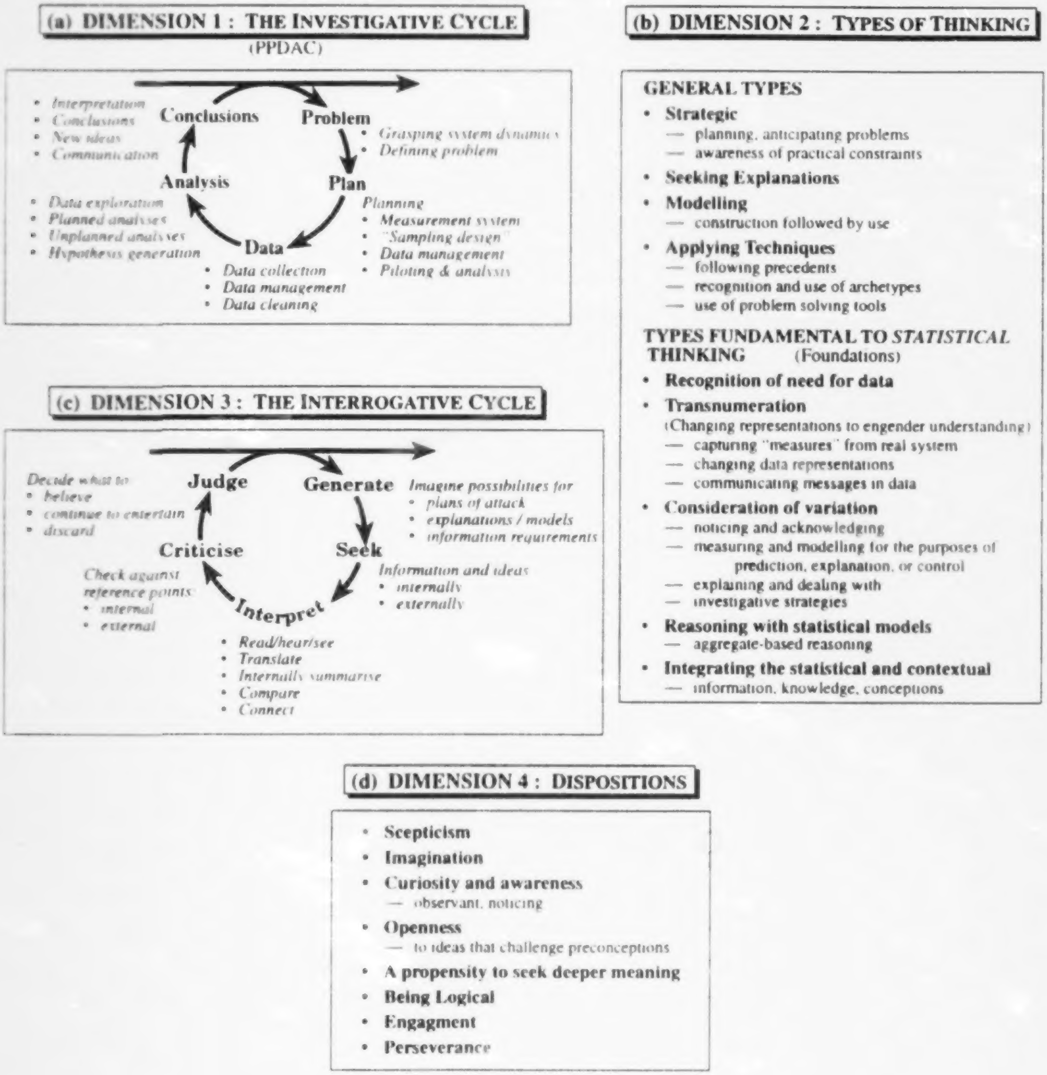

(a) DIMENSION 1 : THE INVESTIGATIVE CYCLE
(PPDAC)
• Interpretation
• Conclusions
• New ideas
• Communication
Conclusions
Problem
• Grasping system dynamics
• Defining problem
Analysis
Plan
• Data exploration
• Planned analyses
• Unplanned analyses
• Hypothesis generation
Data
Planning
• Measurement system
• "Sampling design"
• Data management
• Piloting & analysis
• Data collection
• Data management
• Data cleaning
(b) DIMENSION 2 : TYPES OF THINKING
GENERAL TYPES
• Strategic
— planning, anticipating problems
— awareness of practical constraints
• Seeking Explanations
• Modelling
— construction followed by use
• Applying Techniques
— following precedents
— recognition and use of archetypes
— use of problem solving tools
TYPES FUNDAMENTAL TO STATISTICAL THINKING (Foundations)
• Recognition of need for data
• Transnumeration
(Changing representations to engender understanding)
— capturing "measures" from real system
— changing data representations
— communicating messages in data
• Consideration of variation
— noticing and acknowledging
— measuring and modelling for the purposes of prediction, explanation, or control
— explaining and dealing with
— investigative strategies
• Reasoning with statistical models
— aggregate-based reasoning
• Integrating the statistical and contextual
— information, knowledge, conceptions
(c) DIMENSION 3 : THE INTERROGATIVE CYCLE
Decide what to
• believe
• continue to entertain
• discard
Judge
Generate
Imagine possibilities for
• plans of attack
• explanations / models
• information requirements
Criticise
Seek
Check against reference points
• internal
• external
Interpret
Information and ideas
• internally
• externally
• Read/hear/see
• Translate
• Internally summarise
• Compare
• Connect
(d) DIMENSION 4 : DISPOSITIONS
• Scepticism
• Imagination
• Curiosity and awareness
— observant, noticing
• Openness
— to ideas that challenge preconceptions
• A propensity to seek deeper meaning
• Being Logical
• Engagment
• Perseverance

Figure 1. A four-dimensional model for statistical thinking in empirical enquiry

Consideration of variation. Making a judgement from data requires an understanding of variation during the process of statistical enquiry. It is a type of thinking that starts from noticing variation in a real situation, and then influences the strategies we adopt in the design and data management stages through determining whether we will ignore, plan for or control variation. It further continues in the analysis and conclusion stages through determining how we act in the presence of variation. If we accept that statistics is about making predictions, seeking explanations, and finding causes, then we will be looking for variation in data. It affects all thinking through every stage of the investigative cycle.

Reasoning with statistical models. The predominant statistical models are those developed for analysis of data. These models allow us to summarise data in multiple ways dependent upon the nature of the data. For example, centres, spreads, clusters, outliers, confidence intervals, and p-values are all read, interpreted and reasoned in an attempt to find evidence on which to base a judgement. According to Konold et al.'s (1997) research, students find it difficult, when dealing with data, to make the transition from thinking about and comparing individual cases to thinking about and comparing group propensities. Reasoning with statistical models requires the ability to do both aggregate-based and individual-based reasoning and to recognise the power and limitations of such reasoning across a variety of situations. This aggregate-based reasoning is fundamental to statistical thinking. For example, in mathematics one counter-example disproves a theory whereas in statistics one has a theory about group propensities and one counter-example (an individual case) does not disprove the theory. There is also a need to develop more statistical models for reasoning in the other stages of the investigative cycle. Quality management is one field of statistics that is attempting to address such model development.

Integrating the statistical and contextual. Although the above types of thinking are linked to contextual knowledge, the integration of statistical knowledge and contextual knowledge is an identifiable fundamental element. The statistical model must capture elements of the real situation and thus the resultant data will carry their own literature base (Cobb & Moore, 1997). Information about the real situation is contained in the statistical summaries and therefore a synthesis of statistical and contextual knowledge must operate to draw out what can be learnt in the context sphere. This synthesis operates throughout the investigative cycle and enables some judgements to be eventually formed on the real situation.

Thus the question is raised as to whether this theoretical model will be useful in analysing assessment items and teaching situations for determining the type of statistical thinking that is being invoked. An exploratory analysis, based only on the fundamental types of statistical thinking, is now presented for each situation.

An Assessment Situation

Tertiary students in an introductory statistics course were given, as part of their assessment, a problem (Fig. 2) involving the construction and comparison of graphs. After the assessment they were given model solutions (Department of Statistics, 1999).

Analysis in terms of promoting statistical thinking

Recognition of the need for data. The data gathered and the questions asked do not promote this type of thinking, but illustrate its need, especially as the results are counter-intuitive.

Transnumeration. In this assessment problem this type of thinking is not done by the students, since the "measurement problem" has already been "solved" and the students are asked to construct particular plots. Comments are also not sought on which of the two plots give a better representation for communication about the real situation.

Consideration of variation. The first three parts of the problem deal with possible sources of variation in the design of the experiment such as gender, observer variability and researcher bias. This consideration of variation reinforces the thinking that, for the two groups of children, the effects of differences among the children and among the observers should be taken into account in the design of the experiment, so that the differences observed can only be attributed to the fact of whether the children were rewarded or not initially. The questions, however, do not get the students to think of other possible sources of variation in the design of this experiment. In the solutions to parts (d), (e) (not shown here), and (f), the interquartile range, the medians, and so forth are calculated (measured) and left. Comments on the plots and summaries, with regards to variation or patterns in the variability for the purposes of prediction and explanation about time spent on drawing, are not sought. Also, in the part (g) question, variation ideas are not triggered for the making of an inference, from the sample, about the population from which it is drawn.

Problem:

A study (Lepper, Greene & Nisbett, 1973, cited in Lockhart, 1998, p. 4) was conducted to investigate motivation in children's behaviour. Thirty-eight 5-year old children, all who had shown at least a moderate interest in drawing as a voluntary activity, were divided into two groups. All the children were asked to make some drawings and were given 6 minutes to do so. Beforehand, the children in one group were told that they would receive a "Good Player Certificate" as a reward for their participation. The second group were told nothing about the reward and received none. One week later, all the children took part in a one hour free play session. Drawing was one of the activities they could choose. Observers measured the amount of time (in minutes) each child spent drawing during this time. The resulting data are presented below:

With Reward: 5.2, 8.5, 1.9, 6.5, 8.8, 5.5, 2.3, 3.7, 16.0, 7.9, 1.9, 6.0, 5.2, 7.2, 0.0, 7.8, 2.4, 4.7, 7.8

No Reward: 8.4, 10.8, 8.2, 9.6, 2.5, 7.5, 12.6, 10.8, 13.8, 11.3, 8.9, 9.8, 10.1, 10.6, 11.3, 9.7, 9.4, 10.5, 11.5

(a) The 38 children used in the experiment were divided into the two groups randomly. What was the purpose of doing this at random?

(b) If the experimenters were worried that the gender of the child may influence the way they behave, how should the design of the experiment be modified?

(c) Can any form of blinding be used in this experiment? If so, briefly describe how.

(d) Construct a back-to-back stem-and-leaf plot of the two sets of data using an appropriate scale.

(e) Calculate the five-number summary for the With Reward data. Show your working.

(f) The five-number summary for the No Reward data is: (2.5, 8.9, 10.1, 11.3, 13.8). Using this and your five-number summary from (b) draw side-by-side boxplots for these two sets of data. Show your working.

(g) Using your plots, in plain English, compare the two sets of data.

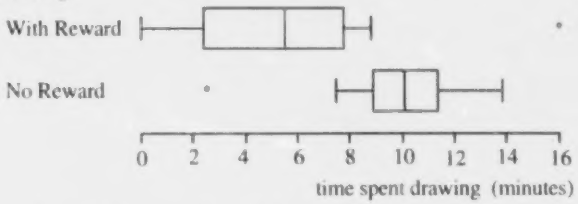
Some Solutions:

(a) The purpose of randomly allocating the children to the two groups was to try and ensure that the comparisons are fair in the sense that two groups are as similar as possible in every way except for whether or not they were rewarded initially.

(b) The experimenter should have blocked the children by gender.

(c) The only form of blinding possible would be for the people recording the observations to have been blinded.

(f) Box plot of time spent drawing



(g) The data for With Reward group is centred much lower and is more spread out than the data for No Reward group. The With Reward group data looks mildly left (negative) skewed. The No Reward data looks slightly right (positive) skewed. There is a possible low outlier in the No Reward group and a possible high outlier in the With Reward group.

Figure 2. Assessment item for introductory statistics course.

Reasoning with statistical models. Part (g) in the solutions is an example of how we would expect students to reason with these models. Skewness and outliers are noticed. The difference in the centres and spreads between the two groups is acknowledged. However, in order to draw out the meaning, in terms of the original situation, from these statistical models it is necessary to use context knowledge.

Integrating the statistical and contextual. If we believe that we undertake statistical investigations to learn more in the context sphere then the questions for this problem take no cognisance of the context of the problem. There is no request to interpret and communicate the information contained in the data about motivation in children's behaviour. There was sufficient context given at the beginning of the problem for students to communicate the information in the data contextually.

Discussion

This problem potentially provides a good base for assessing students' knowledge. It operates in the plan and analysis phases of the investigative cycle. Whilst I recognise that students need directed experiences such as the above item they also need undirected experiences in data-handling. With more consideration given to developing students' statistical thinking I believe that we could rethink the questions we ask and the model solutions we provide. For instance, the recognition of the need for data might be prompted with a question such as: "*Before looking at the data, read the details about the experiment. Use your own knowledge to predict what the results might demonstrate and then state the basis for your prediction. Think of an alternative prediction and justification.*" Transnumeration thinking and the other types of thinking could be promoted by an open question such as: "*compare the two groups explaining what you learn from the comparison/s*". Another question that may be useful for triggering thinking on measurement issues is: "*write down some 'worry' questions you might have about defining the problem for this experiment and explain why*". A question that could be asked to promote thinking about variation could be: "*write down some 'worry' questions you might have about the design of this experiment and explain why*". Questions such as: "*summarise what you learnt from this experiment*" followed up with: "*what would you investigate next if you were the researcher?*" might encourage the integration of statistical and contextual knowledge.

A Teaching Situation

This teaching situation (Shaughnessy, 1999) was observed by me in an undergraduate and a graduate class. Data sets for the time in minutes between blasts of the geyser Old Faithful are cut up into sixteen separate days. Each student is given a strip of data from *one* day of Old Faithful's activity. The students are asked: (1) to construct a graph of the data; (2) to make a prediction about what they think another day of this data would look like; and (3) to swap days with someone else, and repeat steps 1 and 2. They are also asked how long they would expect to wait between blasts of Old Faithful, and to explain their reasoning. As the activity progresses further questions are asked.

Analysis in Terms of Promoting Statistical Thinking

Recognition of the need for data. The giving out of one day's data and seeing that a neighbour's data look different promotes ideas about variability within days and from day to day, and about the need for more data to be able to understand the dynamics of this particular geyser system before making any informed judgement.

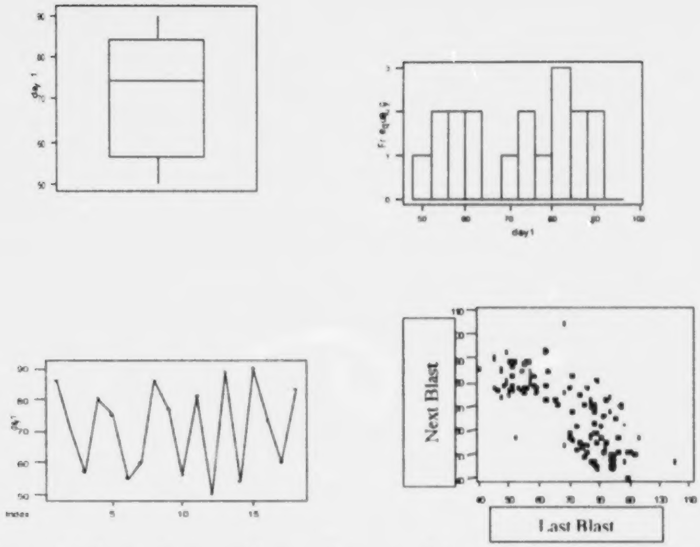


Figure 3. Minutes between blasts – Some possible graphs for teaching situation

Transnumeration. Students start with a strip of one day's data and they need to think how they must change the data representation to facilitate a prediction. The transnumeration thinking is partially done by the students as they are asked to draw a graph but they are not told what to draw. Most students draw a single graph. Students are then asked to share their graphs with the class (see Fig. 3). It soon becomes apparent with data from sixteen days that the dot plots, stem-and-leaf plots, time series line graphs, box-and-whisker plots, the histograms with small class intervals and with large class intervals, all reveal something different about the activity of Old Faithful. The histogram with large class intervals obscures information and hence the students must recognise this and try several different class interval widths. This sharing of graphs promotes the transnumeration type of thinking and the need to look at multiple representations.

Consideration of variation. The box-and-whisker plots reveal the considerable variation within a day and between days even though sometimes the median is approximately the same. The dot plots, histograms with small class intervals, the stem-and-leaf plots and the line plots reveal the bimodal nature within a day's data for most of the days (though not all) and thus the linking of these plots with the boxplots accounts for the observed variability. The students report that the mean time of 70 minutes is how long they expect to wait before the next blast. However after much discussion and looking at the line plot with its oscillating variability and the bimodal histograms gives them the idea that if the last blast waiting time was around 60 minutes then they would expect to wait about 80 minutes for the next blast while if the last blast waiting time was about 80 minutes they would expect to wait about 60 minutes for the next one. Thus the students are beginning to model and measure variation for the purpose of prediction. The question then arises as to whether this prediction is valid. Students must think of a way of representing the data (transnumeration type thinking) in order to validate the prediction. A scatterplot of last blast versus next blast in minutes (Fig. 3) reveals that there is more variability in the next blast prediction when the last blast is greater than 70 minutes than when the last blast is less than 70 minutes.

Reasoning with statistical models. The multiple representations of these data (Fig.3) require the students to be able to read and interpret the graphs and also interrelate the graphs. The oscillating nature of the line plots requires the students to reason that a short blast is often followed by a long blast, that there is an underlying bimodal distribution, that the activity is not random, and the data have an underlying model from which predictions could be made about another day's data. The histograms require the students to reason that there may be an underlying bimodal distribution and there are possibly two distinct clusters of data that are reasonably symmetric with their own underlying means and standard deviations. The boxplots require students to reason comparatively among the days, to reason with group propensities, and to notice that these are summary plots and hence may be obscuring features of the distribution of the data. For predicting the next blast of Old Faithful, the students reasoned that the line plots were the most appropriate graphs for giving measurable predictions which were conditional upon knowing the wait time of the previous blast. Reasoning with the centre, which many students did, is inappropriate for predicting the next blast. Above all, students need to read, interpret, reason, and make sense of the *connections* and *interrelationships* among all the graph representations.

Integrating the statistical and contextual. The patterns seen in the variability stimulates questions about how this geyser works. The data is telling these students about properties of the eruption of this geyser. At this stage more scientific information is needed on geysers so that the statistical knowledge gained can be integrated with the contextual knowledge about geysers. It may be that this type of pattern is repeated in other geysers and that there are good reasons, concerning the dynamics of geysers, for this pattern to occur. Other questions naturally arise as to what other data are needed, such as the duration time of the blasts, to further understand and to be able to predict the waiting time between blasts.

Discussion

This rich teaching situation actively develops and promotes statistical thinking according to the Wild and Pfannkuch paradigm. It could be changed in three ways, although from a teaching perspective I would not recommend these changes for this particular activity. The following changes are only mentioned to highlight that students need some other experiences with data to further promote statistical thinking. Initially, before the data-sets are given out, the students could be asked to predict the minutes between a series of blasts and the bases for their predictions. This may encourage the notion of the need for data. Secondly, each student could be given the same data-set and then it would be up to the students to suggest that more data is needed. Thirdly, there should be no suggestion that they construct a graph. Through not asking them to plot the data the transnumeration type of thinking could be promoted from the beginning of the activity. Thus only the two prediction questions need to be asked. Or taking another step, the students could be given the data for one or several days and asked what sort of questions (i.e., "their notices and wonders" (Shaughnessy, 1997)) they could investigate with these data. This would promote thinking of the variation type and the use of statistical and contextual knowledge.

Conclusion

The assessment situation is an illustration of a structured task whereas the teaching situation illustrates an unstructured task. Rich statistical experiences can be given to students by choosing tasks that portray how statistical thinking is applied to particular statistical situations. Students need to practise that new knowledge and the way of thinking in structured questions. However, if students are to independently model statistical practice and thinking they must be presented with unstructured tasks during teaching and assessment. This will enable students to create for themselves the linkages and connections between what they have learnt and the statistical situation presented. This would suggest that in every assignment and its equivalent teaching unit there should be at least one unstructured task that promotes statistical thinking.

The Wild and Pfannkuch model was an attempt to make implicit practitioner thinking in statistics more explicit. The above analysis was an exploration of whether we could understand our teaching and assessment from a statistical thinking perspective. The analysis demonstrates how this model may be used for assessing the presence of the broad fundamental statistical thinking skills. The model may also identify improvements for teaching and assessing, and clarify why tasks are particularly good for developing thinking. Most introductory tertiary statistical courses have commonalities for the content that should be covered. However, teaching such courses requires an understanding of how to develop the content and thinking that exists in statistical practice. More discussion is needed in the statistical community to develop commonalities for understanding, communicating, articulating, and teaching statistical thought. The new curricula in the future should interconnect, align, and focus on both content and thinking as the statistics discipline evolves with new technology and new statistical practice.

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The Perceptions and Practice of Assessment in Mathematics Education of Teachers of Years 3 and Year 6 Students

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This is a report on part of a larger study about teacher assessment practice in New Zealand. The interviews with teachers of Year 3 and 6 students are analysed. These interviews indicate that teachers are using a variety of assessment strategies for a mastery-based system of assessment. The judgement of students' mathematical performance appears to be dominated by the belief that all students must feel that they are achieving. It is conjectured that a system effect determines teachers' assessment in the school environment.

Recent changes in mathematics curricula have clearly signalled that changes in teaching approaches and assessment approaches are desirable. The literature would suggest that much of the curriculum innovation has failed to take root in the classroom mainly because of system and teacher factors. The system factor dominates and determines classroom practice in assessment since the methods and purposes of assessment are deeply embedded in the school system of a country. Thus changes in assessment cannot be effected unless "old styles of thinking" (Romberg et al., 1990, p. 26) are divested in the institutional system. It is believed that this current lack of cohesion between the goals of the mathematics education curriculum and assessment philosophy and practice inherent in the system, has stifled changes in teaching. However "assessment involves a multitude of deep and difficult theoretical and practical problems" (Niss, 1993, p. 27) which will not be easy to resolve.

The teacher factor identifies teachers' perceptions, beliefs and conceptions of mathematics and assessment as playing a critical role in classroom practice (Watson, 1998; Kyriakides, 1998). According to Thomas (1998, p. 6) teachers' knowledge of mathematics and of their students strongly affects their teaching. "It shapes how they select activities and resources, how they present material in class, how they interact with students and how they assess student progress." Furthermore there is still a limited understanding of teachers' beliefs about assessment and the ways in which they practise and evaluate their assessment methods (Philippou & Christou, 1996; Warren & Nisbet, 1999). Bjorkqvist (1997) also believes that teachers' understanding of assessment in mathematics is shaped by teacher interaction with other teachers, by government policy and by society which evolve into a sociomathematical norm. Thus the system factor and teacher factor are complexly interconnected. Therefore any study on teachers' perceptions and practice of assessment in mathematics must take into account the school and system culture.

Background to Study

In 1992, for the first time in New Zealand, a mathematics curriculum (Ministry Of Education, 1992) was developed for the whole school age cohort. This curriculum was divided into six content strands and eight levels of achievement. Achievement objectives for each level and each strand were specified. In order to achieve these objectives suggested learning experiences and sample assessment tasks were included as exemplars for teachers. The introduction of this new curriculum was promoted by politicians as providing accountability through specified objectives, and by educators as a means of changing traditional mathematics teaching and assessment. These pronouncements caused members of the Mathematics Education Unit at The University of Auckland to wonder if this curriculum

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or future curricula would produce the desired outcomes in the classroom. Furthermore when the Unit was asked to comment on whether the changes had improved the teaching of mathematics, we had no data on which to compare and evaluate the effects of the new curriculum. Thus it was proposed that a 'snapshot' of four areas – technology, the teacher, the student and assessment - would be taken at ten-yearly intervals at the Year 3, 6, 8, 10 and 13 levels (i.e. 7, 10, 12, 14 and 17 year old students). This paper reports on the first 'snapshot' of the assessment practice of teachers of Year 3 and 6 students as the new curriculum was being implemented in the classroom.

Method

Research (e.g., Philippou & Christou, 1996) indicated that what teachers reported did not always match with what they actually did in practice. A decision was made to interview teachers and gather documentation, as my aim was to partially understand the whole assessment process practised by a teacher. The protocol for the interview was based on the assessment framework developed by Niss (1993). This framework is a structural analysis of assessment in mathematics education and is grouped into the two categories of purposes and modes. According to Niss the purposes of mathematics assessment are "the provision of information, the establishing of bases for decisions and actions and the shaping of social reality" (Niss, 1993, p. 6). A given assessment mode includes the whole spectrum of components that model the assessment process. Thus these components, which include the subject, the objects, the items, the occasions, the procedures and circumstances, the judging, the recording, and the reporting, formed the basic structure for the interviews and the collected documentation. Questions were also asked about the rationale for changing assessment practice in mathematics.

Table 1
Subject and School Data

Subject Code	Year	Gender	Ethnic Group	Age Group	Years Teaching	School Status	School Size
3.1	3	F	NZE	20-29	2	M	M
3.2	3	F	NZE	50-59	15	L	M
3.3	3	F	NZE	50-59	20	H	M
6.1	6	F	NZE	30-39	12	M	M
6.2	6	F	NZE	40-49	16	L	M
6.3	6	F	NZE	50-59	30	L	M

Note. Ethnic Group: NZE=New Zealand European; School Socio-Economic Status: L=low, M=middle, H=high; School Size: S=small(<400), M=medium, L=large(>1200).

In order to obtain a cross-section of assessment practice from the schools in the greater Auckland area a random sample of three teachers at each level was selected. From November 1994 until September 1996 interviews were sought and conducted with three teachers at each of the designated levels. A one hour in-depth interview was audio-taped and samples of the teacher's assessment tasks, record book and report form were collected. In some cases the documentation was not made available even though some of it was seen by the interviewer. The interviews and documentation were analysed independently by me and another researcher with primary teacher experience. We subsequently arrived at a consensus about our interpretation of the interviews.

Since participation in this study was voluntary the teachers in the sample may not be representative of the teacher population. Demographic details about the subjects and their schools are listed in Table 1.

Results of Interviews

The results are based around the Niss (1993) framework in order to present the linkages and connections within the assessment system of a school. This is not a full report of the results.

The Report to Parents

All the school reports to parents had a mathematics achievement rating on a three-point scale which could be described as reflecting the normal curve by categorising students as below average, average, or above average. However, the schools used their own terms for these categories. For example, School 6.1 used S=strength, Comp=competent, ED=experiencing difficulty, while School 6.2 used A=excellent, B=satisfactory, C=having difficulty. Two schools (3.1, 6.1) gave mathematics achievement ratings in the three areas of number facts, applications and problem solving whereas the other schools gave one global rating.

Record Keeping

This three-point rating scale was reflected in the teachers' mark books with the predominant use of a tick, dot and a cross for the three ratings. Some marks were recorded for the assessment of number facts. Comments on individual student achievement and progress ranged from detailed comments in the mark book margins to no comments. The mark books, in all cases, had a list of curriculum objectives, broken down into 'manageable' parts for a mastery assessment procedure. These curriculum objectives were developed by either the school, the teacher, or a commercial publisher based on their interpretations of the mathematics curriculum objectives. On remarking that some of the curriculum objectives had been broken down to into smaller parts, 6.3 replied:

> Oh yes they have got to be in and some of the teachers have been complaining because they want these [the objectives] to be broken down even more because some children don't show any progress.

This was a dominant theme that the objectives should be sufficiently broken down so that every student was able to demonstrate progress. It was not clear always to the interviewer how the teachers obtained a global rating from their mark books for the report. However 6.3, when pressed, agreed that she made a visual judgement of the number of ticks, dots and crosses in her mark books. Three teachers (3.3, 6.2, 6.3) also had a portfolio of mathematics work for each individual student.

The Assessment Task

The main forms of assessment were written, observation and interview. For the written assessment many gave the same pre- and post-test. The dominant assessment task was the pencil and paper test based on the objectives, with some practical tasks particularly for measurement, statistics projects and construction of an object. Most of the tasks were photocopied from or based on commercial resources. Very few were designed by the teacher. The actual assessment tasks concentrated on facts, procedures and applications. Very few tasks invoked higher order thinking or could be described as problem solving. Units of work and thus the assessment were based around one content strand. Many assessed tasks were an integral part of the teaching. Students would be given mathematical activities for a content strand and if the teacher informally observed the ability of the student to achieve that task it would be noted, sometimes in their record book, sometimes in their head.

> 90% of it is mental notes, I mean it would be really nice if we could record everything but I've tried something with this jolly thing on my knee and a piece of note paper and trying to teach the kids at the same time ... (3.1)

Criteria for Judgement of Students

All the teachers operated in their classroom by putting students into three or four ability groups. The pre-test or the teacher's professional judgement determined which group a student was placed in for a unit of work, although in reality the nature of the groups did not change much throughout the year. None of the teachers used the students' previous class results or placed value on the other teacher's judgement for determination of their ability group structure as they believed students could change or forget their work in the six week holiday period. Comments such as the following show a strong belief in their ability to judge the appropriate group for a child.

> I don't really need what the teachers from the years before said because I can tell so quickly where a child is at. (6.3)

> No, to tell you the truth I never looked back to see what happened last year because they change so much. I just do it on the first couple of lessons ... I will put out a level 1 and a level 2 activity and just see which ones they cope with and I assess from there which group they will have to work in. (3.2)

The over-riding criteria for all teachers is what they called their 'professional judgement'. The teachers chose the commercial resource to assess their students. This resource could be modified to more suit their students or to enable their students to show progress.

> In this pre-test for algebra – it's not a straight test from the topic book. It's been changed because we wanted to cut out as many words as we could. (6.3)

There was a general belief that the assessment task should be suitable for their students, that students must feel they are succeeding, not failing.

> I couldn't give them all the same test at the end of the day, no. Because otherwise they've got that feeling of failure which we don't want them to get because they must enjoy maths and feel they have achieved something. (3.2)

Generally each ability group mastered an objective on a differentiated level of achievement since teachers admitted they had different expectations for the different groups. Thus two students in the same class who were said to have mastered an objective could not be considered to be able to do similar tasks. The criteria or benchmarks for achievement of objectives were professional judgement, against a background knowledge of textbook examples and in two cases BSM (Beginning School Mathematics) benchmarks. However the overriding criterion was that students must feel that they are progressing and hence the assessment appeared often to be against the perceived potential of the students. For instance, 3.1 realised during the interview that she taught her highest group (diamonds) differently to the lowest group (triangles). On questioned about differentiated and higher expectations in assessment she replied:

> Yes I suppose I would, if I teach them differently, I'd assess [differently] ... definitely ... for a child in the diamonds I might say 'oh, they could do a little bit better than that' ... whereas with the triangles it might be 'oh, at last, and right let's forget it'.

Four teachers (3.1, 3.3, 6.1, 6.3) worked in syndicates (with other teachers) and either used standardised assessment tasks developed by the syndicate or used their own tasks. In both situations there was no discussion on whether their judgement of work was similar to another teacher's judgement. It seemed that teachers shared assessment tasks but did not share outcomes. Two teachers (3.2, 6.2) worked alone. Thus achievement of objectives by students probably could not be used for comparison of students across rooms (the same year level) in a school.

> Yes, I would compare a child with the children in my room but I haven't thought to compare across the rooms ... you very much stick to your own little [world] ... we [the syndicate] all know we are doing statistics for five weeks and if someone says 'hey, this is a really good activity try this one' then we share ideas like that but we very seldom share the outcomings of the children's work. (3.1)

In fact the teachers stated that the standards they expected from students would be quite different to standards expected by another teacher in the same school and definitely would be different from school to school.

> We do talk to each other but I wouldn't guarantee that our standards are the same. I think, no they wouldn't be I am sure. There could be a big difference ... presentation and everything, yes. A big difference. (6.2)

They agreed that the school report could be differently interpreted by teachers since this was an aspect that was not discussed.

> I've never discussed it with another teacher. We write our own reports ... they [school hierarchy] just said this is the report format. (3.1)

The Rationale for Changing Assessment Practice in Mathematics

All the teachers were able to rationalise the changes from a classroom practice perspective. For example:

> Well, I think something needed to be done about the assessment of maths ... I was teaching it before and it was very airy fairy ... something was done and we've taken it on board and gone with it (3.3)

One teacher (6.2) said that some new assessment methods including how they analysed students' responses in traditional tests enabled her to learn more about students' learning. Some believed that the drive for change was emanating either from the school hierarchy, BOT (Board of Trustees), ERO (Education Review Office), parents, or politicians. They believed that there was more accountability sought now.

> Since the new curriculum came in ... I have to show concrete facts ... to BOT and ERO whereas before I didn't have to do that. (3.2)

There was a general feeling that an external source such as the syndicate leader, the curriculum, or ERO dictated that there would be a change in assessment, told them how to do the assessment and they did what they were told. However there was an underlying belief that the changes might not improve the performance of the students.

> I think it has been driven from above, from the ministry ... and we think 'oh we've got to do this' ... I think we are all afraid that it's going to be imposed from above because we feel we know what's best for our children. (3.2)
>
> From that book [curriculum] I guess they just want to improve the children's standard of work ... but is that really important, do we really need to notice? ... we can't really do much about their brains. (6.2)

School policy stated that 3.3 had to assess using the curriculum objectives.

> Because I've had to and they've been dictated to me ... Well, they're trying to make a uniform system where we are all doing the same thing.

None were able to articulate a mathematics education research or an international mathematics assessment perspective. Their understanding of what assessment was actually measuring or what they sought to measure in mathematics learning was at a practice level rather than a theoretical level.

Discussion

Two themes have emerged from this particular analysis. The first theme is the school effect, the second is the education of the teacher. Both these are system effects, not teacher effects.

The first theme of the school effect expresses itself in the requirement for reports, the way teachers operate in that school's system, the way teachers work in their classrooms and the value the school places on having an educated work force. The way teachers operate in their school system is based around a system of either working in a syndicate or working alone. Working in syndicates could provide opportunities for teachers to share assessment tasks, to discuss expected outcomes, and to share students' assessment output for discussion of judgement criteria. It appears teachers only share assessment tasks in this system. Also the decision makers in the school determine the form and scope of the report information. There would appear to be no discussion on what was being assessed, why it was being assessed and why it needed to be reported to parents.

The way these teachers work in their classrooms is dominated by the belief that assessment must show that each child has progressed or achieved success. Therefore each student is an individual and the achievement rating reflects the teacher's assessment, partially benchmarked against what she feels is the student's potential. Thus an achievement rating cannot be used for comparison of students within a class, between classes in a school or between schools.

The school must provide accountability to parents, ERO and the community about the standard of mathematics achievement of individual students and groups of students. Therefore teachers' assessment is focussed on producing evidence of student progress (e.g., the pre- and post-test, the mastery of objectives) and producing a global rating on a three-point scale in order to demonstrate student achievement in mathematics for the system. This achievement rating is largely based on an individual teacher's professional judgement of the tasks to be used and the interpretation of the students' responses. These criteria for judging student achievement should be challenged, particularly when professional development in assessment does not seem to include discussion and comparison of teachers' judgements on incidents or tasks that are assessed. According to Watson (1998) teachers need to be critically aware of their judgement processes and how such factors as their first impressions of the mathematical abilities of students and their view of mathematics may influence their perception of students' achievement in mathematics. Watson furthermore stated that, once teachers' beliefs about students had been established, the teachers reinforced these beliefs by only remembering favourable incidents, and in spite of mounting evidence to the contrary there was a reluctance to change original opinions of the students' abilities in mathematics. Therefore professional judgement and impartial assessment practice is a concern that perhaps should be addressed within the teaching syndicates as part of the ongoing professional development of teachers.

All the schools were up-to-date with the new curriculum in that they were using the curriculum objectives and using a variety of assessment tasks. However there was no questioning of the new assessment practice. There was a sense that teachers felt they were being dictated to by someone else, be it the school hierarchy, ERO, BOT, parents, politicians, the curriculum, or mathematics resource teachers. This sense of powerlessness was expressed as *show me what to do and I'll do it*. There was no sense of energy, debate, evaluation of, and judgement on the assessment practice advocated in the mathematics curriculum. In fact teachers felt overwhelmed by the administrative demands of objective-based assessment and the ongoing changes in all curricula areas. Providing assessment task resource material and showing teachers how to use it and mark it gives procedural knowledge or empirical experience. What appears to be lacking is the theoretical knowledge of why changes should be made to assessment practice and, for example, of the difference in lower, middle and higher order thinking tasks in mathematics (de Lange, 1995).

The second theme arising from the analysis is the issue of professional judgement, which brings into question the education of teachers. Professional judgement is based upon a person's understanding of the nature and purpose of a mathematics assessment task grounded in an understanding of the nature and purpose of mathematics and mathematics education. Many studies (e.g., Ellis, Miller-Reilly & Pfannkuch, 1998) on teacher development aimed at improving or changing teaching state that knowing about new teaching strategies is not sufficient. An important ingredient, among others, is that teachers must discuss or study the theoretical basis or rationale for the new teaching method. The same criterion should apply to the introduction of new assessment strategies.

Bodin (1993) refers to the notion of professional test developers supplying teachers with good assessment devices that will "lead them to a better understanding of the knowledge of each student" (Bodin, 1993, p. 140). Whilst agreeing that assessment tasks would be improved greatly by publishers and test developers understanding and taking cognisance of the research in mathematics education, this research suggests that unless teachers understand the theoretical underpinnings of the assessment they will continue to choose, modify and alter such tasks to cater for their perception of mathematics and for their perception of their students. At the initial stages of implementing the curriculum there appears to have been a failure in addressing the theoretical knowledge associated with the introduction of new mathematics assessment strategies and with mathematics education itself. This is a system failure, not a teacher failure.

Warren and Nisbet (1999) indicate that a relationship between primary teachers' beliefs about the uses of assessment and their actual assessment practice exists. My interviews with these teachers tends to confirm such a relationship. However the similarity in beliefs about assessment and in the practice of assessment and also what is shared and not shared amongst teachers suggest that the school system enculturation process formulates how teachers operate in the school environment. In this 1995 "snapshot" of assessment in mathematics education, one learns that teachers have responded to the requirements of the curriculum by assessing students using a variety of assessment strategies and through learning objectives. The adoption of a mastery-based assessment approach has been the requirement of the system to enforce accountability. This means that the original aims of the writers of the mathematics curriculum to provide an integrated mathematics experience for students, which would allow higher order thinking skills to flourish, probably has been thwarted. But also perhaps thwarted is the politicians' aim to improve accountability, since the teacher in the system makes her own judgements. The question remains as to what the 2005 snapshot will reveal. Will teachers move away from objective-based assessment practice? Will students be experiencing an integrated mathematics assessment approach? Or will national or regional tests be driving assessment in the classroom?

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Primary School Students' Statistical Thinking: A Comparison of Two Australian States

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A framework for teaching and assessing statistical thinking comprising four constructs and four levels for each construct, has been developed and the framework validated using data from 20 US students in Years 1 – 5. The same validation procedures were implemented in two different cohorts, totalling 40 subjects, of Australian students in Years 1 - 5. Lower levels of coherence were found. This paper reports the Australian data, seeks to address reasons for the differences and compares the levels of performance between the Australian and US students.

Background

International pressure for reform in statistical education (e.g. Australian Education Council, 1994; Lajoie & Romberg, 1998; National Council of Teachers of Mathematics, 1998; School Curriculum and Assessment Authority and Curriculum and Assessment Authority for Wales, 1996) has stimulated research on statistical thinking. This has occurred largely in the primary grades, where the focus has been mainly on graphing rather than on broader aspects of data handling and analysis (Lajoie & Romberg, 1998; Shaughnessy, Garfield, & Greer, 1996). Mokros and Russell (1995) have investigated areas such as data organisation, while data modeling has been studied by Lehrer and Romberg (1996) and graph comprehension by Curcio (1987), and Friel, Bright, and Curcio (1997).

A study by Jones et al. (1998) has developed a framework of students' statistical thinking that could be used to inform instruction. To date, the validation of this framework has been carried out with data from US elementary school students. The study reported in this paper extends the validation by using data from Australia to: (a) validate the statistical thinking framework from an international perspective; and (b) compare the statistical thinking of Australian and US students in Years 1 - 5.

Theoretical Perspectives

The Statistical Thinking Framework (Jones et al., 1998) incorporates four key constructs adapted from Shaughnessy et al., (1996): describing data, organising and reducing data, representing data, and analysing and interpreting data. It is based on previous research (Aberg-Bengtsson, 1996; Curcio, 1987; Friel, Bright, & Curcio, 1997; Mokros & Russell, 1995) and has been validated in the US study (Jones et al., 1998).

The first construct, *describing data*, involves finding explicit information in a visual display, recognising graphical conventions, and making direct connections between the original data and the display (Curcio, 1987). The second construct, *organising and reducing data*, incorporates mental actions on data such as ordering, grouping, and summarising (measures of central tendency and spread) (Moore, 1997). The third construct, *representing data*, involves the construction of visual displays that represent different organisations of data. The fourth construct, *analysing and interpreting data* includes what Curcio (1987) referred to as "reading between the data" and "reading beyond the data" (p. 384). It involves recognising patterns in the data, and making inferences, interpretations, and predictions from the data.

The existence of four levels of statistical thinking across each of the four constructs was demonstrated in the US validation study which used a case study approach with 20 students from Years 1-5. Level 1 is associated with *idiosyncratic* thinking that is often unrelated to the given data and focuses on students' own personal data banks. Level 2 is seen to be *transitional* between idiosyncratic and quantitative thinking with students beginning to recognise the importance of quantitative reasoning and trying to make sense of the data. Level 3 reveals evidence of *quantitative* reasoning as a basis for statistical judgments with the beginning of analysis of data from multiple perspectives. Level 4 incorporates *analytical* and numerical reasoning in data exploration and shows evidence of being able to make connections between different aspects of the data (Jones et al., 1998).

Method

The two Australian samples each consisted of 20 students with four students in each sample being purposefully selected by class teachers from each of the cohorts of Years 1–5. Teacher assessment of student achievement in mathematics were the bases for selection with two students from the middle 50% and one from both the lower and upper quartiles of each grade level. Different schooling patterns resulted in variation in the students' ages among the two samples. The first Australian sample (Aus 1) (mean ages by grade - 5.5 years through 9.7) came from a private school in a provincial city in north Queensland. The second Australian (Aus 2) sample (mean ages by grade - 6.0 years through 10.2) was selected from a suburban public school in Sydney. Seventy-five percent of the students in the latter school came from families with a non-English speaking background, whereas the students in the private school were from families with English speaking backgrounds.

The process used to validate the framework for these two samples was similar to that used in the earlier study (Jones et al., 1998). It involved three components: (a) interviewing and analysing target students' responses to a Statistical Thinking Protocol based on the Framework and comprising tasks from three separate contexts (see Appendix 1 for tasks from one of the contexts); (b) examining the stability of these students' thinking over the four constructs; and (c) elaborating the distinguishing characteristics of each thinking level. This validation compared the statistical thinking of students in each of the two Australian states and separately with that of students in the USA. Qualitative analysis was used in all three aspects of the validation.

The Statistical Thinking Protocol [STP] comprised seven questions associated with describing the data, seven with organising and reducing data, three with representing data, and six with analysing and interpreting data. It was administered individually to each of the 40 students by a member of the research team. Students' responses to the STP open-ended questions and probes were audio taped and transcribed, and student drawings and graphs were also collected. As well, notes were taken by each interviewer.

Jones et al. (1998) developed a coding rubric in the US validation study and this was used by the two Australian researchers. A double-coding procedure (Miles & Huberman, 1994) was followed in which the two Australian researchers independently coded all questions for each student's interview protocol. The coding rubric led to each question being coded by construct and the student's level of thinking. The two Australian researchers then met to compare and negotiate thinking levels on each question. Following this negotiation, each student's dominant level of thinking for each construct was determined by identifying the student's modal level of thinking for all questions associated with that construct. The reliability in coding between the Australian researchers on determining levels was 88% which is comparable to the US figure of 85%.

Results and Discussion

Tables 1-4 show the median levels by constructs within each grade for the three samples of primary school students. In using the median as the centre for each set of student data, we assumed that the levels data were interval, that is, that the intervals between thinking levels are equal. Given that the levels are consistent with Biggs' and Collis's (1991) developmental theory model, this assumption seems reasonable.

Table 1
Describing Data Displays: Median Statistical Thinking Levels

Sample/ Year	One	Two	Three	Four	Five
Aus 1	1.5	2	2	3	2.5
Aus 2	1	2.5	1.5	2.5	2.5
USA	1.5	1.5	3	3	3

Table 2
Organising and Reducing Data: Median Statistical Thinking Levels

Sample/ Year	One	Two	Three	Four	Five
Aus 1	1	1	2.5	2.5	3
Aus 2	1	1.5	2.5	2	2.5
USA	1.5	1.5	2.5	3	2.5

Table 3
Representing Data: Median Statistical Thinking Levels

Sample/ Year	One	Two	Three	Four	Five
Aus 1	1.5	1.5	3	3	3
Aus 2	1	1	3	3	2
USA	1.5	1.5	3	3	3

Table 4
Analysing and Interpreting Data: Median Statistical Thinking Levels

Sample/ Year	One	Two	Three	Four	Five
Aus 1	1.5	2	2	2	2
Aus 2	1	1.5	1.5	2	2
USA	1	1.5	2	2.5	3

Growth of Thinking Levels

From the four tables it can be seen that (a) the median thinking levels on all four constructs for the Australian and US samples generally increased or remained constant with increasing grade levels, although this was not always a smooth transition, especially in the Aus 2 sample; (b) the median levels for the Australian and US samples in Years 1 and 2 were similar on most constructs with the Australian samples being slightly higher by Year 2 on describing data displays but lower than the US sample in Year 5 on this same construct; (c) the pattern of median levels across the five grades for the Aus 1 and US samples was identical for representing data; (d) the median levels for the US sample in Years 3, 4 and 5 were similar to or higher than their Australian counterparts in the same grades; and (e) the Aus 2 sample generally performed at a slightly lower level than the Aus 1 or US samples.

From the overall profiles for the Australian and US students there appears to be similar growth patterns and trends. Differences between the data of the Australian and US samples may be attributed to age differences between corresponding grades (in Australia students are about a year younger than their counterparts in the same grade in the US), different curriculum emphases at each grade (for example, sample Aus 2 does not meet probability in its curriculum, perhaps significantly decreasing the students' potential exposure to data handling), and differences in the sampling, such as the high number of non-English speaking background students in the Aus 2 sample.

Stability Across Constructs

With respect to stability across the four constructs, the raw data for all 40 Australian students were examined to determine the numbers of students who exhibited the same level of thinking on at least three of the four constructs. An example of this consistency can be seen in Figure 1 which shows the levels for the Year 1 students in sample Aus 2.

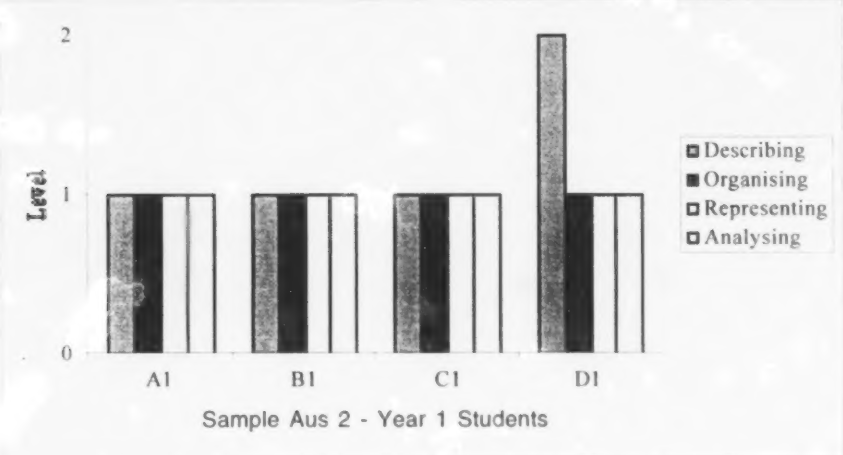

Figure 1. Levels on each construct for Year 1 students in sample Aus 2.

The consistency figures (at least three levels equal) for the Australian samples were: Aus 1 - 65% and Aus 2 - 45%. The corresponding consistency data from the US study (Jones et al., 1998) was 80%. Hence, while the levels for sample Aus 1 were relatively stable and coherent across all four constructs, those for sample Aus 2 were less so. In the case of the

Aus 2 sample, there were a number of language and cultural issues which need to be considered when interpreting these data. Figure 2 shows these less consistent levels for the Year 2 students' in sample Aus 2.

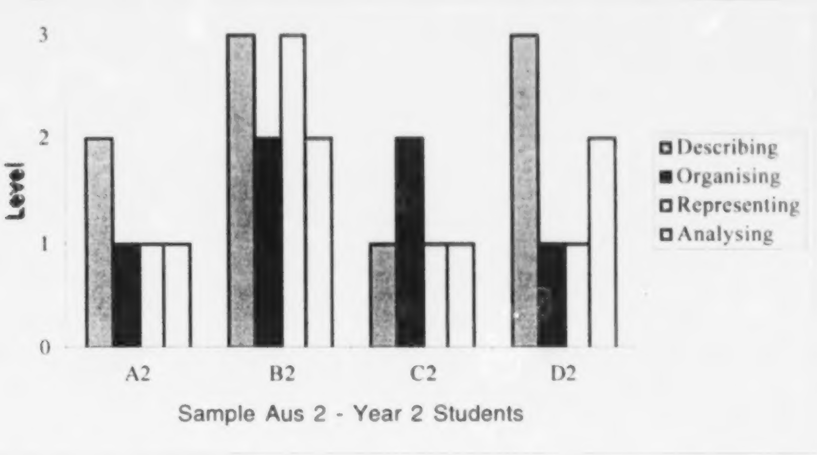

Figure 2. Levels on each construct for Year 2 students in sample Aus 2.

Exemplification of Levels of Thinking

Analysis of the students' statistical thinking across all three samples exemplified the distinctive features of each level of the Statistical Thinking Framework. These are illustrated in responses of students from the different samples.

Level 1 - Idiosyncratic. In answer to the question "About how much did Pete score each day?", a Year 1 student from the Aus 1 sample said "Ten ... maybe he likes ten", while another from Aus 2 suggested "Some days - on Monday, Tuesday, Wednesday, Friday and Thursday, they go home and play after school". When asked "About how many friends would you expect to come to Sam's place each week during the summer holidays?", two Year 1 students from the Aus 1 sample suggested "Two ... because his Mum wouldn't be so busy" and "One ... because ... like you're having too much friends over".

Level 2- Transitional. In response to "What's the average number of Beanie Babies for each child?", a Year 4 student from the Aus 1 sample answered "four … 'cause there's two people with four". This student seemed to have a developing concept of average as the mode. In response to "How many Beanie Babies does each child have?", a Year 5 student from the Aus 1 sample explained that "they should have around 4 or 5 or 3 ... cause if the kids are little, instead of having really hard toys and hurting themselves, they all have soft toys". Here we see a mixture of idiosyncratic and quantitative responses with this student trying to make sense of the data as best she could.

Level 3 - Quantitative. In response to the question about Susie's average score, a student in Year 2 from sample Aus 2 concentrated on the mode and said "3, 3 is the most number in there". A good example of a quantitative response was given by a Year 5 student in the Aus 1 sample in answering the question "About how many friends came to visit Sam each day?" She said "About 3. ... Well lots of numbers are close to three. Its about an average of three". This is approaching a level four response as it does exhibit some analytical and numerical reasoning.

Level 4 - Analytical. In response to "What was Pete's average score?", a Year 2 student from Aus 2 said "Made the average, plussing all together and divide by 5 - I learned that at Vietnamese school".

Educational Importance of the Study

A National Statement on Mathematics for Australian Schools (1991) emphasised Chance and Data as one of the strands that should be part of mathematics curricula in Australia. Various state education departments have developed or are developing revised curricula to reflect recent research into enhancing students' learning of areas such as Chance and Data. While there has been a small number of research studies into how children think about statistical concepts, there is a need for further research with a focus on informing instruction (e.g., Fennema et al., 1996). As well as seeking to address the void in research-based knowledge of students' statistical thinking, this study investigated students' statistical thinking across different cultures in accord with Shaughnessy's (1992) recommendation.

The Australian sample Aus 2 contained a range of children from different ethnic groups whose mathematical experiences were likely to be more varied and who would have had English as a second or third language. This is likely to have accounted for the greater variation in the results for this group compared with sample Aus 1 and the US sample who were mainly first language English speakers. One aspect of this study that needs to investigated further is the relevance of the different contexts in which the probes are embedded. The Beanie Babies and the Bean Bag Game may have been less relevant for the students in the Aus 1 and Aus 2 samples although there are similar toys available in Australia. One of the members of the research team is planning to conduct a similar study with children in a remote Indigenous community in Australia to investigate their statistical thinking and to explore the appropriateness of the tasks.

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Appendix 1: The Beanbag Game and Questions

Susie and Pete play a beanbag game each day after school. In this game they try to throw a beanbag into a circle. Each takes 5 turns every day. The number of times each child landed the bag in the circle last week is shown below.

Scores for Susie and Pete

Day	Susie	Pete
Monday	3	3
Tuesday	4	0
Wednesday	3	5
Thursday	2	3
Friday	3	4

Describing data displays

Q. 1 What do these two sets of scores tell you?

Analysing and interpreting data

Q. 6 How are Susie's scores and Pete's scores alike? Explain.
How are Susie's scores and Pete's scores different? Explain.

Organising and reducing data

Q. 1 About how much did Susie score each day? How did you figure that out?
What was Susie's average score? How did you figure that out?

Q. 2 About how much did Pete score each day? How did you figure that out?
What was Pete's average score? How did you figure that out?

Q. 4 What set of scores (Susie's or Pete's) has the greatest spread, or do they both have the same spread? Explain.

Appendix 2: Statistical Thinking Framework

Construct / Level	Level 1 Idiosyncratic	Level 2 Transitional	Level 3 Quantitative	Level 4 Analytical
Describing Data Displays (D)	•gives a description that is unfocused and includes idiosyncratic/irrelevant information; has no awareness of graphing conventions [e.g., title, axis labels] of the display •does not recognize when two different displays represent the same data OR indicates some recognition but uses idiosyncratic/irrelevant reasoning •considers irrelevant or subjective features when evaluating the effectiveness of two different displays of the same data	•gives a description that is hesitant and incomplete, but demonstrates some awareness of graphing conventions •recognizes when two different displays represent the same data, but uses a justification based purely on conventions •focuses only on one aspect when evaluating the effectiveness of two different displays of the same data	•gives a confident and complete description and demonstrates awareness of graphing conventions •recognizes when two different displays represent the same data by establishing partial correspondences between data elements in the displays •focuses on more than one aspect when evaluating the effectiveness of two different displays of the same data	•recognizes when two different displays represent the same data by establishing precise numerical correspondences between data elements in the displays •provides a coherent and comprehensive explanation when evaluating the pros and cons of two different displays of the same data
Organizing and Reducing Data (O)	•does not group or order the data or gives an idiosyncratic/ irrelevant grouping •does not recognize when information is lost in reduction process •is not able to describe data in terms of representativeness or "typicality" •is not able to describe data in terms of spread or gives idiosyncratic/relevant responses	•gives a grouping or ordering that is not consistent OR groups data into classes using criteria they cannot explain •recognizes when data reduction occurs, but gives a vague/irrelevant explanation •gives hesitant and incomplete descriptions of data in terms of "typicality" •invents a measure--usually invalid--in an effort to make sense of spread	•groups or orders data into classes and can explain the basis for grouping •recognizes when data reduction occurs, and can explain the reasons for the reduction •gives valid measures of "typicality" that begin to approximate one of the centers (mode, median, mean); reasoning is incomplete •uses an invented measure or description which is valid, but the explanation is incomplete	•groups or orders data into classes in more than one way and can explain the bases for these different groupings •recognizes that data reduction can occur in different ways and gives complete explanations for the different reductions •gives valid measures of "typicality" that reflect one or more of the centers; reasoning is essentially complete •uses the range or an invented measure that has the same meaning as the range
Representing Data (R)	•constructs an idoisyncratic or invalid display when asked to complete a partially constructed graph associated with a given data set •produces an idiosyncratic or invalid display that does not represent or reorganize the data set	•constructs a display that is valid in some aspects when asked to complete a partially constructed graph associated with a given data set •produces a display that is partially valid but does not attempt to reorganize the data	•constructs a display that is valid when asked to complete a partially constructed graph associated with a given data set; may have difficulty with ideas like scale or zero categories •produces a valid display that shows some attempt to reorganize the data	•constructs a valid display and when asked to complete a partially constructed graph associated with a given data set; works effectively with scale, zero categories,.. •produces multiple valid displays, some of which reorganize the data
Analyzing and Interpreting Data (A)	•makes no response or an invalid/irrelevant response to the question, "What does the display not say about the data?" •makes no response or gives an invalid/incomplete response when asked to "read between the data" •makes no response or gives an invalid/incomplete response when asked to "read beyond the data"	•makes a relevant but incomplete response to the question, "What does the display not say about the data?" •gives a valid response to some aspects of "reading between the data" but is imprecise when asked to make comparisons •gives a vague or inconsistent response when asked to "read beyond the data"	•makes multiple relevant responses to the question, "What does the display not say about the data?" •gives multiple valid responses when asked to "read between the data" and can make some global comparisons •tries to use the data and make sense of the situation when asked to "read beyond the data;" reasoning is incomplete	•makes a comprehensive contextual response to the question, "What does the display not say about the data?" •gives multiple valid responses when asked to "read between the data" and can make coherent and comprehensive comparisons •gives a response that is valid, complete, and consistent when asked to "read beyond the data"

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They 'Did the Sheet' but didn't 'Feed the Mind'

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This study describes the effect mathematics newsletters had on the home learning environment of two six-year-old boys with low achievement in mathematics. One boy's mother maintained a high level of contact with the school, the other a low level. The mothers of the low achievers appeared to use the newsletter as a means of assessing progress rather than a stimulus for mathematics experiences in the home. These results are contrasted with an earlier study undertaken with high-contact parents.

Rationale

In 1999, recognising the role the critical role of parents, the New Zealand Ministry of Education launched a large scale public information campaign targeted at low income Maori and Pacific Island families. The children of these families are over-represented in the lowest achievers, and parents often have low levels of contact with schools (Wylie, Thompson & Lythe, 1999). The "Feed the Mind" message of the campaign sought to involve parents in simple activities at home which would support the development of early literacy and numeracy skills. The campaign was reported as successful, according to measures of increased awareness of numeracy and literacy activities amongst the target audience (Gerritsen, 2000), but whether it had an *effect* on school achievement or activities inside the home remains unknown. The beneficial effect that home–school contact has on academic achievement has been widely documented (Davies, 1996; Epstein 1987; Ramsay, Harold, Hawk, Marriott, & Poskitt, 1992). In the context of mathematics, Merttens summarizes the situation with the comment that: *We know from the evidence of both research and practice that the single biggest factor in children's educational success is their parents.* (1999, p79).

Researchers have identified a number of factors which may contribute to create increased academic achievement of children of high contact parents. Teachers may be more motivated to spend their time and energy on the children of parents with whom they have frequent contact (Goldring, 1991); and there is some evidence that teachers limit their expectations for children whose parents have little contact with the school (Wylie & Smith, 1995; Wylie, Thompson & Lythe, 1999). Wylie and Smith (1995) also hypothesized that being able to discuss early indications of future difficulty, before a real problem arose, enabled parents to more effectively support their children's learning.

Parent's ability to support their child's learning was the focus of Tizard and Hughes (1984) research which contrasted support for language development at home and in preschool. They argued that the problem of low achievement was not due to a lack of motivation or resources in the home environment, but a lack of familiarity with the culture and content of the formal educational programme. They discuss a "split" between home and school learning that prevents home learning from being utilized by the child at school. This split is less extreme for middle-class children, but when the home culture is significantly different from the dominant culture in the school *"School learning" may become increasingly separated from any learning which takes place outside the classroom.* (Tizard & Hughes, 1984, p. 265).

Homework is often the only link between home and school for mathematics learning and the "right kind" of homework has many benefits (Anthony & Knight, 1999). However, Merttens cautions that:

> *how* it is done is more important than *that* it is done, because the "how" will make the difference between supporting children's learning and facilitating the collaboration of their parents, or it becoming yet another element in an education system in which the benefits are differentially available, according to socio-economic class, gender or ethnicity. (Merttens, 1999, p. 79)

Responding to reports that parents preferred a newsletter as a means of finding out about school activities (Cattermole & Robinson, 1985) and suggestions that newsletters were the best first step in increasing parental involvement in mathematics (Cooper, 1996); Savell (1998a) developed and trialled a regular newsletter in the junior primary school. The newsletter informed parents about the mathematics being taught in the classroom and suggested related activities that could be done at home. After interviewing parents to determine the effect of such a newsletter Savell (1998b) questioned the value of traditional mathematics homework. She found that parents were able to individualize mathematical activities within a context that was personally meaningful to their child in a way impossible for a teacher to achieve within the constraints of the classroom. The newsletters also generated increased levels of "maths talk" which benefited siblings, and parents kept newsletters for future reference. Parents welcomed the opportunity to share in their child's mathematics education, and believed that the support they could provide (given the information in the newsletter) would enhance their child's school performance.

Savell (1998b) suggested that giving parents information about school mathematics in newsletters was an effective way of decreasing the "split" for low-contact parents. However, respondents to Savell's study were all parents with relatively high levels of contact with the school (and their children were above average in their mathematics achievement). There is a clear need to find out more about how such mathematics information is used in the homes of low-contact parents.

In this paper two case studies are used to contrast the reactions to mathematics newsletters within the home environments of two children who have difficulty with school mathematics. Qualitative research allows the researcher to gather more detail about a situation than would be possible from quantitative methods (Rogoff, 1987). When case studies are used and subjects are selected who differ markedly on a particular dimension, the researcher is often able to understand the significance of unique events rather than having to generalize norms to predict future behaviour (Liebert & Wicks-Nelson, 1981). This technique has been particularly illuminating when considering the relationship between home environment and mathematical competence in young children (Young-Loveridge, 1989; Leder, 1992).

In this study, although both children are boys from similar socio-economic backgrounds who have difficulty with mathematics; their mothers have opposite levels of home-school contact. One boy's mother maintains a high level of contact with the school, and the other boy is from a home with a low level of home-school contact. The findings from these two case studies are compared with the results of Savell's (1998a) research involving high-contact parents with above-average to high-achieving children.

Method

Mathematics newsletters for parents, describing classroom Beginning School Mathematics (BSM) activities (Savell, 1999) were distributed semi-regularly to all the children in three junior primary school classrooms for a period of nine months. After this, teachers were interviewed about the effect of the newsletter and asked to classify children according to their mathematics achievement and the level of contact teachers had with the parents. Two boys from a single class were identified as being at opposite extremes in terms of the amount of contact their parents had with the school.

The researcher interviewed the mothers of these two children in their own homes. Interviews were semi-structured to ensure coverage of key points, but also allow parents to comment freely on the aspects of the school mathematics programme or home support that most interested them. The interviews took between 40 and 50 minutes and were transcribed by the researcher.

The two boys, Tony and Simon, were both described by their teacher as "weak in mathematics". Tony is the elder of two children. His mother was classified as "high contact" both by her own evaluation and by the teacher's. The teacher reported that Tony appears to be achieving at a level "not too far under average", but *only because of the level of interest his Mum has*. Simon was the youngest of three children. His mother was classified as "low contact" both by herself and Simon's teacher.

Results and Discussion

Irrespective of levels of school contact, both mothers appeared to view the newsletter in a similar way to a sample of work that their child might have brought home. They talked about going over the work and seemed to see the sending home of either newsletter or completed worksheets as a method the teacher chose to communicate with them about their child's mathematical competence. Simon's mother talked about using the newsletters in a similar way to the work samples: *They would come out of his book bag and I would always look at them. We went through them and he showed me what he did. I asked him about them and he showed me what he did.*" When asked about the latest newsletter, Tony's mother confused the newsletter with a sample of work: *"I just pulled out of his book bag and he had got them all right actually. I said " Wow! You did really well on these.*"

This was in contrast to the reaction of the parents in the previous study. The parents of high achieving children had interpreted the newsletter as general information for them, rather than something specific to "work through" with their child. All the parents in the previous study had kept the newsletters, and mention was made about them being useful for younger siblings. In the homes of these low achieving children, only Tony's mother had kept the newsletter, and she did not anticipate referring to it with his younger sister. "[I kept it] *just for ongoing education for him. So that I can see whether he is up to speed on his maths and that sort of thing.*"

These differing perceptions of the purpose of the newsletter were reflected in the way the parents had used the newsletter. In the previous study most parents said they had tried specific activities from *Maths News*, and adapted suggestions to suit their resources and personal interests of their children. However they reported the greatest effect of the newsletter had been to increase the visibility of mathematics in their daily lives. All parents in the previous study reported an increase in their "maths talk" as a result of the newsletter.

In the homes of Tony and Simon, the effect of the newsletters was limited to the discussion of their content as they were removed from the book bag. When Tony's mother was asked how she supported his mathematics at home, it was clear that she viewed her support as limited to checking up on his progress.

> Well I guess I am the sort of person that unless I get something like the Maths News that I wouldn't really have any ideas. Going just by those Maths News he seemed to be just keeping up with it ...
> *Did you actually try out some of the activities?*
> Yes and he seemed to know the answers and he seemed to get them right so ...

Similarly, Simon's mother reported that, other than "going through" the newsletters as they came out of the book bag, they had not affected what she did at home with Simon in any way. She hadn't tried any of the suggested activities.

In contrast to the parents in the previous study, neither of the low achievers' parents mentioned any increased awareness of the classroom mathematics programme as a result of the newsletters. Given that Tony's mother was classed as high-contact it is perhaps surprising that her answer to the question "What do you think you might see if you went into the mathematics classroom?" showed a view of mathematics which was limited to early number work: *"Probably just counting with beads and things. I'm not too sure really. I'd guess just any sort of small object that they might be putting into sets"*. In contrast, Simon's (low-contact) mother was more confident in her answer and demonstrated a broader view of junior mathematics:

> I would see rods, those coloured rods. They would be counting, measuring ...measuring with feet and then a tape measuring tape ... worksheets; ones like a picture of a cat and another cat equals two cats. I'm not sure what else. I think it would be that picture sort of maths.

In answering this question, Simon's mother may be drawing on her previous experiences helping in the classroom with Simon's older brothers. Although she readily identified herself as low-contact, she indicted that she had been more involved with older siblings.

> The homework sort of slacks off when you get to your third, you know. Like with your first you are really into everything. With your second you are a bit more relaxed and with your third you feel like you just don't do anything. It is a time thing now. It really is with homework.

During the interview she contrasted the welcome that was extended to parents in the school that her eldest son had attended with Simon's current school where she felt less welcome.

> It is made quite clear that you are not allowed to waltz in and out of the classroom ... [The teacher] doesn't even want you sitting on the seat outside waiting. ...She says that if the child sees you that distracts them and she doesn't want that happening. To the children she is wonderful ...[but] you certainly don't feel you can go in there and raise any concerns.

In fact both parents were quick to compare the culture and content of the current mathematics classroom with others that (for Simon) had been experienced, or (for Tony) had been described by close friends. Each of these parents felt that there was something not-quite-right as far as the mathematics programme was concerned. They were concerned that the faults (as they saw them) with the current school's system were affecting their child's mathematics performance, but neither had discussed their concerns with their child's teacher.

> [At another School] They seem to do a lot more maths. More maths and more variety too. From what [my friend] is saying, this child is about six months older, and she can count backwards from twenty and that sort of thing. She has a lot of skills which he doesn't seem to have or be learning... I guess it depends on the individual teacher doesn't it? I do think that communication with this teacher isn't the best.
> *The communication you get from her or the communication she gets from you?*
> It is both really, because she is not always approachable.

> I've often wondered when they start doing things like maths. Because in the juniors it is always the reading that comes home...and then all of a sudden when they are in Year 4 or 5 they have maths they've got to learn in Year 4 and 5. The earlier they bring in the maths the easier it is. ...[In junior school they should] just make a few additions, sort of everyday homework, and it will pay off by the time they reach Year 6. Maybe if it was just something. Like they have to draw a triangle, a square and a circle and just draw them into their notebook. Measure the sides of it or something. Simon loves his homework.

The above quotes (from Tony and Simon's mothers respectively) hint at the difference in the perceptions of their role between the parents of these two low-achieving children and the parents of the previous study. Simon and Tony's mothers clearly put the responsibility for the mathematics learning of their children onto the shoulders of the classroom teacher. In the previous study, parents of the high achieving children believed their children would benefit if they took an active role and were eager to share an educational partnership with the teacher.

Conclusion

While no suggestion is made that the findings of two case studies are generalisable to the homes of all low-achieving children, there appear to be some significant and interesting findings that are worthy of future research. Perhaps the most surprising thing about these two case studies involving parents at opposite ends of the spectrum for home-school contact was their similarities. The research evidence would have led us to expect marked differences between the environments of the high-contact and low-contact parents, yet none were found. The lack of distinction may be because both boys come from similar middle-class European homes. In addition, although Simon's mother was classified as low-contact by the teacher, she reported that she had been more involved with earlier children.

Yet these case studies of the homes of low-achievers did reveal a very different reaction to the newsletter than the earlier research. Consequently the study provides a valuable insight into the differential in the effect of the newsletter for these high and low achieving children and their parents.

- The parents of the two low-achieving children seemed to believe that the place for mathematics teaching was the school. The home contribution to their child's mathematics learning was limited to positive encouragement and interest. Parents of high-achievers were also interested and positive, but they saw value in reinforcing school learning with related activities that they personalized for their child's interests and abilities.
- In the families where the children experienced difficulty with mathematics the newsletters were seen as a 'test' of the child's progress or an exercise to be worked through. In the high achieving homes the newsletter was interpreted as a prompt for the development of mathematical activities and discussion of mathematics in the family's shared daily lives.
- Newsletters generated "maths talk" and greater visibility of mathematics in the homes of high achievers. No effect was reported in the homes of low achievers.
- In the homes of high achievers newsletters were kept for future reference and valued as a guide for assisting younger siblings. In the homes of low achievers, no reference was made to the newsletters after their initial reading.

It is worth noting that a "high-contact" classification by the teacher did not necessarily equate to familiarity with the mathematics programme. Even parents who have a high level of contact with the junior primary school may never inquire or observe *mathematics* teaching. Similarly, this case study revealed that the "low-contact" parent is not necessarily out-of-touch with the classroom.

As with earlier case study research, the parents of low achievers did not appear unmotivated, both mentioned the emphasis that they felt the school (rightly) placed on reading rather than mathematics. Nor were they uninterested in their children's mathematics, rather they had a different view of their role.

The findings of this study suggest the possibility that it may *not* be the level of school contact that determines the quality of the support children receive from their parents. Rather parents' perceptions of the role they have in their child's mathematics education may be the critical factor; and this could cause the positive correlation between home-school contact and achievement. Parents who believe they have a valuable role to play in their child's mathematics education, and confidently assume a "teaching" role, are positively affecting their children's mathematics achievement in school. Generally such parental beliefs are associated with high levels of school contact, hence the correlation between parent-school contact and school achievement. On the other hand, parents who presume that teaching mathematics is the sole prerogative of the school give their child no such advantage, so their children are often amongst the low achievers. These parents would often have lower levels of contact with the school in general, and would see little value in discussing mathematics content or activities with the teacher or their children.

Implications

This study has important implications for schools seeking to improve mathematics achievement. The parents of these two low-achievers reported that they did not feel comfortable "dropping in" to chat with the teacher about their child. In fact the low-contact mother reported that she had been told that the presence of parents outside the classroom was considered a disruptive influence. Lower levels of school contact result when school policies restrict parental access to teachers to pre-arranged appointments after school hours. Such regulation of parent-teacher contact sets up barriers to communication. While allowing the teacher to work more "efficiently" without the "interruption" of parents, the policy sends a clear signal to parents about the relative importance of teacher and parent's roles in educating their children. Parents may respond to this signal by assuming they are not expected or encouraged to take an active part in their children's mathematics learning. Yet in this study it was the assumption that what they do is important and worthwhile that appeared to be a critical difference between the parents of the high and low achieving children.

Perhaps if the teacher encouraged informal contact, there would be more opportunity for the sharing of anecdotes of mathematics progress. The joint discussion and subsequent elimination of small problems, before they become large hurdles, has already been identified as a possible reason for the advantages high-contact parents enjoy. However, this study suggests that the beneficial effect of such discussions may actually be attributed to the unspoken message they send to parents, rather than the solutions discussed. When a parent is welcomed into the classroom, and the teacher shares a child's minor problems with them, the parent receives the message that the teacher values their input and believes they have an important part to play in their child's mathematics education.

Although a newsletter is preferred by parents and is often recommended to schools as a first step, it seems that its effect in the homes of low-achieving children may be limited. It was not that they were unmotivated or disinterested. It was not that the language or format of the newsletter that made it difficult to access the necessary information. These parents both read through the newsletter with their child and questioned them to assess their competence in the skills described. What they were unaware of was the critical teaching role that parents play in their child's educational success. The parents of these two low-achievers saw mathematics learning as the domain of the school, so the newsletter alone did not prompt them to increase the visibility of mathematics in the home or create personalized activities for their child.

For these two low achieving children, the "Feed the Mind" slogan (with its associated messages that parents have an important role to play in their child's education and anyone is capable of assisting their child with literacy and numeracy) appears to be the component that is missing from their homes environment.

In the previous study Savell (1998b) argued that newsletters (which enabled parents to individualize mathematics in meaningful contexts for their children) may be more effective than "homework" exercises or tasks that are essentially the same as "schoolwork". This study suggests that parents of low-achieving children may not be able to gain such advantages from a newsletter. It is possible that rather than leveling the playing field (so that high- and low-contact parents can all be informed about classroom programmes) the mathematics newsletters may have a differential effect: They may be advantaging some children and not others, thereby actively widening the gap between high and low achieving children.

Priority must be given to investigating the effect of the "Feed the Mind" campaign in the low-income homes and schools that it was targeted at. Did the extensive advertising have the desired result? Or are parents still failing to comprehend the power they have to affect their child's school performance? This study suggests that the next question for further research into parental involvement seems to be not "How can we get low-contact parents more involved in their mathematics learning?" but "How can we demonstrate to the parents of low-achieving children that the mathematics learning is not something that happens only at school?" "How can we convince parents of the value in the mathematics they share out-of-school with their children?"

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The Use of Investigative Methods in Teaching and Learning Primary Mathematics in the Northern Province Schools (South Africa): A Case Study

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Research was conducted in the Northern Province Primary Mathematics Projects schools. In view of the high failure rate among matriculation students, the researcher believed that it was necessary that ways to improve mathematics understanding should be devised from the foundation level of schooling. He further believed that children learn better in a cooperative non-threatening classroom environment where they are free to participate. Research was conducted to evaluate the use of constructivist and investigative teaching and learning methods as employed by teachers in a number of primary schools in an area in one province in South Africa, the poorly performed Northern Province. A pilot study was conducted with two experimental schools and two control schools. The experimental schools were operating under the PMP (Primary Mathematics Project) and were supported by expert key teachers who attended courses at Leeds University in England while others were trained locally in South Africa. The methods employed to conduct the research were both qualitative and quantitative, embracing questionnaires done by 174 teachers, interviews with 55 people, written tests by 350 pupils and observations in classes. The results revealed that student performance in the experimental schools where a constructivist and investigative approach was used was better than the control schools. They indicate that children from PMP schools developed a better understanding of mathematics.

Background

Mathematics is a subject that is regarded as a prerequisite for many careers. Because of its status every child must study this subject up to grade 9 or grade 10 level in South Africa. The methods used in teaching should equip the child with the skills to cope with any relevant numerical or mathematical situation outside the classroom. Our traditional ways of teaching place the teacher in a pivotal position in teaching, with the underlying assumption that the child is an empty vessel. The methods of teaching are teacher-centred, with a teacher acting in a way that serves to 'fill up' the students' empty minds. Traditionally, children are not encouraged to question the teacher. Their only participation is perhaps writing down what the teacher tells them to do. Yet even after such 'expert' guidance by the teacher, children are failing. Mathematics performance and results in schools remain poor, while the skills acquired in the classroom do not seem readily transferable.

The theories of educationists like Haire et al (1978) and Howe (1988) suggest that children learn better when they are actively engaged in learning mathematics. How 'active' should this engagement be? At the very best, it should involve pupils talking together about what they are trying to learn, the theory being that meaning is negotiated through discussion (Sebela, 1990). Piaget (1970) and Skemp (1979) have written about a constructivist model for learning. They suggest that knowledge is not transferred ready-made to children, but that children are active participants who construct their own meaning to form new ideas from their own understanding.

A project had been initiated in the Northern Province to address the problem of students failing in mathematics. The underlying rationale of this project is the constructivist postulate that children learn mathematics from their own experiences, out of which they build meaning for themselves. They assemble their own knowledge that arises from a range of problem solving experiences, and through collaborating in discussions they make mathematical sense of it (Ernest 1989, Lakatos 1976). What has become known as the investigative approach is being used to see whether it can alleviate the problem our teachers and pupils are experiencing. The purpose of the study was to find ways of helping parents, teachers, lecturers and pupils/students to develop a positive attitude towards mathematics. Some of the research questions that were to be investigated:

Were the children who are taught through investigative teaching able to develop better understanding of mathematics than those who were taught through the traditional methods where they only listen and react to the teacher's instructions?

Did the attitudes of parents, teachers and lecturers in areas where the investigative approach was used change?

Were the teachers inspired to use group work in their lessons?

Were the teachers encouraged to use discussion and investigation during their mathematics lessons?

The Setting

Constructivist Theory, Cooperative Learning and Investigative Teaching as Followed by Teachers in the Project Schools

Two experimental and two control schools were involved in the study. In the experimental schools teachers were encouraged to change their traditional teaching to incorporate constructivist methods, using ideas such as those of Suggate (1995) and Hoines (1974). Teachers were guided to do the following as constructivist practitioners:

- Not to preach or dictate to the children about what they wanted them to know, and to change their role from that of teller or authority. To change their role from that of teller or authority of all the knowledge children should know or should not know.
- To use problem-centred approaches to teaching, basing their teaching on tasks or problems to which children can relate, to give children opportunity to construct their own mathematical understanding.
- To encourage children to create new mathematical knowledge by reflecting on their physical and mental actions. To make what they learn or acquire meaningful by integrating it into their existing structure of knowledge.

Learning is a social process in which children grow into the intellectual life of those who are around them (Bruner, 1986). A mathematics classroom should be seen as having a culture where children are involved in negotiation, explanation, evaluation and the sharing of ideas. Olivier (1989) also supports the idea that students are not supposed to be seen as passive receivers of knowledge.

Students learning through constructivist approaches are seen to be autonomous and self-motivated, and they in turn acquire the understanding that they do not acquire mathematics knowledge from their teacher but from their own explorations, thinking and participation in discussions. The role of the constructivist teacher is to guide and support the student's invention of viable mathematical ideas rather than to transmit "the correct" adult ways of doing mathematics (Clements & Battista, 1990, p. 35). Teachers using this method must be able to provide taks and opportunities for dialogue that bring about appropriate conceptual reorganisation in the pupils. Mason (1989) emphasises the same idea by indicating that pupils make sense of the world by assembling fragments of their experiences into some sort of story. This is again stressed by Gadanidis (1994) that

> ...students acquire new knowledge through an active process of assimilation and accommodation, where new as well as existing knowledge is transformed as students construct more inclusive schema of understanding. This theory contrasts the view that students acquire new knowledge through a passive process of transmission, where knowledge is passed unchanged from teacher to student. Even in learning situations that are considered passive, such as a lecture, students construct their own understanding. In the constructivist view of mathematics learning, the question is not whether students construct understanding of mathematics concepts but rather how good are their constructions. Thus, a constructivist teacher's emphasis is on creating learning environments that help students create good schema of mathematics understanding. (p. 93)

In this approach pupils are encouraged to exchange points of view rather than accept an imposed idea of correct and incorrect methods. They are seen essentially as scientists who constantly test out hypotheses about the world in which they are living as social beings (Piaget, 1970).

An important aspect of reconstructing education in South Africa is the transformation of classroom practices to include approaches to learning and teaching that are "learner-centred and non-authoritarian and encourage the active participation of students in the learning process" (ANC, 1994, p.69). Presently group work is beginning to appear in our classes. Group work and co-operative learning are essential in constructivist teaching and learning. Cooperative learning is thought to work well in mathematics because it pulls pupils of differing abilities together. Pupils of different backgrounds are placed in situations where they can all participate equally in learning. In his discussion of peer group cooperative learning, Brodie (1995) suggested that peer groups tend to provide more equality in interaction and to allow pupils more control over the learning situation and the knowledge developed. A report by Lilford (1995) supports cooperative learning by quoting from the Bible: "two are better than one, because they have a good reward for their toils. For if they fall, one will lift up his fellow, but woe to him who is alone when he falls and has not another to lift him up" (p.124). Lilford quotes a grade 11 girl, Jennifer Holem, who says: "Working together and having the chance to give your opinion boosts your confidence and encouraged you to work ever harder. My marks went up by five percent." (p. 126) The five essential elements of cooperative learning as suggested by Oberholtzer (1992) are as follows: positive interdependence, face-to-face interaction, individual accountability, interpersonal social skills, and analysis of how the groups work.

In groups, pupils are dependent on one another. The interaction is a verbal exchange among them. They are themselves responsible for learning to use the material. Children learn to work together with other people just as they need to in life as a whole. They are also developing their social responsibility. Davidson and Maher et al (1990) have argued further that small groups provide a social support mechanism for the learning of mathematics, while Brombacher (1995) states that the notion of cooperative learning comprises more than just group work. This is understood to mean that group work is where individuals are working with a particular person dominating, that is, where only one member is accountable and the rest are depending on him/her. Cooperative learning means that all group members are accountable and they reach agreement through mutual argument and consensus while learning from one another. In cooperative learning each member is used as resource.

The investigative teaching approach requires the teacher to have confidence in him/herself and be flexible in his/her approach. Pupils are given the freedom to interrupt the teacher. This approach (investigative) discourages teachers from being simple imparters of knowledge (Lerman, 1983). Instead they become facilitators of the pupils' activities, so as to allow learners to construct meaningful mathematical knowledge for themselves (Ernest 1989; Lakatos, 1976). The approach opens room for back stage participation that allows the children to talk and debate; the teacher stands aside and becomes the observer of what the children are doing, children are more actively involved than the teacher, whose role is nevertheless to facilitate the participation within the groups. Tonkin (1995) supports the idea that discussion between pupils is essential in the learning of mathematics, and suggests that investigations provide ideal opportunities to promote group work and interaction between pupils.

In the study teachers were encouraged to adopt James' (1992) model of investigations which proposes steps:

- The Do and Talk model - where learners do practical activities and the teacher feeds in the necessary language.
- Towards Recording model - children write down what they have been discussing.
- Practice and Consolidation - they use the textbooks to practice what they have learnt to perfection.

Learners from the experimental schools were trained on problem-solving strategies used by different educationists like Polya and Simmons. Polya's (1945) strategies are: draw a diagram, examine the special cases, introduce notations to, and noticing quantities that increase or decrease together. Simmons' (1993) problem-solving process includes the following steps: situation, problem, mathematical problem, using mathematical knowledge, techniques and insight, and the solutions.

Teachers from the experimental schools were encouraged to be facilitators in their classes and to play the following roles:

- encouraging personal responses, introducing controversy, summing-up and generalising,
- tolerating noise or silence, clarifying, informing and supporting.

Key Teachers' Roles

Key Teachers support the schools in their ways of introducing the new approaches to teaching. Some of these people were college lecturers from different regions in Northern Province.

Methodology

Both quantitative and qualitative methods of research were used in this study. Observations of classes and interviews with learners and parents were used. As well, questionnaires were given to teachers.

Teachers' Questionnaires and their Structure

A questionnaire was given to teachers from both project schools and the control schools in order to establish a sound comparative basis. It was comprised of a group of sixteen multiple-choice questions, followed by a second group of five that required detailed responses.

The multiple-choice questions were to be answered on a five point scale ranging from 1 (I disagree strongly) to 5 (I agree strongly). The 16 questions were grouped into four subject areas, each with four questions. The four subject areas were Standards, Support, Teaching Approach, and Attitudes. Examples of the questions may be found in Figure 1.

Section A; Standard

3. The mathematics results in my class are better than in the previous years. 1 2 3 4 5

Section B: Support

5. I am supported by my principal. 1 2 3 4 5

Section C: Teaching approach

12. The investigative approach work as well for weak students as for bright students. 1 2 3 4 5

Section D: Attitudes

15. The students like to work in groups. 1 2 3 4 5

Section E: (Detailed Responses)

17. What do you consider as a major area of need within the Mathematics Education at your school?

....

....

18. What sort of things do you do in class when you use the investigative approach?

....

....

1. What do you like about the new approach?

....

....

Figure 1. Examples of questionnaire items.

The objectives of designing the questionnaire items were:

- To check whether teachers have sufficient support from their colleagues and the principals for the approach to be used in their classes.
- To find out whether investigative teaching and learning has really changed the pupils' attitudes to mathematics learning.
- To identify problems that occur when the approach is employed in the classroom in mathematics lessons.
- To find out what teachers like or dislike about the new approach to teaching.

Children's Test Writing in Mathematics

Tests were given to the learners as a way of finding out whether the methods used can really help children to pass their examinations. A pre-test and a post-test were given to both experimental schools and control schools. The tests were written by grade 2, grade 3 and grade 4 students. The nature of the tests was planned in such a way that students can use any method they have been trained with before. They were to use their own methods or the prescriptive methods given by their teachers.

Results

Reflections Made by the Teachers on their Work in the Different Schools

- Children must be given chance to express and formulate word problems from their environment using concrete objects.
- Colour should be used, because it can help to understand the concept "set", but it should be realised that colour should not interfere with addition of concrete objects.
- Children should be allowed to write what they say.
- A student when giving a report should be positioned so s/he is visible to all the class.
- The teacher must take care with individual learners dominating the classroom discussion.

Test Results

In all cases the results favoured the experimental schools irrespective of class size. These tests convinced the researcher that children taught through investigative teaching and cooperative learning methods performed better than the children who are taught using traditional methods. Constructivist and cooperative learning with investigations are the methods that could be used in our schools.

Summary of the Responses

1. The test results: These indicate that the investigative approach produces results so superior that the approach is to be recommended in all schools in the province.
2. Principals, teachers, advisers and inspectors' interviews: They recommend that in-service training courses be organised to train teachers on how to implement the constructivist approach and how to control and deal with pupils learning through group work.
3. Teachers' questionnaire: Overall, responses to the questionnaire evinced a positive feeling that the standard of mathematics teaching, the teaching approaches, the support for mathematics and the attitude towards mathematics have all changed for the better since the implementation of the new approach.

Primary children are said to be influenced by the teacher's motivation and love for the subject. From this research we are led to believe that children love mathematics. Can't this love be extended upward into the next levels of schooling? What remains a source of surprise and concern is the children's failure when they go on to secondary schools.

Conclusions

The Research Findings

- Children who are taught through investigative teaching methods are achieving higher test scores than children who are taught through the use of traditional methods.
- Most of the teachers who have been exposed to the approach in courses or through literature are in favour of it. These teachers are inspired to use group work. They recommended its implementation in all schools in the province.

Implications for Mathematics Education

- The research findings indicate that many changes need to be effected in mathematics education. Teaching styles and methods need to be reviewed. Colleges of Education also need to change their pre-service and in-service training methods with respect to mathematics teaching and learning.
- Management in schools and classrooms needs to be studied. Principals and teachers should be encouraged to do research on how to manage the changing classroom situation, and also on how to manage the school as a whole, in conjunction with community involvement.
- The curriculum should be changed to allow room for the use of investigative methods.

Limiting Factors of the Study

- Lack of literature on science and mathematics education in local libraries.
- School boycotts caused the researcher to have difficulty in visiting the schools to collect data. (i.e., on some days there were no children nor teachers in schools).

Recommendations

- Schools should be structured so as to reduce the number of children in each class to be a manageable number; that is, primary schools less than 40; and secondary schools a maximum of 30.
- More in-service training courses be provided for all mathematics teachers at the beginning of the year, with follow-up courses each quarter. Such follow-up courses could be organised for each district.
- Teachers who attend in-service training courses be given support by key teachers to help them implement what they have learned at the courses.
- Seeing that the primary school children enjoy learning through the use of learning aids, it is strongly recommended that teachers should be trained on how to produce their own materials and equipment.
- Children in a mathematics learning situation should not only be seated in a way that they all face the chalkboard; they should rather sit in a way conducive to conversation or dialogue without interruption.
- Expert mathematics teachers should not be obliged to change subjects frequently (in some schools principals are allocating new subjects to teachers every year).
- Provincial libraries should be equipped with books on mathematics education.
- School-based INSET (In-service Training) should be supported and implemented in the different schools and districts as this will empower the teachers.

Some further research questions identified by the researcher were

- There is a need to conduct research on the effect of interviews and dialogue in mathematics teaching and learning.
- Long-term research should be done to find whether the attitudes of parents, the community and students towards mathematics is improving, and to look for strategies for developing positive attitudes.

The results of this research are being communicated with the department of education such that implementation of constructivist methods can proceed in all schools in the province. This is being reinforced by Curriculum 2005 that demands teachers to change their chalk-and-talk methods of teaching.

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Researching Numeracy in the Middle Years

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Numeracy has become a key priority of government policy in recent years generating a number of numeracy-related projects. This paper will provide an overview of the *Middle Years Numeracy Research Project* commissioned by the three employer groups in Victoria in 1999 and report on the design and development of a *Student Numeracy Profile* incorporating rich assessment tasks and involving teacher judgements in the assessment of student numeracy performance. Preliminary analysis of the *Student Numeracy Profile* instrument suggests that it is an effective measure of numeracy at this level with the potential to provide useful insights into instructional strategies for middle years' students.

Numeracy has become a major priority area for all Federal, State and Territory governments in recent years. This is most clearly seen in the *National Goals for Schooling in the Twenty First Century*, which includes the goal "that every student should be numerate and be able to read, write, spell and communicate at an appropriate level" (MCEETYA, 1999). It is also evident in the *National Literacy and Numeracy Plan (DEETYA, 1998)* and the *National Literacy and Numeracy Benchmarks* for Years 3, 5 and 7 (National Numeracy Benchmarks Taskforce, 1997). While literacy has always been a high priority of government, the focus on numeracy is relatively recent. Prior to the 1990s, numeracy was primarily viewed as a sub-set of literacy and largely confined to the non-school sector (that is, industry training, adult education and labour market programs). This heritage is reflected in *Being numerate – What counts?* (Willis, 1990) and *Numeracy in Focus*, a joint publication of the Adult Literacy Information Office and the Adult Basic Education Resource and Information Service (Riordan & Tout, 1995).

While the first use of the term *numeracy*, is generally credited to the authors of the Crowther Report in 1959 (Cockroft, 1982), current Australian usage owes more to the view presented in *Maths Counts* (Cockcroft, 1982, p.11) than it does to the view expressed in the Crowther Report or the more recent *United Kingdom National Numeracy Strategy*, which quite clearly privileges number over other aspects of mathematics.

> Numeracy is defined as more than knowing about numbers and number operations. It includes an ability and inclination to solve numerical problems, including those involving money and measurement. It also demands familiarity with the ways in which numerical information is gathered by counting and measuring, and is presented in graphs, charts and tables (quoted in Rhodes et al., 1998).

A broader, more encompassing view of numeracy is evident in *Numeracy = Everyone's Business* (AAMT, 1997), the Report of the Numeracy Education Strategy Development Conference held in Perth in February 1997, which cites the following policy positions.

> To be numerate is to have and be able to use appropriate mathematical knowledge, understanding, skills, intuition and experience whenever they are needed in everyday life. Numeracy is more than just being able to manipulate numbers. The content of numeracy is derived from five strands of the mathematics curriculum … as described in the National Statement and Profiles (from *Numerate Students, Numerate Adults*, Department of Education Tasmania, 1995).

> Numeracy involves abilities which include interpreting, applying and communicating mathematical information in commonly encountered situations to enable full, critical and effective participation in a wide range of life roles (from *Literacy and Numeracy Strategy 1994-8*, Department of Education Queensland, 1994).

> Numeracy is the effective use of mathematics to meet the general demands of life at home, in paid work, and for participation in community and civic life. ... the National Numeracy Benchmarks will refer to the contribution that school mathematics and other areas of learning make to the development of students' numeracy. They will incorporate the development of students' understanding and competence with number and quantity (ie, measurement), shape and location and the handling and interpretation of quantitative data. (National Benchmarking Taskforce, 1997)

These views are variously reflected in the statement developed as a consequence of the meeting in Perth.

> To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work and for participation in community and civic life. In school education, numeracy is a fundamental component of learning, performance, discourse and critique across all areas of the curriculum. It involves the disposition to use, in context, a combination of: underpinning mathematical concepts and skills from across the discipline (numerical, spatial, graphical, statistical and algebraic); mathematical thinking and strategies; general thinking skills; and a grounded appreciation of context (AAMT, 1997, p. 15).

This view suggests that numeracy involves a much broader range of knowledge skills and attributes than can be simply characterised as "basic number facts and skills", that numeracy is simultaneously both bigger than and smaller than school mathematics, and that numeracy has a powerful role to play in facilitating communication and decision making. Perhaps more importantly, it acknowledges that numeracy is *relative to context*. That is, an individual is neither numerate nor innumerate, simply more or less numerate depending on the context in which he/she is expected to operate (Willis, cited in AAMT, 1997). This means that different views of numeracy requiring different assumptions and approaches will be needed for different levels of schooling and contexts (Scott, 1999).

In the early years, the focus is predominantly on the development of counting and number, especially the development of place-value understanding, and the disposition and strategies needed to apply these ideas and skills in relevant contexts. This is evident in *Count Me in Too* (NSW), the *First Steps Program* (WA), and the *Early Years Numeracy Research Project* (Victoria). Although some of these key ideas and strategies need to be consolidated and extended in Years 5-9 (e.g., to support an appreciation of fractions and decimals), numeracy education at this level needs to focus on the development of a broader repertoire of knowledge, skills and dispositions. It needs to equip students to deal effectively with a much larger range of more complex problems involving applications of measurement, data, chance and space. At this stage, cross curriculum links and cross curriculum learning become more relevant in developing strong foundations in numeracy.

Different assumptions and approaches to numeracy at this level are also needed as the socio-cultural context of teaching and learning in the middle years is substantially different to the early years (see Hill & Russell, 1999). Disengagement, low-self-esteem, absenteeism, and poor learning behaviours, concomitant with the pressures of adolescence and school transition, present significant challenges to teachers in the middle years. Trying to uncover what works and does not work in this context is also compounded by the enormous variation in student (and to some extent teacher) knowledge and confidence. Attempts to improve numeracy in the middle years will need to consider not only the contribution that school mathematics might make but also how to impact entrenched classroom cultures, scaffold discourse elements and engage learners more effectively. While existing research in mathematics education provides some guidance, translating these into consistent, coherent advice appears to be more problematic than it does in the early years.

Background and Aims of the Middle Years Numeracy Research Project

In Victoria, as in other Australian States and Territories, there has been a significant increase in government funded literacy and numeracy projects in recent years. The success of the *Early Literacy Research Project (1995-97)* in adopting a coherent, school-wide approach based on the design elements described in the *General Model of School Improvement in Literacy* (Hill & Crevola, 1997), led to the requirement that all future projects be framed in terms of the nine design elements of the model. That is, beliefs and understandings, leadership and coordination, school and classroom organisation, structured classroom teaching program, standards and targets, monitoring and assessment, intervention and special assistance, home, school and community partnerships and professional learning teams.

The *Middle Years Numeracy Research Project: 5-9* (MYNRP) was commissioned by the Victorian Department of Education (now DEET), in partnership with the Catholic Education Commission of Victoria (CECV) and the Association of Independent Schools of Victoria (AISV) in June, 1999. The project is one of a number of other projects commissioned between 1998 and 1999 under the auspices of the DEETYA funded *Successful Interventions: A Secondary Literacy and Numeracy Initiative*. Within this context, the MYNRP was conceptualised by the Victorian industry partners as the second stage of a planned, three-stage research program on numeracy in Years 5 to 9.

As described in the *MYNRP (Stage 2) Project Brief* (DoE, 1999), the aims of the current research project are to:

- inform the development of a strategic and coordinated approach and advice for schools about the teaching and learning of numeracy for students in years 5-9;
- trial and evaluate the proposed approaches in selected Victorian schools; and
- identify and document what works and does not work in numeracy teaching including those students who fall behind.

In addressing these aims, the MYNRP will build upon the work of the *Early Numeracy Research Project* (ENRP), the *Middle Years Research and Development* Project (MYRAD), the *National Numeracy Benchmarks* (DETYA, 1999) and the design elements of the *General Model of School Improvement* (Hill & Crevola, 1997). The MYNRP was specifically required to investigate the role of structured mainstream classroom programs, additional assistance, and the role of parents, mentors, peer support, and professional development, in improving numeracy outcomes at this level.

Key research questions include the following.

- To what extent can numeracy be assessed by the use of structured, rich assessment tasks involving teacher judgements?
- What aspects of current practice appear to be associated with successful and unsuccessful numeracy performance at this level?
- To what extent does the implementation of the *Draft Numeracy Strategy* contribute to improved numeracy performance?
- What characterises the learning-relevant experiences of students at either end of the student numeracy performance spectrum?

The view of numeracy adopted by the Perth conference (AAMT, 1997) has been used to inform the design and implementation of the MYNRP. Although, it is expected that models of 'good' numeracy teaching will principally be found in mathematics classrooms, the project will also explore the contributions made by other curriculum areas and students' 'out-of-school' experience to improved numeracy performance at this level.

Research Design and Methodology

Given the duration and largely explorative nature of the project, it was felt that the most appropriate research design was a quasi pre-pre-post design involving a representative sample and a structured sub-sample. In this case, data was collected from a relatively large sample at the outset to obtain baseline data on student numeracy and some insights into what appeared likely to impact numeracy performance at this level. A smaller, structured sub-sample was selected to participate in a trial phase aimed at finding ways to improve numeracy performance. This sample was selected on the basis of the student numeracy data (high and low) and the extent of evidence concerning supportive school-wide policies and practices (rich and poor) with a view to determining what worked, where and why.

An action research methodology is being used in the Trial schools consistent with a socio-constructivist view of learning (e.g., Lerman, 1998; Crawford & Adler, 1996). This approach is based on the *Maths in Schools* model (Siemon & Ferguson, 1993; Montgomery, 1996) which is premised on the view that success will be greatest where teachers and schools are supported to work on what they believe to be the most appropriate strategy for their situation (Siemon, 1989). This approach was chosen as the research could not know in advance what numeracy-specific strategies would work most effectively in what settings at this level. More particularly, it could not know what would work in combination with the more general strategies suggested by recent middle years research. Specifically, the need for far fewer teachers at years 7 to 9, the importance of sustained and uninterrupted periods of time to engage in meaningful learning experiences, and the need to provide a curriculum that is relevant, negotiated and integrated (see Hill & Russell, 1999; Venville et al., 1998; Cumming, 1996).

To achieve the aims and address the research questions described above, the MYNRP was designed in terms of four broad phases involving the collection of quantitative and qualitative data and the development, implementation, trial and evaluation of a *Draft Numeracy Strategy*. The four phases are briefly described below.

Phase 1: Benchmarking (September-November 1999)

The aim of this phase was to build on the Stage 1 environmental scan (MAV, 1998) by collecting large-scale, sample data on student numeracy performance as well as data related to the design elements under consideration. This phase also included a review of relevant literature and related policies and projects, particularly the *Early Years Numeracy Research Project* (ENRP) and *the Middle Years Research and Development* Project (MYRAD). A formal expression of interest process was used to select a structured, representative sample of 47 schools (27 Primary and 20 Secondary) to participate in the first phase of the project.

The numeracy assessment instruments were developed on the basis of the *National Numeracy Benchmarks for Years 5 and 7* and recognised 'best-practice' models of assessment, that is, 'rich assessment tasks' (e.g., Clarke et al., 1996) and performance assessment (Callingham, 1999; Griffin, 1998). Tasks were sourced to reflect the three strands of the *Benchmarks* and chosen or adapted to ensure they provided an opportunity to demonstrate both content and process outcomes. While the majority of the short tasks were sourced and/or adapted from *Effective Assessment in Mathematics* (Board of Studies, 1998), task-specific scoring rubrics were developed by the project team. Parallel forms of a written test comprising 5 open-ended tasks were developed for Years 5-6 and Years 7-9 respectively. An extended classroom task aimed at evaluating all student understanding of pattern and relationships, *Street Party*, was sourced from the Tasmanian INISS Project (see Callingham, 1999). Teachers administered the *Student Numeracy Profile* (i.e., a form of the written test and the extended classroom task) over two 40 to 50 minute sessions. Scoring rubrics were provided for all tasks and teachers were asked to assess the student's responses using a computer-readable score sheet. Even though it was a difficult end-of-year period, complete data sets were obtained from just over 7000 students in Years 5 to 9.

Data related to the design elements described in Hill and Crevola (1997) were gathered via an auditing process, that is, school principals were asked to complete a questionnaire and then meet with project team members to verify and/or clarify the school's response as relevant. Schools were invited to submit a portfolio of relevant policies and programs if they desired. Complete data sets were obtained from all 47 schools.

An initial professional development day was held to introduce school contact people to the project and to provide some training in the use of scoring rubrics.

Phase 2: Development of the *Draft Numeracy Strategy* (December 1999 – February 2000)

The aim of this phase of the project was to develop draft advice for trial schools about what appeared to be working in relation to numeracy education in Years 5 to 9. This advice was prepared on the basis of the knowledge and data obtained from Phase 1 and was framed in terms of the nine design elements of the Hill and Crevola (1997) model for school improvement. A Briefing Conference for Trial Schools was held in February 2000 to elaborate the *Draft Numeracy Strategy* and introduce the trial phase of the project including the action planning process to be used. Twenty schools, a sub-set of the original sample of schools, were selected to participate in the Trial Phase on the basis described earlier.

Phase 3: Trial of the *Draft Numeracy Strategy* (March-October 2000)

The aim of this phase of the project is to evaluate the effectiveness of the *Draft Numeracy Strategy* by monitoring the design, implementation and effectiveness of the school-based action plans in relation to student numeracy performance. This will be monitored by the collection of 'missing cohort' data in March and whole cohort data in October using parallel forms of the *Student Numeracy Profile*. The data obtained will enable some degree of within-school comparisons to be made and some limited between school comparisons. School visits, teacher journals, classroom observations and student reflections will be used to monitor the school-based implementations of the *Draft Numeracy Strategy*. Individual case-studies involving student 'outliers' (those identified at either end of the *Student Numeracy Performance* spectrum) will also take place during Phase 3 to explore the specific classroom factors that impact numeracy performance.

Phase 4: Preparation of Advice (September-November 2000)

The aim of this phase of the MYNRP is to revise the *Draft Numeracy Strategy* on the basis of the data and experience derived from Phase 3 with a view to preparing advice that will inform numeracy improvement in the middle years of schooling.

The Student Numeracy Profile

The aspect of the MYNRP that will be reported here concerns the effectiveness and potential of the *Student Numeracy Profile* (SNP) to assess student numeracy performance across Years 5 to 9 and to inform classroom practice. As described above, the two components of the SNP were administered and assessed by classroom teachers using previously supplied scoring rubrics and computer-readable score sheets. The data was analysed using SPSS and *Quest*, a Rasch modelling tool developed by Adams and Khoo (1993). While it is inappropriate to report the aggregate data at this stage, it is appropriate to explore the findings in relation to the SNP itself. Figure 1 shows the *Quest* analysis of item estimates, or thresholds (p = 0.5), for all students across all items.

```
------------------------------------------------------------------------
                                       |    cd  .4
                                       |
  3.0                                  |
                                  X    |
                                  X    |
                                  X    |
                                  X    |
                                 XX    |     sprb.2 frcc.2
  2.0                            XX    |
                               XXXX    |     sp8b.2
                              XXXXX    |     trvc.2 docm.3
                             XXXXXX    |      sp10.2 wlka.2 frca.2 frcc.1 cd  .3
                            XXXXXXX    |     frcb.2
                         XXXXXXXXXX    |     sp4 .3 sp9    docm.2
  1.0                    XXXXXXXXXX    |     sp8a   wlka.1
                    XXXXXXXXXXXXXXX    |      sp8b.1 dayb.2 cd  .2 bird.2 medb.1
                     XXXXXXXXXXXXXX    |      trpb.2 sp4 .2 sp10.1 frcb.1
               XXXXXXXXXXXXXXXXXXX     |     busb.2 trvb   spna.2 meda.2
              XXXXXXXXXXXXXXXXXXXX     |     trvc.1 frca.1
                    XXXXXXXXXXXXXXX    |      trpa.2 sp5b.2 sp6 .2 wlkb.2 daya
   .0          XXXXXXXXXXXXXXXXXXX     |     calc.3 docm.1 meda.1
                   XXXXXXXXXXXXXXXX    |     busa.2 trpa.1 spnb.1
                    XXXXXXXXXXXXXXX    |      busb.1 sp6 .1 wlkb.1
                  XXXXXXXXXXXXXXXX     |     calc.2 sp5b.1 dayb.1
                         XXXXXXXXXX    |
                        XXXXXXXXXXX    |     trpb.1 sp5a   sp7
                           XXXXXXXX    |     calc.1 bird.1
 -1.0                         XXXXX    |     trva
                            XXXXXXX    |
                              XXXXX    |     sp4 .1 cd  .1
                                XXX    |     busa.1
                                XXX    |     sp3c
                                 XX    |     spna.1
 -2.0                           XXX    |     sp3b
                                  X    |     sp3a
                                       |
                                 XX    |
                                       |
                                       |     sp2
 -3.0                             X    |
                                       |
                                       |
                                  X    |     spl
                                       |
 Each X represents 26 students (22 cases perfect scores, 26 cases 0 scores)
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Figure 1. Quest analysis of item estimates (Thresholds), all-on-all (N = 6861, p = 0.5).

The coded items on the right refer to a particular part of each task (**sp2** for instance refers to question 2 of the *Street Party* task, possible score 0 or 1). The location of the coded item indicates the point at which students scoring at this level (indicated by the X's immediately to the left) have a 50% chance of satisfying the scoring criterion indicated by the number

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following the full stop. For example, **spna.1**, indicates part (a) of the *Draw a Spinner* short task. Its location indicates that those students whose total score relative to the whole population is represented by the X's on the left have a 50% chance of achieving a score of 1 on this part of the task (possible scores 0, 1 or 2).

Of all the items used in the SNP, only one task, *How Far to Walk*, lay outside the boundaries set by the Rasch item fit analysis suggesting that all the others were measuring a similar construct. This outcome is heartening as it suggests it is possible to measure a complex construct such as numeracy using rich assessment tasks incorporating performance measures of content knowledge and process (general thinking skills and strategies). Further trialing is needed to determine the reliability and validity of the SNP over time. Another encouraging feature of the overall item analysis presented in Figure 1 is that the degree of difficulty of the SNP appears to be appropriate for the cohort tested.

The most promising result is that the scaling indicated by the location of the items in Figure 1 suggests that it will be possible to generate a Numeracy Profile with rich descriptions of distinct developmental levels of numeracy performance based on the content and process analysis of the items included in the SNP. This has important implications for the design of structured, numeracy-specific teaching and learning materials which not only support students to acquire the necessary content knowledge and skills but also scaffold a hierarchy of skills, strategies and dispositions concerned with mathematical thinking and problem solving. Callingham (1999) has reported a similar developmental pattern for the *Street Party* task which she has described using the SOLO taxonomy. Further analysis is needed before this can be reported in more detail.

Quality feedback on student understanding has long been recognised as a powerful factor in teachers' decision-making concerning classroom practices (Fennema et al., 1998; Griffin, 1998). A major task within this phase of the project will be to evaluate the impact that feedback, in the form of school and individual reports which are referenced to the emergent numeracy profile, has on the teaching and assessment approaches of Trial schools.

Conclusion

The *Student Numeracy Profile* appears to provide a sound basis for the development of a Numeracy Profile that provides rich descriptions of the key stages in the development of numeracy at this level. While the Numeracy Profile will be informed by further trialing of the SNP, it will be used in the Trial phase as a framework to guide the design and implementation of school-based teaching materials and assessment tasks. It is hoped that such an approach will encourage schools to adopt a multi-disciplinary approach to locally relevant problems or issues as a means of increasing the engagement of middle years' learners (see Venville et al., 1998). During the Trial phase it is also planned to collect data to help frame advice concerning the design elements under consideration. That is, structured mainstream classroom programs, additional assistance, the role of parents, mentors and peer support, and the role of professional development in improving numeracy outcomes. It is envisaged that this will involve a description of the principles of best practice in the development and implementation of numeracy programs within school mathematics, across the curriculum and in the context of a whole school approach.

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Exploring the Nexus between Assessment and Instruction in Mathematics

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This paper is the result of a school/university partnership that was formed to enhance teachers' professional development. The partnership explored pedagogical practices that promoted students' mathematical thinking and sense making. The links between assessment and instruction are described using 'accounts of practice' that are grounded in classroom contexts and provide guiding principles for mathematics educators seeking to make assessment more student centred and purposeful. The findings contribute to a growing body of empirically based knowledge related to the nexus between assessment and instruction.

The current emphasis on integrating assessment and instruction is present at a state, national, and international level (Australian Education Council, 1991; Clarke, 1996; National Council of Teachers of Mathematics, 1995; Mathematical Sciences Education Board (MSEB), 1993; NSW Department of School Education, 1996). This emphasis has evolved due to the increase in knowledge about the way students learn, and the way mathematics should be taught (Bransford, Brown, & Cocking, 1999). In essence, mathematics needs to be seen as a way of ordering and explaining our everyday lives. This occurs when teachers plan learning experiences that are relevant and meaningful to students, build on their prior knowledge, and require thinking power rather than the simple recall of known facts and procedures (Fraivillig, Murphy, & Fuson, 1999).

There is common agreement that with a change in the focus of how mathematics should be taught, there should be a reciprocal change in the way we assess students' achievement in mathematics (Bransford et al., 1999; Clarke, 1997; Kulm, 1994; Romberg, 1995). Similarly, De Lange (1995) concluded that in measuring the outcomes of the new forms of mathematics curriculum, educators needed to structure their assessment practices on the same principles. A review of the current literature related to assessment in mathematics provides a conceptual framework that highlights the following principles. Assessment should be:

- integral to, and naturally derived from good instructional practice (Clarke, 1997; MSEB, 1993; Yackel, Cobb, & Wood, 1992);
- accessible, relevant and meaningful to all students so they can show what they know, understand and can do (Lajoie, 1995; MSEB, 1993);
- on-going and cyclic in nature (Bransford et al., 1999; NSW Dept of School Education, 1996); and
- an indicator of students' thinking as well as content knowledge (Bransford et al., 1999; Clarke, 1996; De Lange, 1995).

Clarke's (1997) guiding principles of "constructive assessment" provide further insight into the importance of merging assessment and instruction. He reminds us that locating students' learning at the heart of the assessment process distinguishes constructive assessment. Clarke maintained that assessment should provide opportunities for students to display their mathematical understanding through constructed rather than remembered responses. This notion is symbiotic with authentic assessment (Kerr-Stenmark, 1991; Kulm, 1994; Lajoie, 1995) which describes any assessment that engages students in tasks that reflect "real" contexts. By its very nature, constructive, authentic methods of assessment require a merging of student centred assessment and instruction.

Sullivan (1999) suggests one way of achieving a collection of rich assessment information is through the use of content-specific open-ended tasks that, among other things:

- require a higher level of thinking, creativity and engagement;
- have more than one possible solution;
- are suitable for students at different stages of development;
- are focused on curriculum content; and
- collect rich assessment information.

According to Sullivan (1999), content-specific open-ended tasks cater for different stages of development, promote creative thinking and are an outcome of student centred learning rather than teacher directed learning. It could be argued then, that work samples constructed by students and derived from open-ended tasks provide an authentic method of assessing students' learning that is easily integrated with instruction.

Background to the Study

This paper aims to highlight opportunities for integrating assessment with instruction through accounts of authentic assessment practices from two case studies. It reports on aspects of a larger study completed for an Honours thesis that involved a school/university partnership (Bobis, 1998). The investigation documents the third and final phase of a study that generated accounts of practice taken from four case studies of teachers in a rural central school (K-12) as they incorporated the Working Mathematically outcomes (NSW Board of Studies, 1998) into their classroom practice. The seven primary classes were structured in "Stages" that are reflected in NSW syllabus documents. The teachers were all striving to promote students' mathematical thinking and sense making, characteristics that were reflected in the new outcomes. For this paper, only two of the case studies are reported.

Method

This study adopts a socioconstructivist perspective on learning (Yackel et al., 1992) where learning is seen as an active, social construction of concepts that builds on existing knowledge. This perspective sees the participants in this study as co-learners along with the researcher and the students themselves. One of the participating teachers eloquently described this perspective during a post-lesson interview when she stated:

> What we are actually doing as teachers is exactly what we are asking the kids to do. You know, reflect on what we've done, think about your thinking…verbalise your thinking.

This verbalisation of thinking forms the notion of "accounts of practice" that have been used in this case study. Simon and Tzur (1999) outline "accounts of practice" as an extension of case study research when they explain that:

> The generation of accounts of practice is not an anthropological approach, an attempt by a neutral observer to describe the life of the classroom teacher. Instead, it is an attempt to understand teachers' practice in a way that accounts for aspects of practice that are of theoretical importance to the communities of mathematics education researchers and teacher educators (p. 254).

This paper presents these "accounts of practice" using participant voices taken directly from pre- and post-lesson interview transcripts, teacher surveys and field notes from classroom observations. The accounts are then analysed using the conceptual lenses of a researcher that guide the interpretations of the data. A similar study by Fraivillig et al. (1999)

focused on strategies that promoted children's thinking. They also relied on an in-depth analysis of observed and reported data from skilful teachers in the field. In the present study, the principles for assessment that have been outlined from a review of current literature are used to interpret teachers' perceptions and classroom observations related to the links between assessment and instruction. Pedagogical practices that exemplified the integration of assessment and instruction were identified by using a line by line analysis of transcripts and field notes, combined with annotations that referred to concepts related to the conceptual framework for assessment (Simon & Tzur, 1999).

Results

Case Study One: Kim

Kim wanted to focus on the strategic use of work samples, derived from open-ended tasks and written procedures, that would enhance students' portfolios. She is relatively new to the teaching profession and has three years experience. Kim was teaching a Stage Two/ Three class (Years 4 and 5) during the project, and constantly sought to improve her practice by reading widely and trying new strategies. During this phase, Kim wanted to focus on developing authentic assessment tasks that would generate evidence of students' thinking and metacognitive processes. This evidence, in the form of work samples, was placed in the students' Progress Books, a form of portfolio, and was used during three-way interviews later in the year.

Open-ended tasks. Kim identified open-ended tasks as a key element in developing students' thinking and providing evidence of their thinking processes. One example of an open-ended task used by Kim for a unit on Volume was to Design, Make and Appraise (DMA) a container that would hold three litres. This task was accessible to all students, and directly related to a real context, which gave purpose to the assessment task. In a post-lesson interview, Kim confirmed that:

> The open-ended questions have been the biggest focus for me. I didn't do a lot of these before. The book you gave me helped to make good work samples that I could put into their Progress Books. They were practical because before I was only putting in worksheets and they were boring and didn't show their thinking and how they went about doing the task.

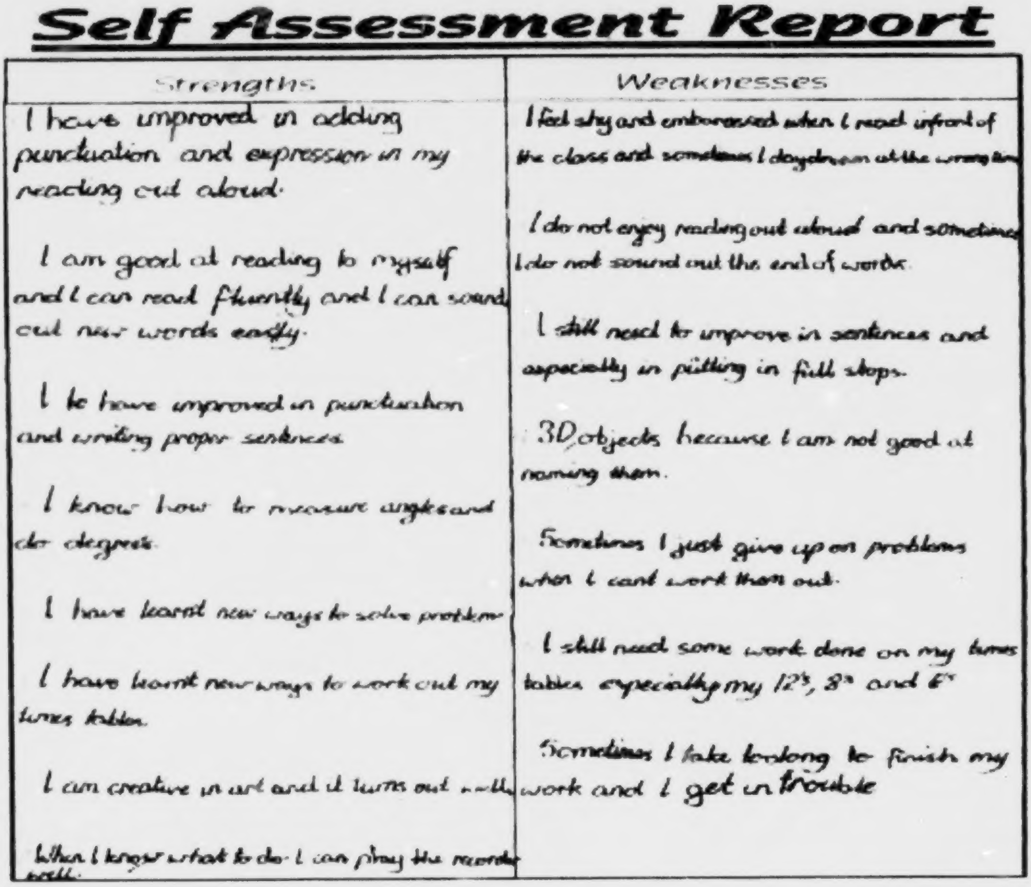
This excerpt demonstrated Kim's aim of developing tasks that would make thinking more visible while providing evidence of student achievement. The book Kim referred to (Sullivan & Lilburn, 1997) proved to be a valuable starting point for professional development in this area. The link between assessment and instruction was reflected in the rich, content-specific open-ended tasks that were developed by Kim and clearly linked with instruction in a post-lesson discussion when she explained that:

> When I'm programming I put in an open-ended task and I identify which tasks they will be and use these as assessment tasks...I think of assessment as I program.

This notion of planning for assessment while programming was also identified by Anna and hence Clarke's (1997) idea of constructive assessment was incorporated with Sullivan's (1999) content-specific open-ended tasks. It appears that assessment tasks that are naturally derived from learning experiences will be more likely to occur when identified during the programming and planning process.

Writing Procedures. Kim also identified the use of writing procedures as an effective assessment task that promoted student language. She cautioned that the confidence of slower learners was an important factor to monitor when asking them to write down a procedure, but it was a "good way to show their thinking". These procedures were used in an attempt to make thinking more visible and assessed students' process and communication skills. They were directly linked to learning experiences and clearly defined aspects of student achievement for assessment purposes while providing opportunities for reflective and metacognitive skills to be developed.

Sharing the purpose and role of assessment with students. Sharing the goals and purpose of assessment with students was evident in Kim's classroom practice. She placed a strong emphasis on self-assessment that involved reflective and metacognitive skills and believed that making the purpose of assessment explicit assisted the students to know which direction they need to improve on. Figure 1 provides an example of Hannah's self-assessment that was placed in her Progress Book and referred to during a three-way interview with Hannah, her parents, and her teacher.



Self Assessment Report

Strengths	Weaknesses
I have improved in adding punctuation and expression in my reading out aloud.	I feel shy and embarrassed when I read infront of the class and sometimes I daydream at the wrong tim
I am good at reading to myself and I can read fluently and I can sound out new words easily.	I do not enjoy reading out aloud and sometime I do not sound out the end of words.
I to have improved in punctuation and writing proper sentences.	I still need to improve in sentences and especially in putting in full stops.
I know how to measure angles and do degrees.	3D objects because I am not good at naming them.
I have learnt new ways to solve problem	Sometimes I just give up on problems when I cant work them out.
I have learnt new ways to work out my times tables.	I still need some work done on my times tables especially my 12s, 8s and 6s
I am creative in art and it turns out well	Sometimes I take too long to finish my work and I get in trouble
When I know what to do I can play the recorder well.	

Figure 1. Hannah's self assessment of her progress.

Hannah's self-reflection clearly provided an opportunity for her to assess her strengths as well as the areas she needed to focus on. Kim reflected on these points during a post-lesson interview:

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> I do tell them when I'm assessing their work. I don't know if that's the right thing or not. It kind of goes with our Progress Books and the idea of self-assessment and I think it's been a combined effort. Then if we do something I tell them what I'm looking for and share the criteria for assessing it. It gives them something to strive for and they know which direction they need to improve on...The students had to explain each work sample to their parents and that helped them to revise their learning. If you have to teach or explain something to somebody else, then you know it better.

Kim's explanation emphasised the important role of communication and reflection in students' learning (Brown et al., 1999). The students' perceptions of assessment were also elicited by asking them an open-ended question: How does Ms H. decide what you know in Maths? The words "test" and "assess" were deliberately left out of the question to prevent any bias towards this terminology. Although the majority of responses related to a maths test of some sort, it was interesting to note that there were a number of responses that indicated alternative ways of knowing. Responses such as "she uses a mind map to see what we know", "she looks at our progress books", and "she reads our procedures" significantly validated Kim's attempts to develop and share the purpose of authentic assessment tasks that were naturally derived from teaching and learning experiences.

Case Study Two: Anna

Anna had a strong desire to "collect effective work samples that would get their (the students) thinking on paper". She has had four years teaching experience and was teaching Stage One (a K-2 composite class) at the time of the study. Anna wanted to develop more purpose to her mathematics lessons. She felt that she was doing a "fair" job but began to wonder at the end of a unit if she and the students had achieved anything or found out anything.

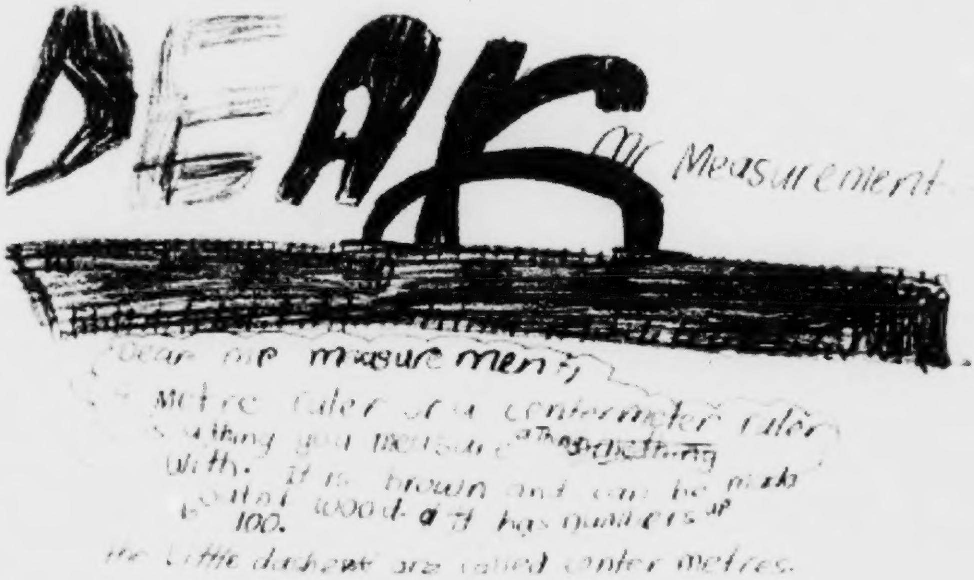
Sharing the purpose of open-ended tasks. Anna's focus for this part of the study was to incorporate open-ended tasks, and develop an achievable purpose for each lesson. She worked on making the purpose of the tasks very clear to the students so they had a direction to work towards and Anna had measurable criteria for student assessment and evaluation of her teaching methods. Anna's desire to elicit students' thinking as part of the learning process fostered the notion of "thinking aloud" which she modelled as part of the teaching/learning process.

Assessment as inquiry. Anna discussed her assessment strategies during a post-lesson interview when she reflected that:

> A lot of the assessment has been hands-on and using their work samples. Lots of observations and questioning and getting them to explain what they're doing and trying to relate them to other things. I use checklists to record the observations and I put comments beside them as well. The open-ended questions help to bring out their misconceptions a lot earlier. It gets the kids thinking about other ways of approaching problems and linking it to other things and see the relation between things. Now my assessment is planned to occur more as you go through.

These reflections highlighted assessment as an ongoing process that occurred naturally during mathematical inquiry. Anna used open-ended tasks at the beginning of a unit of work to find out about the students' prior knowledge and misconceptions. Figure 2 represents a work sample collected at the beginning of a unit of work on measurement. The open-ended task was designed to determine students' conceptual understanding of the metre ruler. The children were told that Mr Measurement (a personal friend) had lost his memory and was highly embarrassed that he couldn't remember what a metre ruler looked like. The students had to draw a picture of a metre ruler and explain everything they could about the ruler to help Mr Measurement get his memory back.

Sharing the inquiry. The work samples were shared as a class group and led to explanations of a metre ruler and an in-depth discussion of how to represent centimetres accurately on the ruler. This process also fostered the metacognitive skills of reflection and monitoring of solutions. These skills were illustrated clearly when another student, Sam, told the class as he shared his drawing that he had made a mistake because he put ten dashes in between each number instead of nine. These work samples and shared explanations indicated many understandings about the metre ruler, but also identified some misconceptions which gave direction for further instruction that matched the students' levels of development.



Dear mr measurement,
Metre ruler or a centimeter ruler
is a thing you measure something
with. It is brown and can be made
out of wood. It has numbers up
to 100.
the little dashes are called center metres.

Figure 2. Annabel's explanation of a metre ruler.

The students in Anna's class were encouraged to explore open-ended tasks and were reminded that there was more than one way to get a solution and possibly more than one solution. Anna confirmed this when she stated:

> That's one good thing about this project, it's highlighted the fact that maths is not just a right or wrong answer. The open-ended questions cater for the differences much better. It keeps them much more interested and more challenged.

Both Anna and Kim used open-ended tasks as assessment opportunities and acknowledged these tasks as an important factor contributing to the students' increased interest and enthusiasm. They suggested that these tasks were more challenging for the students and catered for the differences in their ability. The use of these tasks at the beginning of a unit of work represented a change in pedagogical practice for both teachers and was identified as an effective method for deciding further direction for instruction.

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Conclusion

This paper has highlighted the nexus between assessment and instruction by providing accounts of practice derived from teachers exploring their assessment practices during a school/university partnership. These accounts have highlighted the impetus for integrating assessment with instruction in a number of ways. Firstly, validity is increased by collecting evidence of student achievement that is naturally derived from, and related to everyday instruction because it reflects the intention of the teaching and learning activities. Kim exemplified this notion when she stated that:

> I've always tried to integrate my assessment with instruction. You can't assess them on something they've never done in class. I think the more assessment strategies you use you get a better picture. The more evidence you have, the more accountable you can be.

Secondly, Anna and Kim both identified using open-ended tasks at the beginning of a unit to assess students' prior knowledge as a valuable strategy for directing instruction. Thirdly, both participants in the case studies highlighted the notion of planning for assessment during the programming stage. This strategy assisted them to plan purposeful assessment tasks that were directly linked to learning experiences by nominating which classroom tasks would provide opportunities for assessing students while programming. Fourthly, work samples derived from open-ended tasks were systematically included in student portfolios for use in three-way interviews when reporting to parents. This saved valuable time during the reporting process and provided opportunities for promoting student reflection and self-assessment. Finally, the participants in this study identified the strategy of sharing the purpose of assessment tasks with the students. This involved explicitly outlining the criteria for assessing a task, which gave the tasks more meaning and purpose.

Theory into Practice. Of equal importance to teachers are the practical strategies derived from the data that illuminate how instruction and assessment can be integrated. The four principles for effective assessment that were introduced at the beginning of this paper, and the related strategies from the case studies are presented below. The strategies derived from the data are in *italics* and highlight implications for teachers that are grounded in practice. According to the current literature, assessment should be:

- Integral to, and naturally derived from good instructional practice – *developing content-specific open-ended tasks that produce work samples for assessment; seeking explanations and justifications during lessons through student modelling;*
- Accessible, relevant and meaningful to all students so they can show what they know – *sharing the purpose and outcomes of assessment and instruction; using assessment tasks that are open-ended to ensure accessibility for all students at all developmental levels;*
- On-going and cyclic in nature – *developing portfolios of work over an extended period, using open-ended tasks at the beginning of a unit to find out about learners' prior knowledge and misconceptions, and to guide instruction; planning for assessment during the programming stage*; and
- An indicator of students' thinking as well as content knowledge – *seeking clarification and justification of solutions and reflections; facilitating student modelling of solutions; eliciting work samples such as concept maps, procedures and explanations; using self-assessment and three-way interviews.*

Implications

The findings of this inquiry have shed some light on teachers' perceptions of the impetus for linking assessment with instructional practices and why they see this as an important goal. Strategies such as planning for assessment opportunities while programming, using open-ended tasks at the beginning of a unit to assess prior knowledge, and placing relevant work samples in student portfolios have been clearly identified by the participating teachers. These strategies highlight the nexus between assessment and instruction, and provide accounts of assessment practices that are grounded in real contexts. Valid and authentic guiding principles are documented for teachers seeking to make assessment more student centred and purposeful, clearly a goal worthy of pursuit.

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Portfolios as a Tool for Professional Development: A Basis of Reflective Practice

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This paper focuses on a component of a model to encourage and support teacher reflection in a professional development programme attempting to change the teaching and learning of mathematics. Our goal is to educate teachers to become analytical, self-reflective and adaptive agents in the classroom. The purpose of this paper is to report on the initial attempts by participant teachers to construct mathematical portfolios to promote reflectivity.

Introduction

The launch of a new curriculum in South Africa resulted in major changes in teaching practices. A shift toward reflective practice has become part of the alternative assessment strategies promoted in the new curriculum, Curriculum 2005. Portfolios have been identified as one of the new assessment strategies. Research has highlighted the great potential of portfolios as a tool for reflectivity (Green & Smyser, 1996; Wolf & Siu-Runyan, 1996; Blake et al., 1995).

This study was framed by two important determinants. First, if teachers are expected to adopt multiple assessment methods, then they must experience them. Secondly, to examine how primary school teachers would respond to a shift from traditional assessment practices (testing) to undertaking performance assessment involving alternative assessment strategies to collect evidence of significance.

Assessment and Curriculum 2005

The use of portfolios as part of classroom assessment practice in South Africa was stimulated by the introduction of the outcomes based model of education. Under the old system assessment was traditionally seen as something to be done to learners at the end of a period of teaching and learning. In the old system the teacher controlled the teaching and assessment.

The introduction of Outcomes Based Education (OBE) spurred an important change in how assessment is perceived. The change impacts on teachers' classroom practices, and some of these changes require teaching to be integrated with assessment. Teachers need to develop a wider range of skills in assessing learning outcomes. There is a recognition that there is a need to develop a wider range of methods and approaches to assessment, recording and reporting.

Why Portfolios?

The use of the mathematics portfolio within the course was a way to give in-service teachers a first hand opportunity to experience portfolio construction as a process that they will probably be using in their classrooms.

I saw the use of a portfolio for in-service teachers as an opportunity to document their own professional growth and development and to highlight some of the important ideas they have learned in the mathematics education course. This idea is supported by Blake et al.

(1995), who agree that the inclusion of reflective practices enhances professional growth. One way schools can promote reflection and self-assessment is to encourage the use of teacher portfolios.

Significant in the preparation of a teaching portfolio is the opportunity that allows teachers to reflect on the meaning and evidence of reflective teaching. Green & Myser (1996) describe teacher portfolios as a means to integrate all the aspects of teaching so that teachers can see for themselves where they are and where they are going professionally. My intention with the portfolio therefore was to inspire this type of reflection. Reflection allows us to learn from our experiences; it is an assessment of where we have been and where we want to go next.

Planning and Implementing the Portfolio Process

The portfolio task was introduced during the students' first visit to the University. The teachers received a selection of readings on portfolios and these were discussed and analysed.

The portfolio was proposed as a collection of carefully selected items accompanied by short reflection statements. Topics included self-assessment undertaken to critically assess own teaching; reflection on student learning was required as evidence of what makes their teaching effective in their own environment.

The portfolio also included reflections from students, that is, evidence of student self-assessment. The idea was to allow students to reflect on their strengths and weaknesses and to be active participants. In this process students become responsible partners in documenting their learning.

Each portfolio comprised three parts. The first part referred to the mathematics education course during the first professional block of the teacher education programme. It consisted of reflective statements, and journal entries. The teachers were asked to analyse their learning throughout this period.

The second part documented their field experience, reflecting on classroom experiences through observations, journal writing, collecting examples of children's work, analysis of own teaching and examples of own material developed.

The third part involved reflecting on the mathematics education course during the second professional block. The teachers were encouraged to include a diverse set of information gathered across a variety of learning contexts. This part assessed whether there was growth in the way they articulated.

Discussion of Portfolio Findings

The portfolios were collected and assessed at the time of completion. The criteria for the portfolios were discussed with the participant teachers on the course. Unedited teacher/learner responses of the portfolio reflections have been included in an attempt to enhance the discussion.

The topics include:

- Feedback from learner's own reflections;
- Analysis of own teaching;
- Material developed; and
- Journal entries.

Feedback from Learner's own Reflections

Student reflection provides teachers with a more meaningful picture of student understanding/misunderstanding or growth. This data can be useful for teachers in instructional decision making. This is evident from the following examples of a multigrade classroom (Grades 5 to 7).

Example 1. Sisanda Mami Grade 6 5 June 1998

> This year I learnt how to help other children, how to ask teacher if there is something that I do not no in maths.I have the problem. My problem is that I do not understand Multiplication.I need more help to multiply by factors. I do not feel happy in maths because it is hard. My worry is that I do not understand if I multiply using factors.

Example 2. Yandiswa Grade 6

> This year I like Maths. Last year I didn't like very much. This year I calculate the sum in different ways. We are talking English in Maths. I understand how to divide in different ways. We do a lot of Maths. If another one has a problem I can help him or her. I ask to another person If I have a problem.

Another student expresses positive emotions concerning mathematics.

Example 3. Boniswa Grade 7

> Last year I didn't understand very much Maths but this year I feel better. I like how to add a sum into different ways and I get the same answer. I like maths because it helps me to think, to do things on my own. Last year I didn't understand how to add a sum into many different ways but this year I try. I like how to divide or multiply using factors. I understand how to multiply into different ways. Last year we didn't do very much Maths but this year we do a lot and we enjoyed to see visitors at my school. If I have a problem I ask to another person to help me. I like Maths because my teacher help me when I have a problem in my sum I'm doing.

Liebars (1999) suggests portfolios give students an opportunity to pull together what they have learned and to document their growth. In Example 3 reflection provides the reader with a sensitive portrait of what the student knows and is able to do. It presents a broader picture of how the student has changed or improved. The students' writing demonstrates clearly how students feel about mathematics which is not usually possible to observe.

Students' writings revealed how students felt about themselves as learners. One student wrote the response below in Example 4 Probably no example is as telling as that of this student. The anxiety is evident in this statement. It gives insight into what many other students must feel when confronted with similar situations.

Example 4. Mzwamdoda Mqhayi Grade 8

> I do not like Mathematics because it is difficult and I fail it. Also my teaches in grade 5 did not explain to us. She wrote answers on the board but we did not understand the answers that is why I hate mathematics and I don't want to do it in grade 10 because I don't want to fail. The Change I want is that I must understand it so that I can like it.

I have learnt a great deal about both the teachers and their teaching by reading what students have to say about their learning. The teacher portfolio has energised the professional status and development of educators. Portfolios offer teachers a wide range of information from a variety of tasks and settings.

Analysis of Own Teaching

As teachers' responses were read I was struck by the realisation of the benefits of self-reflection. Smyser and Green (1996) say that portfolios empower teachers by making them reflective. The following self-assessment by a participant teacher shows how reflections can serve as an assessment of where teachers are and then further refinement of goals to get to the next stage.

Example 5. Self Assessment

I struggled to do cooperative work in so much that I use to find myself doing individual learning. Reflection on my teaching helped me to realise where the problem lies. As time went on I got used to it. So I discovered that it is not easy to move away from something you use to do but patience is the solution as Rome was not built in one day.

With the activities I wanted to do so many activities that will help in promoting my aims of developing eg. + x ÷ & -. I realised that this is not the thing. I can never do all the activities of number sense within a short period of time what I could only do is to try to extend an activity to fit in eg measurement.

I also used to forget to remind pupils about roles because they tend to forget allocating roles for themselves. I just could remember when they were starting to work some having already started. They liked too much to be a scribe. So in my planning I had to put a big NB star so that I can see it and remind them about it.

The following teacher is rethinking her work and feels good about this commitment. She speaks about her new focus on what students are learning rather than on what she is teaching.

Example 6. Analysis of My Own Teaching

On the whole, my teaching has improved since I have been to RUMEP. Firstly I have changed my attitude towards my work as a teacher. Now I am more interested in developing my pupils as a whole rather than focussing on how productive they are, that how they pass tests and exams. Rather I am interested in their daily performance and skills that they have. Now I encourage my pupils to discuss, share ideas and work together as much as they can. I am not syllabus-driven like before when I had to rush and sometimes leaving some of the pupils behind.

Through continuous assessment now I can see that pupils' performance is different, in speed and otherwise, so I am able to accommodate that. However, I still battle with the practice of letting them keep their own portfolios. Either they do not write on the portfolios frequently enough or they are not quite able to express themselves in English, especially the grade 5's. I have somewhat hope that by the end of the year I would have won this battle.

Material Developed

Part of the requirements for the portfolio content was to include examples of own material developed. The following response is a reflection on this process.

Example 7. Reflection on Own Material

Before I went to Rhodes I never thought that I could develop my own teaching and learning material. First of all, I have been able to use "waste material" creatively for example, using waste paper to teach various geometric concepts like symmetry, congruency, etc has proved very useful to me. In the process, my students gained some skills because they had to collect the material, cut out things themselves and then display their work in class. For example, they have been able to make various polygons and these, together with their properties, are displayed in the classrooms.

Secondly, I did give my pupils projects to do and they earned marks by making some apparatus which they used in their learning. For example, I gave pupils tasks to do geoboards using wood and nails. They were able to come up with quite good wood blocks which with my help, were developed into beautiful square shapes and ultimately we had enough geoboards to learn the geometric shapes, etc. they even brought some elastic bands so I did not have to buy these.

The fact that the teachers on the course were requested to reflect on the mathematics material they developed in their own classrooms resulted in positive benefits for the teaching and learning of mathematics. It is true that the change in instructional decision making is greater when the teachers are responsible for their own professional growth.

Journals

For the journal writing process the teachers were asked to respond to certain questions to assist them to reflect on the learning experiences during teaching on the course. According to Burk & Littleton (1995) more is learned from reflecting on an experience than from the experience itself.

Teachers' journal entries revealed that they felt confident now about implementing OBE. They felt good about working cooperatively in small groups, it helped them to feel comfortable and motivated to bring about change in their classrooms. The following reflections are teachers' written responses to teaching events.

1. Personal Concern:
   (Expressing personal matters that had an impact on your class participation)

> Group work and co-operation in group activities helped me in that now I have confidence to express myself in front of the class doing presentation without fear. The co-operation and support and suggestions we made in the groups helped me a lot in that my learners as from next week I'm gonna arrange them in groups of 5 or 4 so that they can learn from one another.

2. Personal Concern:
   (Expressing personal matters that had an impact on your classroom participation)

> The concern I had and of which I think will have an impact on my class is that of allowing them (learners) chance to come up with their own opinions and suggestions when solving problems. Each and every learner has his own way of doing things.

3. Other:

> I enjoyed the topic about reflection because it gives me an idea of what to do and expect. I can now look back and evaluate my learners and also give them chance to express themselves.

Conclusion

The teachers on the course had no previous experience of the portfolio process. The process of creating a mathematics portfolio proved to be a valuable tool for professional development.

I support Green & Smyser (1996) when they state that "the only way teachers can learn about their own strengths and weakness as teachers is to reflect ...". Current reform movement in mathematics education recommends reflective practice in teacher education. Increase of reflective thinking on teaching is seen as primary benefit of teaching portfolios. The preparation of the teaching portfolio challenged teachers to be critical of their own teaching practice. In modelling the portfolio process opportunity is afforded to formalise and structure reflection. This is consistent with Schon (1983) who feels strongly that teachers increase their effectiveness in the classroom by being "reflective practitioners" – those who study and reflect on their actions.

I will continue to use the teacher portfolio as a tool for professional development to encourage reflective practice.

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Change in Attitude Towards Mathematics among Aboriginal Adult Learners

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This study investigated changes in adult Aboriginal students' attitudes during a year-long mathematics unit taught by the researcher. Students completed an attitudinal survey before and after the unit and also kept regular diaries. Data was also collected from a focus group who were observed, interviewed and visited at home on a number of occasions. Although attitudes fluctuated during the unit, the general trend was positive. Factors identified by the students as contributing to this change included the separation of students into ability groups, negotiation of content and learning style, relevance of content, and community visits by the teacher.

The Diploma of Community Management (DCM) is a two-year external course offered by Macquarie University to Aboriginal students. It includes a full year unit called Mathematics and Statistics Appreciation (MSA) which starts with basic skills and leads into applications. It is hoped that, with improved mathematical skills and knowledge of how to apply them, the graduates will be in stronger positions at work and in the community. MSA was taught by the first author from 1992 to 1997.

Mathematical autobiographies gathered during 1992-1995 revealed that many of the students had negative experiences of mathematics at school. The MSA unit was therefore redesigned in 1996 with the specific aim of fostering positive attitudes towards mathematics. The study reported below was an investigation of the success of the redesigned MSA in bringing about the desired attitudinal changes.

Background

Fostering Positive Attitudes to Mathematics

Many adults suffer great anxiety when faced with the prospect of doing mathematics. Classroom experiences (Cobb, 1986), teachers' attitudes (Sullivan, 1987) and a host of other factors such as a lack of success can help to form negative attitudes. Hembree's (1990) meta-analysis of 151 studies on mathematics anxiety showed that it is a learned condition linked to poor test performance and avoidance of mathematics. Anxiety is just one reason for failing to reach full potential or to succeed in mathematics. Low ability cues such as communicating pity following failure, offering praise following success on a task perceived as easy by the student, and unsolicited offers of help by the teacher may set the student on a course towards failure (Graham & Folkes, 1990). Inaccurate perceptions of mathematical capability, rather than poor performance or lack of skill, have been shown, in many cases, to be responsible for the avoidance of mathematical careers or courses (Lent & Hackett, 1987).

Ma and Kishor's (1997) meta-analysis of 108 studies found that the relationship between attitude towards mathematics and achievement in mathematics was positive and reliable but not strong. African American students in particular demonstrated a significant positive relationship between attitude and achievement (Ladson-Billings, 1977). Empirical evidence suggests that exposure to new mathematical experiences can lead to significant attitudinal changes to mathematics. The creation of a non-threatening, cooperative environment with an emphasis on group work and assignment-based assessment was found by Gooding & Kaye (1993) to increase self-confidence, motivation, and enjoyment for many of the students.

Educating Adult Aboriginal Students

According to Christie (1985), encouraging Aboriginal students to make associations with their own culture and experiences when studying mathematics can help make the subject meaningful to them. Goals need to be negotiated, and the purpose and relevance of each activity needs to be clearly stated, providing for regular feedback once learning begins. If the fundamentals are not mastered then neither will anything else, so it is best to allow plenty of time in the initial stages before moving on. Educators need to realise that how they teach and how they act may be as important as what they teach (Sommerlad, 1976). Effective teachers of Aboriginal students have been found to be patient and sensitive while maintaining high expectations of attendance and achievement (Harris, 1990; Flowers & McDaniel, 1995).

Valuing and building on the existing knowledge, experience, and goals of adult learners helps students to accommodate new information. If the content has no meaning for them then they will not engage in learning (Caffarella, 1994) and will feel they are wasting their time. Finding a common purpose through negotiation with the students can fulfil their needs without compromising the overall educational goals.

The MSA Unit

The design and content of the program was negotiated with the students and the university's Aboriginal Programs Unit. The content was linked to students' needs and interests.

All students kept a diary throughout the year. The diary comments were used to instigate changes in the unit and alert the teacher to students at risk.

The students were placed in three ability groups—called Level 1 (lowest), Level 2 and Level 3—based upon their performances on an arithmetic test and where they felt most comfortable.

Assessment for the unit was broken into small parts so that a pass in the unit appeared achievable even for the most mathematically anxious student. All students had the opportunity to achieve the highest possible grade (A), based upon the work set for their level. Students were required to complete skill-based exercises from a reference book, submit practical assignments and worked solutions to problems, and give a group presentation to the class.

The Present Study

In 1996, a study of the effectiveness of MSA was carried out (Spielman, 1999). In the present paper, we summarise results from this study which are relevant to the following research questions:

1. What feelings do MSA students have about mathematics, and how do these feelings change as a result of studying the unit?
2. What motivates the students to learn mathematics?
3. What characteristics of the unit influence the growth of students' confidence?

Method

All 31 students (24 women and 7 men) enrolled in the MSA unit in the academic year 1996 agreed to participate in the study. They were all asked to complete an attitudinal survey (based on Aiken, 1974) at the start of the unit (February) and near the end (October). They were also required to keep personal accounts of their mathematical progress in the form of a monthly diary divided into five components: what mathematical concepts they had found to be important; what mathematics they still found difficult; how they felt about studying mathematics; their strategies for improvement; and comments on their progression.

The attitudinal survey was administered to all students in February. Unfortunately attendance in October was poor and only 15 surveys were completed.

Six students were selected (from nine female volunteers) to form a focus group. All of the focus group students resided in New South Wales. They agreed to participate in community visits, to give two or three taped interviews, and to be tape-recorded during some problem-solving sessions. The researcher visited the focus group students in their communities twice. Potential daily experiences (such as quilting, darts, drink driving, get-rich-quick schemes, discounting, and budgeting) served as content for mathematical teaching and learning.

Qualitative data from the focus group were coded by the researcher and an independent coder and analysed using the NUD•IST 4 program (Richards, 1997). Themes emerging from at least two students and at least two types of data were identified and used to plot changes among the entire group throughout the unit. Data from the attitudinal surveys and the researcher's field notes were also incorporated into the analysis.

Ethical Considerations

Unfortunately, Aboriginal people have been the subjects of destructive research. Safeguards are now in place to ensure that research benefits the researched and future students. One principle is that "indigenous participants should essentially own the process and be an integral part of all aspects of the inquiry and dissemination of results procedure. ... Indigenous people should have the final say about what is public domain knowledge" (Smith, 1997, p. 28). After negotiation with the staff and student representatives of the Aboriginal Programs Unit, the university ethics committee, and the students to be involved in the research, several measures were agreed in order to empower the students and give them the confidence to participate. Some of these were:

- Videotaping was regarded as too intrusive and was not used.
- All interview questions were shown to the student representatives and the focus group at least a day prior to interviewing.
- The focus group students were given copies of the interview transcripts and could request sections be edited out.
- After completing the research, a session involving the whole group and interested family members was held in which the researcher explained the outcomes of the research and answered questions.
- The thesis was scrutinised by the Aboriginal Programs Unit staff and student representatives before being made accessible to the public.

Results

The qualitative analysis indicated that five core categories influenced students' attitudes towards mathematics:

1. Anxiety (caused by perceived prejudice, difficulty with the unit, shyness in asking for help, and the pace of learning)
2. Motivation (challenge, need to pass the unit, lack of motivation, relevance of content)
3. Confidence (self-concept, willingness to give and seek help)
4. Understanding (one-to-one teaching, resources, time)
5. Enjoyment (group, individual, of the curriculum)

Instead of stating the results for each of these categories and the attitudinal survey separately, we summarised them in terms of the three research questions noted earlier.

Changes in Students' Attitudes

Half the students had had negative schooling experiences that affected their confidence, career paths, and levels of anxiety about doing mathematics. Having returned to study, however, these students were motivated to learn and were interested in what the unit had to offer. The diaries and interviews indicated that most students became more confident and less anxious as the course progressed with all but one student claiming to enjoy mathematics at the end of the course. An exception occurred in September, where students' entries indicated anxiety was high — either because of uncertainty in being able to complete the course due to proposed cuts to AbStudy, or because many students had fallen behind and were unsure if they could successfully complete the course in time.

Many of the students found the unit difficult at first, commenting that they needed individual help and much more practice in order to master fractions, decimals and percentages. Although a number of these students said they felt "dumb", they were encouraged because most of the others in their group were also struggling. The diary entries indicated that students had a better idea of their weaknesses than their strengths and had little insight into how to overcome these difficulties.

Bobis and Cusworth (1995) suggest that significant people, such as the teacher, can play important roles in helping students confront their attitudes towards mathematics. Once the MSA students felt comfortable with the teacher, they became more relaxed and began to ask questions. Acceptance of the teacher by the rural and isolated students was especially evident from the diaries and interviews. For this group, enjoyment of mathematics increased by an average of 11 percent (on a 48-point scale) over the course of the unit. This finding confirms the assertions of Ocean & Miller-Reilly (1997) that, in order for learning to occur, the teacher needs to be sensitive to the students' beliefs and feelings, and must create an environment in which students feel safe.

Having the time to learn was another factor contributing to enjoyment. Five students with demanding jobs, who did not have the time to put into the unit, became more negative in their feelings towards mathematics between April and September. Their assignments became overdue and were left until November to complete. Three of the students were able to set aside the time to complete the assessment tasks in November, and wrote in their diaries about how relaxed and happy they felt that they had had time to understand and practice the concepts. Bressant (1995) also found that college students believed learning and mathematical enjoyment would improve if given the opportunity to master one concept at a time.

According to the survey results, the value that the students assigned to mathematics increased significantly more for the women than the men ($p < 0.05$). Similarly, the enjoyment and value of mathematics grew significantly more for those students educated in the rural and isolated areas compared to those schooled in the city ($p < 0.05$). Change in enjoyment was strongly correlated with the students' unit grade ($p < 0.01$). Students who were up-to-date with their work and succeeding in the unit expressed feelings of enjoyment and pride in their interviews and diaries.

Motivation to Learn

When the content was perceived as relevant, the students became more interested in mathematics and their enjoyment of the subject increased. Relevance was a common theme running through the diaries and interviews, and it was also identified as a key to the students' understanding and enjoying mathematics. On completing the unit many students commented on how they had applied, or would now be able to apply, what they had learned in the unit to their work or leisure. The increase in interest is an important result in view of the finding of Schiefele and Csikszentmihalyi (1995) that interest contributes significantly to achievement and can be used to predict future grades. Ladson-Billings (1997) also found providing relevant, meaningful content to be the key to success in mathematics among African-American students.

Anecdotal evidence gathered by Eckermann (1988) suggests that Aboriginal people are strongly motivated by the mastery of specific tasks rather than final grades. This was supported by the personal diaries and interviews provided by the focus group. Although they were all keen to pass, the final grade was not as important to five of them as the understanding of specific concepts.

Another great motivator, according to students' reports, was the need to help their own children with mathematics. Two focus group students in particular, drew a great feeling of accomplishment and enjoyment from their mathematical experience on this account. Fitzsimons (1993) provides a similar example of an adult who, before learning mathematics, had low self-esteem but afterwards experienced emancipation when she discovered that she was capable of helping her children with their mathematics.

Unit Characteristics Influencing Student Confidence

Students' comments suggest that their increased levels of confidence can be largely attributed to the ability grouping, the use of concrete materials, and the supportive nature of the teacher's actions. They felt that they learnt best in these circumstances.

Ability grouping. Many students commented that working with one or two other students of similar ability was much better than working in a large group of students with mixed abilities. All but one student preferred the streaming of the groups and the older students in particular did not want to be in with "those young ones that know how to do it". As a result, they felt more confident to ask questions and even present their findings, as a small group, to the rest of the class.

Grouping according to ability also enabled tasks to be more appropriately challenging and the workload demanding but achievable, as recommended by Flowers and McDaniel (1995). Indeed, the more advanced students expressed the need for more challenging mathematics and were bored when the whole class was together in the first few weeks.

Despite the perceived advantages of working in small groups, the importance of individual learning, especially when aided by a tutor, cannot be ignored. A high proportion of students initially felt the need for individual assistance but towards the end of the course, when they were feeling more confident, preferred to work unaided.

Use of concrete materials. Many of the students stated that the opportunity to visualise concepts through using concrete materials helped them understand the concepts better. Group interactions during these activities were also reported to be more enjoyable. However, students' comments also made it clear how important it is for teachers to take care not to embarrass adults by allowing non-group members access to the classroom or the results of their efforts.

Teacher support. The actions of the teacher were very important. Sitting with those having difficulties helped students to start and gave them the confidence to go on. When the teacher could not give this individual time, the students talked about losing confidence. Catering for different learning styles and being sensitive to cultural differences were other important aspects in terms of student achievement and acceptance of the teacher by the group. Students tended to be very supportive if the teacher made cultural mistakes, providing the teacher acknowledged the mistakes.

Other factors. The textbook was valued for developing confidence and understanding of mathematical concepts, especially since most of the work was carried out off-campus. However, with answers at the back, many felt that the temptation to cheat was too great. The older students in particular refused to look up the answers. It may be better to provide hints on how to get started instead of answers.

Providing assessment tasks that could be done individually or in groups seemed to relieve the anxiety for some of the students. It was noticed that older students tended to prefer to work alone, in order to assure themselves that success was due to their own efforts. The clear assessment timeline also helped the students to relax and concentrate on completing the set tasks.

Discussion

Limitations

This study has found associations between students' attitudes and classroom practice that are consistent with findings in the current research literature. However, the results summarised above are from a non-random sample of motivated students and some of the findings may be specific to this group. It is important to remember that indigenous Australians are a diverse group and any trends emerging from this research are not necessarily representative of all Aboriginal or Torres Strait Islander students.

There are other limitations of this study. The results of the attitudinal surveys are biased because more than half the students who completed the initial survey did not complete the final survey. Further analysis shows that the 18 students who did not complete the final survey initially enjoyed and valued mathematics less than those who did. Since their diary entries indicated that they were feeling relaxed and enjoying mathematics more towards the end of the unit, the surveys may well have underestimated the degree of attitudinal change.

The qualitative analysis relies entirely on responses from women. However, the relatively small proportion of men in the group is typical for Aboriginal students attending the DCM. As the DCM is attracting more students each year, further research may provide a better insight into the male perspective.

Having a researcher who was also the teacher and sole assessor of the unit may also have biased the findings. As students were keen to pass, they may have said things to please the teacher rather than risk criticising her. If, however, an independent researcher was to undertake a similar study, he or she would need to liaise frequently with the lecturer so that appropriate changes could be made to the curriculum, course structure, and teaching style. In support of the dual teacher-researcher role, Guba (1981) found that greater credibility can be given to a qualitative study when the researcher is so much a part of the environment that she does not present a threat to the participants.

Implications

Perhaps the most important implication of this research is that Aboriginal adults can readily learn mathematics at university despite cultural differences, negative schooling experiences, and the different cultural background of the teacher, provided certain approaches are taken. This study has also shown that effective, beneficial research can be carried out on the education of Aboriginal students even when the researcher is not Aboriginal.

Despite the limitations of the study, the emerging trends suggest some important and relevant recommendations to other educators of Aboriginal adults.

Providing streamed classes when numbers are high is recommended, but care needs to be taken with the structure and assessment of these groups. The more advanced students respond well to group problem solving and skills application, while the more educationally disadvantaged students prefer a clear structure with an emphasis on practising basic skills (Clute, 1984). Some students prefer to work alone rather than in a group, and should be given the option of submitting their own solutions instead of a group report. Reward should be given to students for prior knowledge so that they are motivated to achieve higher grades.

Care must be taken to introduce problem solving through the use of meaningful and relevant contexts. Most of the students in this study did not want to waste time learning mathematics that could not be immediately applied to their lives. The more mathematics became relevant, the more motivated they seemed to become.

Rather than develop a prescribed course prior to meeting the students, educators should negotiate the content, pace, and teaching style with the students. If this recommendation is not followed, students may leave through boredom or an inability to cope. Making learning enjoyable can also have a great impact on attitudinal change and achievement, increasing their chances of success.

Regardless of how busy teachers are, time must be taken to sit with individual students and work through their mathematical problems. This seemed to be an important factor influencing confidence and understanding, regardless of how long the teacher spent with a student.

Community visits to students are important for a number of reasons. The teacher gained a deeper insight into their students' demands, mathematical needs, interests, backgrounds, and problems they were experiencing in the unit. Once the students knew that the teacher was interested in hearing about their past and present situations, and helping them achieve their goals, a greater empathy developed between them.

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Research-Led Policy Change for Technologically-Active Senior Mathematics Assessment

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This paper describes a new research project, which aims to investigate how computer algebra systems can be introduced into the Victorian Certificate of Education mathematics subjects. Lessons drawn from the experiences of the introduction of scientific and graphics calculators are reported along with current moves in some other countries. Options for the role to be played by technology in assessment are canvassed. It is proposed that technology should be permitted in all components of assessment but that explicit attention will need to be given to including questions that test algebraic insight. A plan for minimising inequity is proposed.

For thirty years, new information technology has been a major force driving change in mathematics curriculum and assessment. In the 1960s the introduction of electric calculators changed the way that arithmetic was done in business. As a consequence, the goals of the mathematics curriculum needed to be thoroughly revised. Later, when hand-held electronic calculators became affordable for students at school, curriculum and assessment began to change at all levels — a change involving such a fundamental re-assessment of goals and teaching methods that it continues today. Adapting to the technology of the four-function calculator was, for example, a major theme of the 1998 NCTM Yearbook (Morrow & Kenney, 1998). School systems around Australia are now seriously grappling with the use of graphics calculators in senior school mathematics teaching, curriculum, and assessment. We see this as a transient stage, simply the prelude to the next phase of technology-driven challenges and opportunities for teaching mathematics, which will centre on computer algebra systems.

Computer algebra systems (CAS) can perform all the routine procedures of mathematics normally covered in secondary school and university, including drawing graphs, calculating with vectors and matrices and doing algebra, calculus and statistics. These systems have existed for over twenty years, even on small systems such as home and school computers, but until the recent advent of hand-held machines of moderate cost, they have had little effect on school mathematics. Even at the earliest stages, it was predicted that there would be little impact on school and university teaching until hand-held technology arrived. Wilf, in his 1982 article evocatively entitled "The Disk with the College Education", looked forward to the day when students might have a $39.95 pocket calculator with a long LCD window for display and lots of buttons such as DERIV (for differentiation) and MATRIX, performing operations from the major areas of senior secondary and early tertiary mathematics.

Although the $39.95 version has not yet literally come, CAS "supercalculators" should soon be priced such that all Australian senior mathematics students might own one. The potential effect of the CAS revolution is dramatic because of its broad application across mathematics and because it directly involves the secondary-tertiary interface. As the availability of CAS in schools increases, so does the need for the official curriculum to produce appropriate policy responses. Effective policy development on the possible use of CAS in formal examination-based assessment requires rigorous and well-grounded research, especially because this is associated with the highly sensitive credentialling arrangements for tertiary study.

Researching Possible Curriculum Change

This paper outlines some preliminary issues to be addressed in a new study funded from 2000–2002 by the Australian Research Grant Strategic Partnerships with Industry Scheme. The study aims to investigate the changes that regular access to CAS supercalculators will have on senior mathematics subjects and the associated assessment in Victoria, Australia and to explore the feasibility of offering new mathematics subjects which use CAS extensively. In Victoria, schools provide Year 11 and 12 subjects for the Victorian Certificate of Education (VCE) in accordance with the 'study design' that specifies curriculum content and assessment procedures. Final results, which are used for various purposes including selection to tertiary courses, are derived from a mix of tasks set at the school and external examinations. The Chief Investigators of the project are Gary Asp, Helen Chick, Barry McCrae and Kaye Stacey; and David Leigh-Lancaster is a partner investigator. The four industry partners are the Board and three calculator suppliers and manufacturers: Hewlett-Packard; Shriro (Casio); and Texas Instruments. The industry partners will supply CAS supercalculators to students in three schools for a three year program of classroom based research. With the cooperation of the Board, the content and formal assessment undertaken by these students for Year 12 will be altered, culminating with the trial in volunteer schools in 2002 of a VCE examination using CAS, an alternative to the current Mathematical Methods subject. The major features are summarised in Figure 1. Actual implementation of this requires the Board's continuing approval, which will be informed by the findings of the project in earlier years. The outcome will be substantial policy advice to the Board, practical feedback to the calculator manufacturers, and important research insights into questions of learning mathematics in a technology-rich environment.

For each project school CAS support from one industry partner

2000	2001	2002
One Yr 10 Maths class →	→ Yr 11 Maths Methods →	→ Yr 12 New CAS Maths Assessment: New Board CAS paper
One Yr 11 Maths Methods class →	→ Yr 12 Maths Methods Assessment: Standard VCE + trial CAS paper	
	One Yr 11 Maths Methods class →	→ Yr 12 New CAS Maths Assessment: New Board CAS paper

Figure1. Time-line of research and assessment changes for each project school.

For the calculator industry partners, the project enables their products to be tested seriously within an Australian curriculum context, investigating the suitability of the products' capabilities, interfaces, notations and physical characteristics. Materials for training teachers to use the CAS supercalculators will be enriched by experiences in the project schools. For the Board, the project will result in advice to support the development of policy for curriculum and assessment, covering issues such as:

- which parts of Year 12 assessment could permit CAS, forbid it or require it;
- the protocols for the use of CAS supercalculators in assessment;
- how (and if) examination questions can be set to be fair to users of CAS supercalculators of different brands and models, given that they have different capabilities (the involvement of different industry partners is critical for this); and
- subsequent redevelopment of lower secondary mathematics curricula and suggestions for post-secondary mathematics and mathematics-related studies.

Beyond these practical issues, the scientific importance of the study lies in its advancement of our detailed knowledge of how students learn mathematics and how it can best be taught. This new learning environment provides us with new opportunities for research into the teaching of mathematics. When students have CAS in class and in examinations, the need for memorising routine procedures may be enormously reduced, yet the need for conceptual and structural understanding is almost certainly undiminished and possibly enlarged.

Effective policy development (in particular for assessment used for university selection and pre-requisites) requires rigorous and imaginative research, especially as the possible changes are likely to be substantial and subject to robust debate. We know of no similar study being undertaken anywhere in the world. Thus, the results of this research are likely to have implications for education systems throughout Australia and, indeed, the rest of the world.

In the rest of this paper, we present our preliminary thinking on one of the major policy issues to be resolved through the project: the way in which assessment will use or not use CAS. We recognise that this is likely to be a sensitive and possibly political issue and so careful detailing of arguments and analysis of data will be needed to guide policy.

Senior Mathematics Assessment with Technology

In this section, we review the experiences of adapting assessment in Victoria to graphics calculators and report on some current policies from other countries on adapting assessment to the presence of CAS.

The Graphics Calculator Experience

The CAS revolution (if that is what it proves to be) for senior mathematics comes hard on the heels of the introduction of graphics calculators. Out of reach because of price just a few years ago, schools in many places have now enthusiastically taken them up. Systemic education curriculum and assessment authorities, internationally and nationally have, over the past decade, grappled with policy issues related to graphics calculators (Stacey, Dowsey, McCrae, & Stephens, 1998).

Within Australia, the state education systems have now specified whether graphics calculators are permitted, are banned or are compulsory in examinations and other assessment. As of March 2000, Victoria and Western Australia have adopted graphics calculators, while the other states with an external examination system have not. In Victoria, the policy of the Board is one of "assumed access" to graphics calculators for all year 12 examinations by 2000. The adoption of graphics calculators by the Board required change in the syllabus for each mathematics subject and change in the type of examination questions used. Questions that are "trivial" with a graphics calculator have been replaced by questions that probe conceptual understanding and problem solving with technology. The above changes are being introduced smoothly at least in part because of extensive professional development and strong support of the Mathematical Association of Victoria. However, these changes are just a prelude to the much larger policy issue of CAS.

In Victoria, the transition to using graphics calculators in Year 12 external examinations was made in two stages: firstly (in 1997 and 1998 for the main tertiary entrance subjects) the examinations were set in a "technology neutral" fashion. This meant that a student without a graphics calculator should not be disadvantaged on any of the questions. So, for example, in 1996 before graphics calculators were permitted, one multiple choice question on the Specialist Mathematics examination (CAT 2) asked about the implied domain of the function $f(x) = 1 + \mathrm{Sin}^{-1}(2x)$. In 1997, a similar question asked about the function $f(x) = \mathrm{Cos}^{-1}(x - a)$; calculator neutral but somewhat harder. The transition has now been made to "technology-active" assessment, where not using a graphics calculator on some questions may definitely disadvantage a student. The Board (1999) provided examples of the new range of questions which could be asked, including sketching the graph of the function $v = 100 \tan(\mathrm{Tan}^{-1}(3) - 25000t)$.

Systemic Adoption of CAS

Examining mathematics with access to CAS presents more challenges than examining mathematics with graphics calculators. McCrae (1996) found that access to graphics calculators would impact on only 6% of a 1994 VCE Specialist Mathematics paper (the hardest mathematics subject in Victorian schools), but that about 60% would be affected by the availability of CAS. Similarly, Shumway (1989) reported that about 90% of the exercises in most U.S. textbooks could be computed directly by CAS. The adoption of CAS in teaching is therefore inherently associated with the adoption of CAS in assessment.

As yet, only a few countries around the world have national policies permitting the use of CAS in examinations. In Denmark, CAS will soon be permitted in all mathematics examinations for 15-19 year old students. In France, any calculator, including those with CAS, is permitted in examinations. In parts of Germany, teachers can decide whether CAS is permitted in lessons and examinations. Elsewhere around the world, some countries have now permitted graphics calculators but not CAS (as in Victoria), some (such as Italy and Ireland) are about to introduce pilot projects, and some, often in Asia but also including poorer nations, do not use calculators at all. The unique nature of each country's assessment regime and the national priorities makes research in individual countries important.

One policy likely to be influential in Australia is that of the USA College Board Advanced Placement (AP) Calculus (see website). The long list of calculators permitted includes both graphics and CAS supercalculators, although those with QWERTY keyboards, pocket organisers and pen-input computers are not allowed. From 2000, both multiple-choice and free-response sections will be in two parts: one where some parts require the use of a graphics calculator and one where the use of any calculator is not permitted. The AP web-site notes that: "This change in format is an effort to respond to heightened concerns with equity as more students may use graphing calculators with computer algebra system (CAS) features." It is claimed that the two part format will provide flexibility in the types of questions that can be asked and also ensure greater fairness to students, regardless of calculator used. In addition, it is specified that students can only use a calculator for three operations: solving an equation; finding a derivative; or calculating the value of a definite integral. In these cases, the student must indicate the set-up of the problem (e.g., to write down the relevant definite integral for finding an area before evaluating it by calculator). In all other cases of calculator use, the student must show the "mathematical steps necessary to produce the results". For example, to find a local minimum of a function, normal calculus procedures must be followed to establish the derivative and set it to zero. Many of the capabilities of CAS therefore cannot be used.

Technology-free, Technology-neutral or Technology-active?

Stephens and Leigh-Lancaster (1997) write that there are three possible positions that should be investigated for the use of CAS technology in examinations:

- that assessment (of at least some areas of mathematics) should be *technology-free*, i.e., that students should not use technology (or at least advanced or "new" technology) in the assessment;
- that assessment should be *technology-active*, i.e., that students should be permitted to use specified advanced or 'new' technology in assessment and that some questions should be designed to require its use; and
- that assessment should be *technology-neutral*, i.e., that students not using technology in an assessment should be able to answer questions as easily as those using technology.

As noted above, the Victorian mathematics subjects have in recent years passed from technology-free (with regard to *graphics* calculators, although scientific calculators have long been permitted), to technology-neutral, to technology-active external assessment. This experience has led us to believe that the technology-neutral position is certainly not sustainable in the long-term, as it is very hard to set questions which are genuinely technology-neutral (McCrae, 1996). In this we agree with Kemp, Kissane and Bradley (1996) who note in regard to graphics calculators that:

> "... the use of calculator neutral examinations is an unwise long-term strategy, although it may be seen as helpful in the short term to allay concerns about disparities in student access to graphics calculators. In the long term, such a strategy would send a clear (and incorrect) signal that graphics calculators are not of importance in mathematics, and would discourage both students and their teachers from acquiring either hardware or expertise in its use."

In designing its examinations, AP Calculus has combined the three assessment positions. Technology-free assessment has been chosen for part of the assessment and the other part is technology-active assessment, partially neutralised. It is technology-active in the sense that a CAS calculator is permitted and some parts of some questions require its use. On the other hand, the use is constrained to three operations. They have given two reasons for this choice (http://www.collegeboard.org/ap/calculus/html/exam002.html): firstly, the recent introduction of graphics calculators has placed a large burden on teachers, who need time to adjust their courses. Secondly, the College Board notes that it can develop fair examinations with any specified type of technology, but it cannot develop exams that are fair to all students if the spread of capabilities of the technology is too wide. The College Board has endorsed the use of any technology and promoted it in teaching—accepting that some students will have CAS whilst others have basic graphics calculators—but constrained their use in assessment in order to increase equity.

Our initial position (which may be modified in the light of experience and data) is that our project should aim for technology-active assessment only. The reasons for this initial decision are explained in the next section. However, we expect that this position will be feasible provided only a narrow range of (high) technology capabilities is permitted and if teachers are provided with adequate professional development before they begin to teach the new subjects. Instead of the wide range of calculators permitted by the College Board, we expect to recommend machines with strong commonalities. With time, the capability of these machines will increase, as more advanced technology becomes affordable. Furthermore, if the new subjects are initially alternatives to the current subjects, teachers can decide to move their classes across to the new syllabus when they are themselves professionally ready and when typical students in their school community can afford to purchase an adequate machine. We see serious equity considerations arising from the cost of new technology which cannot be solved, only reduced, in the foreseeable future.

Although we do not intend to investigate technology-neutral assessment (for reasons given above), one of the features of the research design is that it will allow us to fully explore the practicality of *brand-neutral* assessment. As hand-held machines become more and more powerful, the divergence between the capabilities of different models may increase markedly (alternatively it may decrease with "product maturity"). Current experience with graphics calculators shows that questions that are challenging on one machine can be simple on another. For example, an important point of an assessment item may be to test students' understanding of the shape of a graph. Sometimes this can be done by using a graph that does not appear in a standard viewing window, so that the student needs to know properties of the graph to locate a desired feature. However, facilities such as determining complex arithmetic or automatically locating zeroes and turning points are available on some calculator, making certain questions straightforward. The capabilities of different models will need to be fully appreciated in order to set brand-neutral assessment items.

Why Plan only Technology-active Assessment?

Our initial position is to create technology-active assessment, rather than technology-free or technology-neutral assessment. This does not mean that every question will require use of technology, although it would be available for all. Our primary reason relates to values and beliefs about mathematics: mathematics at school should be like mathematics as used outside school. As mathematics outside school changes and the methods of choice change, so the methods of choice at school should also change — to the extent that this is possible. Furthermore, we believe that assessment should be aligned with teaching as closely as possible. This principle has been endorsed by national associations for many years (see, for example, AAMT/CDC, 1987) We believe that a technology-free component of assessment would endanger mathematics remaining a sensible subject where students learn to use up-to-date methods. On balance, we believe that the decision to make senior school mathematics arithmetic-calculator-active has been the right one and that this will also be the right decision for algebra. Our final argument for technology-active assessment only is that this maximises the assessment time that is spent on assessing higher-order thinking skills, rather than routine procedures.

What are the arguments for a technology-free component of assessment? One is the need to provide fair assessment for students with different types, models and brands of calculators

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and we have outlined above our approach to minimising this concern. The research project can provide some data to judge this approach. A second argument is that tertiary mathematics courses may not use CAS and hence students must not rely on it. We expect that this will not be the case for much beyond the time frames of the proposed curriculum change. A third argument is that the existence of technology-free assessment would encourage the acquisition of important mental or by-hand skills and a fourth is that technology-free assessment is better able to test 'true understanding' — what students really know and really understand. These two final arguments are substantial, getting to the heart of values and beliefs about mathematical activity. How can we respond to them?

How can Mental and by-hand Skills be Encouraged?

We take it as axiomatic that an important function of an external examination system is to encourage good learning and teaching. We agree that there are skills (especially algebraic skills) over which students should have personal mastery. Moreover, we agree that the existence of technology-free assessment would encourage the acquisition of these important mental or by-hand skills. Years of teaching with scientific calculators have shown that to achieve the goal of sensible and powerful mathematics, students cannot be only taught how to carry out arithmetic on a machine. They must develop a strong number sense, which enables them to operate quickly and effectively in the world of informal arithmetic (e.g., "Am I being charged about the right amount here?") and also to operate a machine competently, guarding against errors by effortlessly monitoring the results of calculations. At the same time, there has been a re-assessment of what students should know and by-hand procedures such as taking square roots and division using log tables have been consigned to history. Parallel moves will need to be undertaken with CAS — students will need well-developed symbol sense (Arcarvi, 1994), a mastery of some simple procedures (e.g., expanding with the distributive law, doing the same to both sides for equation solving) and some currently taught procedures will not be required (is factorising one of these?) Our initial position, however, is that separate technology-free assessment is not required. If algebraic insight (symbol sense) is made a goal of the assessment, we expect that it can be assessed as well with calculators as without, by clever question design. On the other hand, we expect that mastery of simple procedures is absolutely essential for smooth CAS use and therefore, students who do not have it will not do well on a technology-active examination. This will be implicitly tested, rather than explicitly. It will therefore be important to be quite explicit about these issues in advice to teachers.

In the argument above, we have assumed that only simple procedures will be important to be able to do by-hand as a mathematics professional or user in the future. Exactly what constitutes a simple procedure will vary from person to person, as it does now with arithmetic. This however, may be a vigorous area of debate for many years. Kutzler (1999) in proposing the idea of assessment of "intellectual fitness" is beginning this debate.

Does True Understanding Depend on being able to Carry out the Steps?

The fourth argument for technology-free assessment is that it is better able to test "true understanding" than is technology-active assessment. In fact, it is tempting to believe that what a person 'really understands' what he or she can do from memory or "by-hand". Pierce (1999) has shown how students themselves often believe this. Unfortunately, however, dissatisfied mathematics students across generations have attested to the fact that they did not

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understand procedures that they learned to carry out by hand successfully. Technology-free assessment therefore cannot claim to reliably identify 'true understanding': the question is whether technology-active assessment can identify it and encourage it any better. Experience with assessing with graphics calculators leads us to expect that careful question design may be able to achieve this. However, the question of what will constitute understanding (with possibly different answers for different types of users of mathematics) is a much larger question and we cannot address it adequately in this paper.

Conclusion

Year 12 mathematics is critical to prepare Australian young people for a technological future. Mathematics subjects are the principal pre-requisites for tertiary study in the sciences, engineering and economics, on which Australia's economic competitiveness depends. It is clearly important that our students learn to use the mathematical tools of the future, but also that a cautious approach needs to be adopted so that the right balance between traditional by-hand algebra skills and technology use can be achieved.

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Computational Choice: The Reasons Behind the Choices

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This paper represents some of the results of a study carried out to explore the computational choices made by 75 students in Years 5-7. Data were collected to determine the initial and subsequent computational choices made by students. A record was made of the success rate for various computational choices and students were interviewed to determine the reasons for making particular computational choices. Reasons given by students were placed in four categories of number magnitude, efficiency, knowledge of multiplication facts, and teacher influence.

Introduction

The ability to choose and use a repertoire of computation methods is an important goal of the mathematics curriculum (Australian Education Council, 1991). While this may seem to be a reasonable expectation for primary school mathematics programs, there is not a great deal of evidence to describe how students go about making decisions relating to computational choice. Various models of computational choice have been suggested (National Council of Teachers of Mathematics, 1989; Trafton, 1994; Swan & Bana, 1998), but few have been tested. Seventy-five students in Years 5-7 in two schools in Western Australia were interviewed, based on their responses to an eighteen-item instrument in order to evaluate the computational choice model postulated by Swan and Bana (1998). The results were collated and an analysis was undertaken to determine the patterns of computational choices made by students, the success rates of these choices, and also the reasons given for such choices. The main focus of this paper is on the reasons given by students for making particular computational choices.

Background

Previous studies (Reys, Reys & Hope, 1993; Price 1997) have tended to focus on the computational choices made by students, but not on their reasons for making these choices. Reys, Reys and Hope (1993) found that while written methods did tend to dominate the thinking of many students, mental methods were also employed in many cases, with calculator methods being the least used. While they did not interview the students to determine reasons for particular computational choices, they did consider the nature of the questions and noted that computational choice seemed to be related to the nature of the numbers used.

Price (1997) suggested that teacher presence was a strong variable in computational choice. This was explained by the idea that teachers generally seem to promote written methods of computation ahead of mental and calculator methods. He designed a teaching experiment which sought to test this notion and found that with a teacher present, students chose written methods 56 percent of the time, calculators 26 percent of the time, and mental methods 19 percent of the time. A swing of 10 percent from written to calculator methods was noted when the teacher left the room.

Several authors (Reys & Reys, 1998; Kamii, 1994) have argued that an over-reliance on written methods of calculation interferes with the development of number sense. Thus the argument in favour of increased mental and calculator methods is that they develop a better

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understanding of numbers, generally referred to as number sense. The rather narrow view of computation as the development of standard written algorithms has certainly been challenged (Ralston, 1999; Reys & Nohda, 1994; Trafton 1994). A model of computational choice (NCTM, 1989, p. 9) outlining three main approaches (mental, written and calculator methods) to computation was refined by Trafton (1994) to emphasise the role of estimation in computational choice. Ruthven (1998) challenged this simple linear model of computational choice stating that "... a refinement of the common-sense trichotomy between mental, written and calculator methods was necessary to take better account of different forms and functions of writing within computation" (pp. 29-30). Swan and Bana (1998) developed an alternative model of computational choice in an attempt to better describe its complex nature.

Methodology

Seventy-five students from two schools (one State and one Catholic) in the south-west of Western Australia were asked to solve eighteen computation items, using any methods they chose. An individual, clinical-interview approach was used with each student in the study. For each item the student was observed undertaking the computation; the researcher noted the chosen method(s) of computation; then the student was asked to explain how he/she solved the problem. The interview was audio taped, and the student's written jottings or calculations were collected to confirm observations made by the researcher. For each item the student was also asked why he/she chose a particular computational method. Once the student had completed the item using the preferred method(s) of solution and had explained the reasons for using that particular method, he/she was asked if they could use an alternative method of solution. The interviewer continued to ask the student for alternative methods of solution until he/she ran out of methods.

Results

The instrument and some of the preliminary results of the study have been reported elsewhere (Swan & Bana, 1999). Further preliminary results below present an overview of the choices made by students in relation to particular sets of items, then focus on the reasons given by students for the computational choices that they made.

Table 1 shows that overall, 35 percent of all calculations were performed using mental methods, 27 percent were undertaken using pencil and paper, and 27 percent were performed with a calculator. The remaining 11 percent of calculations involved either mixed methods or no method of solution at all. For most students there was little or no hesitation when making the choice as to which method to use when solving an item. There was little evidence to support the notion that students carefully examine a question before choosing a computational method. On a few occasions it was observed that students, having embarked on a particular method, found that it was inappropriate and abandoned it in favour of another.

Table 1
Percentages of Student Choices of Computational Methods (N= 75)

Mental	Written	Calculator	Mixed or Not Attempted
35	27	27	11

Table 2 lists the items where students preferred to use mental methods. These items involve small numbers, fractions or decimals, and also the two items that were presented in a shopping context. It may seem somewhat surprising to see mental methods as the preferred choice for most of the items involving fractions, but it appears that many students were unable to solve these types of questions using a paper-and-pencil algorithm and did not know how to use a calculator to assist them. Thus they only had one option left at their disposal. It also seemed that some students overestimated their ability to solve items using a particular computational method. For example, some seemed confident that they could solve an item using mental methods when in reality they experienced difficulty obtaining a solution.

Table 2
Items where Mental Methods were Preferred

Item Number	Item Content
1	28 + 37
11	$^1/_2 + ^3/_4$
12	$10 - 4\,^3/_4$
13	$^2/_3$ of 45
14	$1.99 + $ 1.99 (presented in a shopping context)
15	$4.93 + 39c (presented in a shopping context)
16	7.41 – 2.5
18	3.5 ÷ 0.5

Table 3 lists the items that students preferred to solve with paper and pencil. The four items in this category are two-digit multiplication and subtraction computations which closely resemble the types of questions typically given in mathematics textbooks. Students generally spend a great deal of classroom time completing questions of this type. It appears that the cognitive demands of such two-digit items are such that many students in Years 5-7 are unable to complete the computation mentally and therefore need to use another method. Standard written algorithms are designed to be completed by recording interim steps, thus making them inefficient as a basis for mental methods.

Table 3
Items where Written Methods were Preferred

Item Number	Item Content
2	74 – 36
4	36 x 25
6	29 x 31
7	33 x 88

Table 4 shows the five items in which calculator use was the preferred method. "Big numbers", in some students' words, was a trigger that had students reach for a calculator. This was particularly evident in both Item 8 and in Item 17. The use of calculators dominated computational choice in Item 17, possibly because it involved both large numbers and decimals, and it was an unfamiliar calculation to many of the participants. Item 9, which involved the use of percentages was another item unfamiliar to many of the students, with a majority stating that they were unsure of how to proceed. Some of the students who were unsure of what to do, chose to use a calculator simply because they knew it had a percentage button. Many did not know how to use the percentage button and simply assumed that pressing the button in conjunction with the 750 would produce the desired result. Thirty-six percent of students chose to use a calculator method to solve 14 x 9 ÷ 6. This item often saw the use of combined methods of solution, with the first part being solved using one method and the second division part by another. It seems that these students chose to use a calculator because they were unfamiliar with this type of multi-step computation.

Table 4
Items where Calculator Methods were Preferred

Item Number	Item Content
3	369 ÷ 3
8	1000 x 945
9	10% of 750
10	14 x 9 ÷ 6
17	0.25 x 800

It appears that some aspects of a computation can override other considerations when it comes to making an initial choice of what method to use. For example, the "big number" cue seems to have come to the fore in Item 8 for 1000 x 945, where most students chose to use a calculator to complete what could be considered to be a relatively straight-forward mental calculation.

Reasons Given for Making Particular Choices

Previous research on computational choice by Reys, Reys and Hope (1993) reported on the computational preferences of students in Years 5-7, but the students were only required to state their preferences. The researchers indicated the need to interview students to find out why particular choices were made. Students participating in the current research were asked about the reasons for making particular computational choices and were also observed using their preferred method to undertake the computation.

The students in this study were not always able to clearly explain the reasons behind the computational choices they made. Some had problems articulating their reasons, while some appeared to automatically adopt a particular approach without much conscious thought. While some students were able to give detailed reasons for making a particular choice, others made broad statements such as "big numbers" to explain why they used a calculator. Even though a number of students gave their reason as "big numbers", it was most evident that students providing this explanation possessed differing viewpoints as to what constituted a big number. Similarly, other reasons given by students in the study tended to have various shades of meaning. The various reasons given for choosing a particular computational strategy are grouped under four common themes below, with examples from transcriptions of taped interviews with the students. This addition of comments made by the students helps to clarify what was meant when a particular reason was given.

Magnitude of Numbers

Students tended to use mental computation when the numbers involved were small, but often cited "big numbers" as the reason for choosing not to use mental methods. In some cases this observation prompted students to use a written method, while in other cases they used a calculator. In the following extract the student elaborates on the "big number" explanation. Note how the student links the idea of "big numbers" to memory constraints by suggesting they are difficult to remember. It is possible that the cognitive load is increased by the introduction of larger numbers.

I: Water noodles costs $4.93 and two-minute noodles sell for 39c, how much would that cost me altogether?

S: I'd just go $4.93, plus 39c equals, 9 plus 3 threes 2 left so carry the 1, 9 plus 3 is the same, carry the 1, put 2 and that's $5.22.

I: And why did you choose to do that by writing it down?

S: Because $4.93 is a big number to remember. You might just forget and put $4.39, it's better just to write it down so you can remember it properly.

Here is another explanation.

I: What about 369 ÷ 3?

S: 123

I: So you used the calculator there, why?

S: Because it's a big number, it's easier to use a calculator.

I: Could you do that in your head?

S: Yes, I'd do it like I was writing it in my head.

There was no evidence to suggest this student could successfully complete the division item in her head but the comment about "writing it in her head" is interesting. Several students described mental methods based on the use of a typical paper-and-pencil approach. The usefulness of this approach will be discussed later.

Some cautionary remarks need to be made about the "big number" explanation for using a calculator. Students using this reason for adopting a calculator-based approach to solving the item were not totally reliant on the use of a calculator but simply chose it as the most expedient option. The following extract shows an example of a student who chose to use calculator but was also able to complete the same item mentally. Whether the initial use of the calculator made the mental computation simpler is debatable, as her explanation of how to solve the item mentally is most plausible.

I: 70 x 600

S: 42 000

I: And you used a calculator for that one – why?

S: Because it was a big number.

I: Could you have done it another way?

S: I could have gone seven times six is 42 and then put three zeros on it.

I: Would you have done it in a written way at all?

S: No.

Later the same student also used a calculator to solve 1000 x 945. Note the reason she gave for using a calculator and her later explanation of how she would solve the same item mentally.

I: 1000 x 945

S: 945 000

I: Why did you do that one on a calculator?

S: Because I don't really know how to do long multiplication.

I: Could you do that one mentally?

S: Yes, I could have gone 945 x 1 and add three zeros.

The technique of crossing off and adding zeros was used by a number of students for Item 5 and Item 8. In most cases students experienced difficulty applying this strategy and did not appear to have any real understanding of why it worked.

Efficiency

A common reason given for using a calculator involved speed. Expressions like "it's faster", "it's quicker", or "it's quick and easy" were often given in support of calculator use. Speed was also cited as a reason for not using the written algorithm. Comments such as "it would take longer" and "it would take too long" were commonly used to explain why the written algorithm was not favoured. Examples of students using this reason for various items are given below. In the first example the student uses the "big number" reason in conjunction with a comment about the amount of time it would take to complete the item on paper to explain why he opted to use a calculator. The inflection in his voice suggested that the speed factor weighed heavily on his mind.

I: 1000 x 945

S: 945 000

I: Okay, I notice that you used a calculator, why was that?

S: Big sum.

I: Could you do it any other way?

S: On paper, but it would take forever.

I: Any other way?

S: No.

Knowledge of Multiplication Facts

The comments made by students in the study indicate that basic multiplication facts or "tables" feature prominently in their thinking about mathematics and have an influence on the computational choices that they make. The following comments by a student indicate a preoccupation with "tables". The last comment is of concern as it appears as though the student considers that, to perform such a multiplication requires knowledge of basic facts beyond 9 x 9.

I: 14 x 9 ÷ 6

S: 21

I: You used a calculator, why?

S: Because 14 x 9 and divided by 6, I don't know my times table up to that standard.

Here is another case.

I: What about 36 x 25?

S: 900

I: Once again you chose to use a calculator. Why?

S: Because again it was a big number. I don't really know my 36 times table.

Teacher Influence

The comments made by students in the interviews help to answer the question "What influences student choice of computation method?" There were comments from students alluding to the fact that they did it this way in class. For example the influence of the teacher and the teaching of standard written algorithms may be noted in the following extract.

I: All right but you prefer to write it down. Is there a reason for that?

S: I just was taught to do it and I've always been doing it that way. It's easier because when I do it on the calculator I think it's not using your brain and you don't work things out better and you don't get smarter, so I write it down because it's easier and I understand better.

The student also indicates an aversion to using the calculator, based on the notion of "not getting any smarter" as a result of using a calculator. The impact of this thinking is to restrict computational choice.

Conclusion

The examples above represent only a small sample of the responses that were given by students in the study, but they do serve to throw some light on the reasons for students employing particular computational approaches. As Price (1997) has found previously, teachers have a strong influence over the computational choice made by children. Schools and teachers chosen to be involved in this research were in part selected because they allowed students to keep calculators in their desks and because they espoused contemporary beliefs about the need to develop a repertoire of computational strategies. Despite this, students still referred to the teacher having an impact on their computational choice. Clearly there are implications for teachers and teacher educators in this result.

The issue of speed may also be influenced by classroom practice where students can be encouraged to complete as many questions as they can in the time available. It was not surprising to find students mentioning "tables" as a reason for making a particular computational choice, because significant adults in students' lives such as parents and grandparents may often refer to "tables" whenever any mention is made of mathematics. Basic number facts drill is a common practice in many classrooms and, while it was not a significant part of the daily routine in the schools included in this study, there was evidence of "tables" practice having been carried out in the classrooms. The "big numbers" explanation was a broad category used by students to explain the choice of written or calculator methods. Although this explanation was not used consistently by all students in the study, this is possibly due to their wide range of experiences.

Further research is needed to examine the various influences on students' computational choice, particularly that of their previous school mathematics programs. Teachers need to be assisted to help their students to make appropriate computational choices. Shumway (1994) refers to this idea as "metacomputation" where students would be encouraged to consider various aspects of a computational strategy before defaulting to a favoured approach. From the current research it appears, however, that students make a fairly hasty decision based on a limited set of criteria.

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They can run, but can they hide?

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Secondary and tertiary mathematics teachers often bemoan the fact that many students cannot solve complex questions due to misconceptions involving simple concepts. The authors have earlier reported that the incidence of mathematical misconceptions in both bright and weaker first-year tertiary students was reduced by using a technique based on Piaget's notion of cognitive conflict, and that most of this gain persisted. This study sought to determine whether the same technique diminished the frequency of misconceptions in longer, more complex questions.

Introduction

Teachers of secondary school mathematics or tertiary mathematics will be all too familiar with the following scenario. A student is given a problem to solve; the student presents a solution which would have been correct except for a misconception carried forward from earlier years which destroys the student's attempt to obtain a flawless solution.

The authors, having witnessed events such as this over a long period of time, decided to pursue a possible solution to the problem. The various stages of their research have been reported over the last four years. Their first step was to devise a list of mathematical misconceptions which students exhibit. Davis (1984) had previously considered the different types of mathematical misconceptions. Swedosh (1996) recorded a study which examined the nature and frequency of mathematical misconceptions displayed by students entering tertiary mathematics subjects at two Australian universities.

Having investigated the nature of mathematical misconceptions demonstrated by these tertiary students, the authors considered strategies which might reduce or even eliminate these misconceptions. Tirosh (1990) and Vinner (1990) had earlier supported the conflict teaching approach, based on Piaget's notion of cognitive conflict, as being successful in helping students overcome misconceptions. To put the conflict teaching approach into operation, teachers discuss with the students the inconsistencies in the thinking of the students in order to have the students realise that their conceptions are inadequate and in need of modification. Swedosh and Clark (1997) communicated an experiment designed to help a group of bright first year students (as seen from their high secondary mathematics and university entrance scores) at the University of Melbourne (U. of M.) overcome their mathematical misconceptions so that the success or otherwise of the conflict teaching approach could be determined. The authors found the use of this strategy to reduce mathematical misconceptions to be a remarkable success.

The authors then conjectured that it may be possible that, if these students were tested at some later time, some of them may have reverted to their (often long-held) misconceptions. Swedosh and Clark (1998) described an investigation into whether the reduction in mathematical misconceptions exhibited by the group of bright first year students at the U. of M. referred to in the previous paragraph persisted or whether the effect was temporary. The authors' research showed that the improvement resulting from the use of the conflict teaching approach was much more than a short term quick fix, with the students showing that substantial benefits were still evident one year after the initial reduction in mathematical misconceptions had occurred.

The strategy used to this point, based on Piaget's notion of cognitive conflict, was clearly very useful in reducing mathematical misconceptions with the group of bright first year students with whom the investigation was carried out. The question now arose: for students who are less able mathematically, would such an improvement occur? The reason for asking this question was two-fold: The bright students had been found to be capable of recognising the inconsistencies in their previous thinking and then learning the correct concepts quickly, and they had indicated during their discussions with the authors that they were embarrassed by making the errors that they did and they had a strong desire to remedy the situation. The authors were of the view that both of these qualities might be more pronounced with bright students than with less able students. Swedosh (1999) related an investigation to determine the effectiveness of the conflict teaching approach with a group of less able students; the experiment provided strong evidence that a teaching strategy based on Piaget's notion of cognitive conflict can be successfully employed to significantly reduce the frequency of mathematical misconceptions exhibited by less able students.

Having now shown the efficacy of the conflict teaching approach in reducing mathematical misconceptions in students of different ability levels, and that the benefit persists over time, the authors were able to assert that

> The major benefit to be gained from incorporating this strategy, which is extremely simple to implement, into one's teaching, is that not only is it likely that fewer students will have these misconceptions, but, as a result of this, many will directly improve their chances of being successful in their future studies of mathematics (Swedosh, 1999, p. 476).

The investigation reported in this paper considered the problem as to whether a mathematical misconception which had been eliminated or at least vastly reduced in a particular group of students might nevertheless be exhibited if the concept was embedded in some longer, more complex question. For example, consider the question: "Solve for x: $x^2 = 4x$ for which a common misconception is that $x = 4$ is the only solution. Suppose that this misconception has been eliminated, and that students respond $x = 0$ or $x = 4$. Does the misconception re-occur if students are asked a question such as: "Use the fact that $\ddot{x} = \frac{d\dot{x}}{dt}$ to solve the differential equation $\ddot{x} = \dot{x}$"?

Methodology

The methodology was essentially the same as that reported in Swedosh and Clark (1997) with a few small differences. A short test was administered to a group of first year students who were studying Applied Mathematics at the U. of M. in Second Semester, 1999. In the earlier study, a test had been used which was made up of questions similar to those posed in earlier tests at the U. of M. and at LaTrobe University (Worley, 1993) and to those which appeared in the list of misconceptions provided in "Algebraic Atrocities" (Margulies, 1993, p. 41). Each of the questions had previously yielded a high frequency of misconceptions (Swedosh, 1996) so that if students did possess a particular misconception, the question would provide them with the opportunity of exhibiting that misconception. The tests compiled for this study used some of the questions from the earlier study, but also comprised questions in which the concepts being tested in the simpler questions were embedded in more difficult problems so that the concept being tested was not so obvious. Students had no prior warning that there would be any tests.

Questions on all test papers were examined carefully to gain information on the misconceptions exhibited and the nature of each response was recorded. That is, whether the response could be categorised as correct, a misconception, or another wrong answer. The test was administered in the third week of the semester.

Approximately twenty minutes was then spent during a lecture in the fifth week of semester using the conflict teaching approach in an attempt to decrease or eliminate the frequency of misconceptions exhibited on the first test and replace them with the correct concept. For example, if students were to exhibit the misconception that if we were to solve x: $x^2 = 4x$ and answer that $x = 4$ was the only solution, suitable approaches would be to demonstrate by substitution that $x = 0$ is also a correct solution or to discuss the fact that we are dealing with a quadratic and expect two solutions. Reference to the relevant parabola could also be used. At this point, the student (hopefully) would be able to see that the misconception leads to an incomplete and therefore incorrect answer, and would be happy to discard it. The correct concept would then be taught (putting the quadratic on one side of the equation so that it equals zero, factorising, and hence finding both solutions).

The six questions on the first test were used again in the second test but an extra question was added. This was due to a situation which arose in an assignment question in the middle of the semester which the authors thought needed further consideration in that a high proportion of students made the same misconception. The misconception was to solve an equation for a variable squared, and then give the answer as the positive root. In this question, the initial conditions ensure that the correct answer is the negative root and students were expected to consider both roots, and then choose the positive one. The second test was administered to the same class in the tenth week of semester (five weeks after the teaching session previously described). The five week break was important so that students were answering questions on the second test based on their knowledge of the relevant concepts rather than the memory of what they had recently been taught. In the case of Question 7, a teaching session was carried out in week 12.

The frequency of occurrence of misconceptions on each of the questions was then calculated, and these were compared with the frequencies exhibited on the same questions before the teaching session. In the case of the new (assignment inspired) question, the frequency of misconceptions was compared with the frequency exhibited on a similar question on the final exam.

The Sample

The test was administered to students enrolled in first year Applied Mathematics in Semester 2, 1999 at the U. of M. All students in this class had successfully completed a fairly demanding prerequisite mathematics subject in Semester 1 which only about 25% of first year students were permitted to attempt. So that a meaningful comparison of the results could be made, and to ascertain whether the intervention of the teaching approach had caused a reduction in the frequency of misconceptions, a sample of these students who have homogeneous backgrounds are reported on. The students reported on in this study sat both the first test and the second test, and had completed Specialist Mathematics 3/4 (SM) as part of their Victorian Certificate of Education (V.C.E.). There were 57 students in this category.

In Specialist Mathematics 3/4 there are three Common Assessment Tasks (CATs) and students are awarded a grade from E to A+ corresponding to a ten point scale from one to ten. Using this scale, the 57 students in this sample had an average mark for SM CAT 1 (the challenging problem) of 9.91, an average mark for SM CAT 2 (facts and skills) of 9.47, and an average mark for SM CAT 3 (the analysis task) of 9.26. They had an average Equivalent National Tertiary Entrance Ranking, or ENTER (a percentile with the highest possible ranking of 99.95 and with about 23 students for each 0.05 -- 0.05% of students in the state is about 23), of 97.64. Clearly this was an extremely able group of students, even more able than the group which had been considered in a previous study (Swedosh and Clark, 1997) despite the fact that that group was extremely good. In the previous study, the average TER was 93.56. The relationship between the TER, and the new measurement, the ENTER, is that participation rates are now taken into account in the calculation. This has the effect of elevating the rank, in this case by about two points. Those excluded from this study include mature age students as well as interstate and overseas students whose backgrounds were quite different to the 57 students considered herein.

The Tests

The first test consisted of six short answer questions. The first three questions were relatively straight-forward, whereas the latter three were more complex but had the same concepts embedded. The second test contained these same six questions plus one extra question which is numbered 7 below. The exam question is shown as E below. As previously stated, each question "invited" students to make a particular misconception, should that misconception exist. Students were given twenty minutes to complete the test. An analysis was conducted on each question and a count was taken of how many students had the correct answer, how many had exhibited the misconception, how many had another wrong answer, and how many had not attempted the question.

The eight questions used are shown below, and the most common misconception(s) are shown to the right of each question:

1. Solve for x: $x^2 = x$. $x = 1$

2. Given that $\sin\frac{\pi}{6} = \frac{1}{2}$, evaluate $\sin\frac{7\pi}{6}$. $\frac{1}{2}$, $\frac{7}{2}$

3. Indicate whether the statement is True or False. (circle one)

   $k\left(50 - \frac{x}{5}\right)(80 - 2x) = k(250 - x)(40 - x)$ T / F True

4. Use the fact that $\ddot{x} = \frac{d\dot{x}}{dt}$ to solve the differential equation $\ddot{x} = 2\dot{x}$. $\dot{x} = 2x + c$

5. When preparing to sketch a polar graph, you are required to first complete a table of values. Given that when $\alpha = \frac{\pi}{3}$, $\cos\alpha = \frac{1}{2}$, fill in the three blanks in the following table for the relation $r = 2\cos\theta$. DO NOT SKETCH THE GRAPH.

   2, 3, 4; $\frac{1}{2}$, 1, $\frac{1}{2}$

θ	α	2α	3α	4α
r	1			

6. The differential equation governing a particular chemical reaction is
$\frac{dx}{dt} = k(40 - \frac{2}{5}x)(50 - \frac{1}{5}x)$.

Can the solution to this differential equation be expressed in the form
$\log \frac{100 - x}{250 - x} = kt + c$?

Yes

7. Solve the differential equation $\frac{dv}{dt} = \frac{1}{2v}$ subject to $v(0) = -2$. $v = \sqrt{t+4}$

(Your answer should explicitly express v as a function of t.)

E. Solve the differential equation $2xy\frac{dy}{dx} = x + 1, x > 0, y(1) = -2$ $y = \sqrt{x + \log x + 3}$

(Your answer should explicitly express y as a function of x.)

Teaching Method

The teaching took place two weeks after the first test. As specified in Swedosh and Clark (1997),

> The method essentially involved showing examples for which the misconception could be seen to lead to a ridiculous conclusion, and, having established a conflict in the minds of the students, the correct concept was taught (p. 476).

When the correct concepts were taught, care was taken to use slightly different examples to the questions on the tests to ensure that students had to correctly use the concept, not just remember what they had been shown. Some examples of what was taught are shown below.

$x^2 = 9x, x^2 - 9x = 0, x(x-9) = 0, x = 0, 9$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$; $\sin\frac{4\pi}{3} = \sin\left(\pi + \frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$, as $\frac{4\pi}{3}$ is in the third quadrant.

$k\left(10 - \frac{x}{2}\right)(200 - 5x) = k(20 - x)(40 - x)$, (True or False?)

$k\left(\frac{1}{2}\right)(20 - x)(5)(40 - x) = k(20 - x)(40 - x)$; $\frac{5k}{2} = k$ which is only true for $k = 0$.

$$\frac{dv}{dt} = \frac{1}{4v}, v(0) = -3; \int v\,dv = \int \frac{1}{4}\,dt; \frac{1}{2}v^2 = \frac{1}{4}t + k; v(0) = -3, k = \frac{9}{2}; v^2 = \frac{1}{2}t + 9; v = \pm\sqrt{\frac{1}{2}t + 9}.$$

But only the negative root satisfies the initial conditions, so $v = -\sqrt{\frac{1}{2}t + 9}$.

Results

The tables below summarise the responses given by the 57 students to the eight questions shown earlier. It is important to note that in the tables below, "Question 7 Before" refers to the first time this question was attempted (on Test 2) and "Question 7 After" refers to the second time this question was attempted (on the exam). Table 1 shows the frequencies of each category of response given before and after the respective teaching session. Table 2 shows the proportions of each response before and after teaching session for those who attempted the question. In each table, 'Q' is the question number.

Table 1

Response Frequencies Before and After Teaching

Q	Correct		Misconception		Other wrong		No attempt	
	Before	After	Before	After	Before	After	Before	After
1	53	56	4	1	0	0	0	0
2	57	57	0	0	0	0	0	0
3	54	55	3	1	0	0	0	1
4	9	24	47	31	1	0	0	2
5	42	45	7	4	4	6	4	2
6	18	42	12	13	7	0	20	2
7	29	31	24	17	3	9	1	0

Table 2

Response Percentages for those Attempting the Question

Q	% Correct		% Misconception		% Other error	
	Before	After	Before	After	Before	After
1	93.0	98.2	7.0	1.8	0.0	0.0
2	100.0	100.0	0.0	0.0	0.0	0.0
3	94.7	98.2	5.3	1.8	0.0	0.0
4	15.8	43.6	82.5	56.4	1.8	0.0
5	79.2	81.8	13.2	7.3	7.5	10.9
6	48.6	76.4	32.4	23.6	18.9	0.0
7	51.8	54.4	42.9	29.8	5.4	15.8

It can be seen from Table 2 that for every question except Question 2 (where it was impossible), the proportion of misconceptions exhibited decreased. Question 6 actually had one more student exhibit a misconception after the treatment but there were ten times as many students not able, or not willing, to make an attempt on the first test. Many of the gains were not large but many could not be due to the very high baseline set by this group. This was particularly evident on the straight-forward questions. The reason that the "Other error" proportion was high on the exam (Question 7 After) may be due to exam pressure.

Table 3 shows the pattern of student responses from the first test to the second test. CC means that the student gave the correct answer on both tests. CM means that the student gave the correct answer on the first test and exhibited a misconception on the second test. MC means that the student first exhibited a misconception and then gave the correct answer. M M means that the student exhibited a misconception on both tests. Only responses categorised as correct answers or misconceptions are considered in this table.

Table 3

Pattern of Student Responses

Q	CC	CM	MC	MM
1	53	1	3	0
2	57	0	0	0
3	54	0	1	1
4	6	3	17	28
5	34	3	4	1
6	15	3	10	3
7	24	2	5	14

For all questions (except Question 2 again), there were more students in the MC category than in the CM category. Questions 4 and 6 stand out as having had large improvements. The data in Table 3 can be analysed using McNemar's test for correlated proportions which focuses on discordant pairs (CM and MC). This test gives a very significant result for the Question 4 data ($p = 0.003$). The data for Question 6 is significant for a one-sided test ($p = 0.046$) but just fails to be significant with a two-sided test ($p = 0.092$). It is a little disturbing that there are a small number of students who gave a correct answer the first time and a misconception the second. The authors believe that it is likely that many of these students do not have a real understanding of the concept and that the answer given might therefore vary arbitrarily and that they "fluked" the correct answer on the first occasion. In any case, the outcome for those students was not improved.

Conclusions

The students considered in this study were so proficient at correctly answering simple questions involving basic concepts that little improvement could be made on such questions. On the more complex questions in which the same concepts were embedded, there was an improvement in all questions and a substantial improvement in two of the four. It seems that as we expected, it is more difficult for students to recognise exactly what is required and which concepts are involved in more complex questions. It appears that they become so immersed in the larger question, they lose track of the detail required or lose concentration regarding the "nuts and bolts" of the question.

The results indicate that while it is more difficult to reduce the rate of misconceptions when concepts are embedded in more complex questions, there is a clear improvement in student performance on these questions after the conflict teaching approach has been used. While not as dramatic as the improvement found in previous studies on simpler questions, the benefits are still considerable. This method has been shown to be useful in combating the very serious problem of students not being able to solve a non-trivial question without being hamstrung by a lack of mastery of some basic concepts along the way.

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Is the MERGA Conference Refereeing Process Fair?

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This paper examines the extent to which MERGA's 1999 Conference refereeing procedure was fair to the authors. The short answer to this is "yes". But while conceding that the realities of preparing a conference mean that fairness must be balanced against several constraints, the paper shows how some weaknesses may be ameliorated by detailed planning and a little more editorial authority.

MERGA's four-yearly reviews of Australasian mathematics education research (e.g., Sullivan, Owens, & Atweh, 1996) provide good summaries of activities by MERGA members and others, much of which has been presented in some form at MERGA Conferences. But the process by which this research has reached publication status has received little formal attention even though peer refereeing is seen as a *sine qua non* of academic endeavour.

MERGA Conference Proceedings have been refereed since 1993, with acceptance requiring a majority decision from three people described in the annual calls for contributions as "senior researchers". The number of referees used has gradually increased, without any corresponding increase in the number of accepted papers. The 1999 Conference used nearly twice as many referees as in 1998, and not all of them were "senior". This diversification was required by increasing work and publication time pressures, and was welcomed by some members. Here we use findings from confidential data available to us as editors of the 1999 Conference Proceedings (J. & K. Truran, 1999) to assess the balance between widening the refereeing base, maintaining the quality of papers, and fairness to authors of rejected papers. We report in a form which illustrates finer points of the process while preserving anonymity.

The MERGA Conference Refereeing Process

Each year the process is administered by host city editors who have a high degree of autonomy while still subject to MERGA and Conference structures. Their academic role involves matching submitted papers with appropriate referees, checking that referees' reports are appropriate, and seeking advice in borderline cases before making final decisions. Since a wide range of research approaches and paradigms is encouraged, this task is not always easy.

Refereeing is "double blind", but MERGA is small and many authors will be recognisable. There is an "Early Bird" (EB) facility, principally for new researchers, but available to all, allowing referees to recommend revision and resubmission of a paper. But for the remainder, referees may give only "accept" or "reject" judgements. Referees are generally sent no more than three papers, but their work is a labour of love, relying heavily on members' substantial goodwill. They are provided with the criteria authors were expected to address and a pro forma listing seven aspects to address before making a judgement.

Because of tight time-lines, we sent all papers to three referees simultaneously—a break with the previous practice of using a third referee only for hung judgements. This proved to be wise. The experimental probability that any two out of three referees would provide identical decisions was about 0·45. Some differences were academic ones, but others arose from failure to return or acknowledge the receipt of papers, unwillingness to referee a specific paper on ethical grounds, etc. Given that the 0·45 includes some reports which were returned very late, using only two initial referees per paper would have required a time-consuming second mailing for about two-thirds of the papers.

Choosing a Pool of Referees

We estimated that for a projected submission of 100 papers we would need at least 100 referees—a doubling of a refereeing pool which had never been larger than 51 in the past. We had attended most MERGA Conferences since 1992 and knew most long-standing members by sight, but often knew little about their research interests. So we compiled a data base of all current members, all authors of refereed papers to MERGA Conferences between 1993 and 1998, and all referees used (excluding 1997 when all referees were from New Zealand and no list was provided). The MERGA Membership Directory (White, 1998) with its electronic update provided members' reported "major areas of interest" (usually three or less) and "current research" and also lists of who was interested in each research field.

After discussion with the local Programme Committee we decided that the basic criterion for the refereeing pool should be two refereed papers already accepted for MERGA Conferences. This criterion satisfied the Department of Education, Technology and Youth Affairs requirements operating at the time, and was more rigorous by one paper than that used for the international Psychology of Mathematics Education (PME) Conferences.

Of our basic list of 134 referees, 49 had not refereed before. We added another eight members who were distinguished researchers but had been little involved in MERGA Conferences and four distinguished unfinancial members. We did not include a further 32 eligible, but unfinancial, researchers, although five of these did submit papers in 1999. Thirteen former referees did not have sufficient published papers to be included in our list—indeed, eight of these had never presented a refereed paper at a MERGA Conference. So our refereeing pool of about one-half of MERGA's 300 members was constructed in a way which seemed fair to authors because it was based on transparent criteria, and fairer to members, because it made it easier for newer members who had some proven research experience to be recognised as ready for refereeing experience.

Indeed, after we had written to members on our list, several new referees said how pleased they were to be invited, and a few members sent favourable comments about our general approach to selecting referees. Some of those selected were unavailable in 1999, and a small number of experienced members with many MERGA Conference publications declined to be involved, but we were confident that we would have sufficient referees. In practice, other difficulties did arise. Some were unavoidable—study leave, sickness, unexpected other commitments, etc; others arose from human frailty—long-delayed or cursory reports—or electronic problems. Some reports were never returned or acknowledged. It proved necessary to use four South Australians who did not fit the standard criteria, but who had had relevant experience.

We did not find that inexperienced referees necessarily produced poor reviews. Many did not. In the important case of ultimately rejected papers there were only eight new referees concerned with fourteen papers, and only two papers were read by more than one new referee. Some of these had had extensive experience in other areas and, to the best of our knowledge, none of the others was naïve. In any case the vast majority of negative reviews came from the more experienced referees. Using many new referees did not affected the ostensible fairness of the process at its most critical point.

This section has summarised how we doubled the size of the refereeing pool in a systematic way which ensured that all referees had had at least minimal adequate experience for the task. We now need to assess the extent to which this change provided fair judgements for authors.

Questionnaires

Two questionnaires were prepared. Responses will be used at suitable places in the analysis below. The first was given to all conference participants and attracted 18 responses. It included the following questions:

- If you were a referee did this system mean that you reviewed papers that were appropriate to your interests and specialties?
- As an author do you think your paper was reviewed by a person confidant in your specialty?
- If referees were not unanimous in the judgement of your paper how did you respond?

The second was sent to thirteen authors of rejected papers and asked if they considered their referees to have shown a sound grasp of the field and to have made reasonable points. Where the referees were not unanimous we also asked whether the positive review was seen as more valid than the negative ones. We received seven responses, including four from senior MERGA members. ("Senior" from now on refers to a person who has either held a leading academic role in MERGA or has a rank of at least Associate Professor.)

The Fairness of Judgements on Papers Which Were Rejected

In this paper we focus on the fairness of the process to those whose papers were rejected since this is the most overt form of potential error. But acceptance of poor papers is also unfair, certainly to the authors, and also to those who attend the presentations. Indeed, we had received comments about papers which were believed to have slipped through the net in previous conferences; unfortunately, space does not allow further discussion of this point here.

Some rejected authors were understandably upset, and some very quick to withdraw from the Conference. One believed the rejection was the result of paradigm warfare; another sent an unsolicited short list of suspected referees! It was totally inaccurate. Several believed that some of their referees did not understand their work or their paradigm very well, particularly if they were not working in traditional moulds (either quantitative or qualitative).

This important claim was also made by some of the accepted authors. It is difficult to examine and hard to rectify if valid. As editors, we could not have pre-read every paper: many would have been too far beyond our expertise for us to have anticipated difficulties anyway. But some criteria are available.

Acceptance Rates

One simple measure of fairness is acceptance rates, which could show if there has been a substantial, not easily explained, change from previous years.

In 1999 there were 17 EB and 70 standard submissions. Eight EB papers were accepted at once, and the others resubmitted and subsequently accepted, apart from one withdrawn for personal reasons. Four EB and seven standard papers were entered for the Practical Implications Award which was judged separately by a panel of five who also accepted or rejected the papers. Our main editorial task dealt with 80 papers, sent to 96 referees, of whom 90 returned reviews.

Of the 86 final proposals received, 70 were accepted, and 16 recommended for presentation as Short Communications. Seven of these papers, from six authors, were withdrawn, in four cases accompanied by the authors' withdrawal from the Conference. So the acceptance rate for 1999 of 80 percent is similar to that of 84 percent for 1996 (Clarkson, 1996). Data for other conferences are not available, but Clarkson suggested that the rate seemed to be reasonable to him, although he did receive some comments that it was unreasonably high.

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So the acceptance rates suggest no large divergence from a general practice which had been seen as reasonably effective. The rejection of at least five papers by senior authors was not surprising, because a similar phenomenon had occurred in 1997 (Biddulph, co-editor, pers. comm.), and Clarkson (1996) had noted that in some jointly-authored papers "the senior partner had [clearly] given little input into the final development of the paper". Three of our rejected papers may have fitted this situation.

Quality of the Referees

Peer review relies on consensus among experienced researchers for its validity. No editor can be sufficiently knowledgeable to assess fairly all papers and judgements. A high quality of referee is essential. So we shall examine the quality of our review process in several ways as basis for a general conclusion.

Comparison with Other Areas of Mathematics Education

Of the 90 referees whose reports were returned, 28 were either members of the Editorial Board, or in 1999 had refereed for MERGA's *Mathematics Education Research Journal* (MERJ, 11, 2, 156). Eight (out of sixteen Australasian referees) were used by the American *Journal for Research in Mathematics Education* in 1999 (JRME, 30, 5, 598–599), and one (out of five) by the European *Educational Studies in Mathematics* (ESM, 40, 3, v). Referees used by these journals but whom we did not use were either unfinancial, unavailable, not within our criteria, or just not called upon.

Two points arise. Our refereeing base, as in all recent MERGA Conferences, used far more members than do several leading journals. This says little about quality, because the selection of referees reflects editorial knowledge and experience. For example, the small proportion of Australians used by JRME does not reflect their large international contribution to mathematics education.

More importantly, many senior MERGA members were "unavailable". Their reasons were many, some obviously reasonable, some less so. For conference papers there is a need for the greatest expertise possible. Judgements are more absolute and immediate than those on journal papers. Every negative decision affects a university's research budget by about $1000, may affect the Conference's budget, and will have a significant irrevocable personal impact on authors. We had some weak referees, and our widened base would have benefited from the presence of more very experienced researchers.

The Judgements of Specialist Referees

Some evidence for this claim is available because we tried to use referees with specialist skills who were outside (but only just outside) the normal MERGA circle. Three well qualified people co-operated, refereeing six papers from five authors. They rejected all six. In three cases all referees were agreed, in two others the paper was rejected and in one case the paper was accepted.

This small sample cannot provide conclusive evidence, but does suggest that papers sent to an outside expert were more likely to meet a more demanding standard than those sent to people, however well read, with only a general interest in the topic. The fact that in the three non-unanimous cases the "expert" opinion was that the *methodology* of the research was inadequate lends support to this claim because such technical detail is unlikely to available to the general reader. Indeed, two of the methodologically poor papers received perfunctory positive reviews from a long-standing MERGA member with a professed interest in the field, but no relevant published papers, at least at MERGA conferences. The contrast in quality found here provides a good case for some editorial discretion over accepting minimalist reviews (our shortest was one word), especially when they are negative. It also suggests that regular infusion of new blood is likely to lead to an increase in standards over time.

Matching Papers with Referees

Some of these difficulties could be avoided by making a good match between authors' content and referees' expertise. Most respondents felt that the reviewers were competent to review their paper, although one wanted more detailed feedback to be sure that the referee had actually read the paper, and one felt the referee didn't know enough. Equally, most referees were willing to review papers not within their expertise, no doubt aware of the difficulties we were encountering in effecting good matches between our data base of interests and authors' description of their papers in terms of MERGA's categories. This was made more difficult because some information for both authors and referees was not available, forcing us to make some classifications ourselves. as well as making some adjustments because the classifications had changed in the period just before the 1999 Conference.

Assessing the quality of matching is not easy. One-one correspondence will not do, because a paper may not be adequately classified. Thus a paper classified as "Geometry" but with an assessment perspective might lead us to choose one referee with expertise in assessment but not in geometry. Sometimes we made use of personal knowledge not recorded on the database, particularly to ensure that good researchers were fully utilised. Of the fourteen rejected papers we handled, copies were sent to 47 referees. Five went to outside experts, as discussed above, 23 to good matches in the classification system, and seven to mature researchers with a broad interest. Seven went to referees for reasons which are not apparent from the data base ten months later—possibly creative desperation, and five were not returned by referees, requiring a second, perhaps less good, choice of referee. This was the best we could achieve: the work was done thoughtfully and never rushed. But some check was available on poor matches by examining the quality of responses, as discussed below.

One additional problem arose because many researchers work in teams which submit several papers. We decided to send papers from one team to no more than one member of any other, and preferred to send papers to three different states or at least three different universities. These constraints caused special difficulties for smaller research fields. In one field, for example, there were five appropriate referees, two pairs of whom formed research teams. We distributed the two papers from a third research team as fairly as we could, but it happened that for one of these papers one referee with expertise did not respond, another rejected the paper, and the third, with no specific expertise, accepted it. The ring-in, last-minute, local, fourth referee had no specific expertise and was quite aware that a delicate line-ball judgement was being called for.

Proven Experience of the Referees

Did members' expressions of interest match their published papers and proven experience? A full answer would require examination of each referee's *corpus,* which has not proved possible. But a random sample of ten referees was taken to compare expressed interests with refereed papers published at MERGA conferences. This small survey is sufficient to indicate the existence of all five theoretically possible cases. One person had interests and publications which matched exactly, and two had no overlap between them at all. Three had interests which were wider than their published papers, and one had papers not related to their declared interests. Three had both of these: papers outside their interests and interests outside their papers.

Such diversity is not surprising. Our own listed research interests cover three where we have published papers, another where one of us has a major piece of work in preparation, and two where one of us is currently teaching students and is therefore familiar with the literature. No doubt others have expressed interests for similar reasons. But to find such diversity in a small sample is strong evidence that a process of selecting referees based on their proclaimed interests does not ensure that they have *proven* research expertise in those fields, and also fails to pick up expertise which has been proven.

The data presented here show that some of the real difficulties encountered in making good refereeing matches might be overcome by a more detailed data base, both of skills and of past reliability. Weaknesses which remain may be partially addressed by more attention to the quality of the reviews.

Assessing the Quality of Review

We discuss the general efficacy of the reviewing process elsewhere (K. & J. Truran, 2000); here we discuss some hard cases. We have mentioned how hard it is to assess work in unfamiliar fields. Nevertheless, for us good reviews usually conformed with the published criteria, showed awareness of other work in the field, summarised strengths and weaknesses, and made an holistic summary arguing a case for the judgement given. The best ones also suggested improvements and ideas for further work. The poor one contained errors of fact, made unreasonable demands on a short paper, were perfunctory, or offered little of value to the author. Frequently they were also returned tardily.

As a check, one of us read all reviews to check for glaring inconsistencies, and both of us read all reviews of potentially rejected papers. We found two situations which we thought needed attention. The first was when a review did not seem to be factually accurate. For example, one negative review stated that a theoretical paper was not "sufficiently concerned with mathematics education" and "did not offer a significant review of a body of literature [on] mathematics learning". This strong claim matched neither our knowledge of the author nor what we read in the paper, so we sought a fourth opinion on the accuracy of the claim and the acceptability of the paper, because the other referees were divided. Strictly, we went beyond our remit, but we considered that we had authority, after suitable consultation, to over-rule a report on a matter of fact. In our opinion this produced a much fairer decision.

Secondly, we found a minority of reviews which were brief, trivial, rushed, or more than one of these. One example has been mentioned above. We wanted to put these to one side, especially the negative judgements, and to seek further opinions, but time did not allow this. It was a consolation to find that only one of the negative reviews of the rejected papers was close to being unacceptable on these grounds, but this did at least cite clear, testable reasons for rejection, albeit very briefly.

We are not of the opinion that editors should wantonly put any reviews aside. The process is peer review, not editorial review. Several reviews, particularly from strong-minded members, seemed unsympathetic to the authors' approaches, but were carefully argued, and it would have been improper to reject them. But where a review, especially a negative one, provides poorly explicated reasons, it seems to be fairer to seek another opinion. This we felt we could not do, but believe it was detrimental to the fairness of the process.

Conclusions

The confidentiality of the refereeing process means that this paper cannot be subject to some of the normal checks of academic research. And since some aspects of the refereeing process are idiosyncratic to individual editors, it is not possible to generalise our conclusions.

Nevertheless, the results illustrate several features of refereeing which are rarely formally analysed, and provide a research base for establishing principles of wide generality for sound refereeing.

We have not quite answered the claim of some rejected authors that some referees had not understood their papers. Because almost all of the rejection decisions were not unanimous, it is not surprising that authors felt that if one referee held a positive opinion, there were presumably others who would have agreed. We have certainly had the same experiences: trying to estimate one person's understanding of another's ideas must be a task for another paper. What we have done here is to show some of the reasons why there might have been misunderstandings, and to suggest ways in which the likelihood of misunderstanding might be reduced.

We had been told that the refereeing process can never be made foolproof. Here we have shown how the efficacy of the refereeing process rests strongly on the skills of the editors in matching papers with competent referees while working under significant time pressures, and also in making good value judgements about the quality of the reviews which are submitted. We have shown how errors can arise, but also how many potential problems can be reduced by more carefully constructed data bases or by allowing editors a little more autonomy to over-rule, or seek further opinions on, reviews they consider unsatisfactory. Our argument also suggests that there is a case for using permanent conference editors, rather than making use of people from the host city.

Nevertheless, there is little evidence to suggest that the 1999 procedures led to gross unfairness. Some allocations were only moderately good, and some reviews of poor quality, but these weaknesses were less apparent in cases where a paper was rejected than in general. This seems to have been fortuitous, thus emphasising the importance of careful prior organisation. No process is foolproof, but we have shown how the present procedures may be easily tightened.[1]

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Is the MERGA Refereeing Process Improving the Quality of Australasian Mathematics Education Research?

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This paper follows from a belief that the activities of an educational organisation should model good educational practice. MERGA's refereeing practice is examined as a way of improving the quality of teaching and learning about mathematics education research for both authors and referees. While many positive features are already present, some weaknesses or omissions are identified which, it is suggested, are more likely to be addressed if MERGA's educational role is seen as an important focus of its work.

In the calls for contributions to annual MERGA Conferences will be found statements like the following:

> MERGA believes that the review of papers gives conference attendees some confidence in the quality of the publication of proceedings. The refereeing process also provides a mechanism for peer review and critique and so contributes to the overall quality of mathematics education research.

This objective matches the first official aim of the Group: "promoting quality research in mathematics education" (MERGA Constitution, 10 Jul 1997). MERGA, as a professional organisation, wishes to ensure that papers presented and published at its conferences conform to high academic standards. But how are such standards and quality to be achieved? Osmosis is rarely sufficient. We will argue in this paper that when MERGA's activities, in particular its Conference refereeing process, are seen as providing models of good educational practice, then the question being addressed here becomes an important preliminary to identifying ways of enhancing MERGA's current practices.

The review process used is described in J. & K. Truran (2000); here we assess its value at the 1999 Conference for improving authors' papers, especially in the case of Early Bird (EB) submissions where resubmission is possible. This is clearly a teaching process, one conducted at a distance and, quite unusually, one where teacher and learner are unknown to each other, in theory at least. It is an area which has attracted little formal research attention, partly, of course, because it is conducted confidentially. This paper uses confidential data available to us as editors of the 1999 Conference Proceedings (J. & K. Truran, 1999) to examine aspects of this rather special teaching/learning situation. We report in a way which illustrates current practices while preserving anonymity.

The Refereeing Process

Three referees competent in the field review each paper independently and all reviews are then sent to the author to support the editors' acceptance or rejection of the paper. While the review relies on subjective judgements, reviewers are asked to assess the paper against seven criteria, using a broad interpretation of the accepted norms for reporting research, and ensuring that it represents a contribution to mathematics education or research processes.

The success and perceived quality of the papers presented depends on the quality of the referees and their knowledge of the subject that they review. This is an issue attracting some attention in mathematics education research circles. For example, the Chair of the 20th annual Psychology of Mathematics Education Conference reported that some authors of rejected papers at PME conferences were claiming that some of the accepted papers were of lower quality than their own (Gutiérrez, 1998). Our subjective impression after attending five of the

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seven MERGA refereed Conferences is that similar views about poor papers being accepted are also found in MERGA, and not only by authors of rejected papers.

Gutiérrez also said that PME, which also relies on members volunteering to referee for its conferences, had found that some referees seem to confuse *being* expert in a topic with a *wish to be* expert in the topic (italics added). He stressed that in his opinion reviewing papers for PME was not the way to acquire expertise. We have suggested in J. & K. Truran (2000) that a similar confusion may exist among some MERGA members.

Nevertheless, none of us has finished learning about our research interests, and all referees have incomplete knowledge, even about their areas of specialisation. So, while the quality of referee's knowledge and comments is basic to the success of any conference and its publication, "quality" cannot be seen as a dichotomous attribute, but rather as a developing one. This view fits comfortably with one which sees the refereeing process as a broad educational activity.

In this paper we first examine the EB Process which provides an opportunity to see how authors use referees' comments to change their papers. We then look at some of the reviews presented for the standard papers to identify the types of comments made and assess their value for the authors, and we conclude by looking at what seem to us to be some missed opportunities for further educational development.

The Early Bird Process

EB reviews are available to inexperienced authors who feel the need for an opportunity for rewriting if judged necessary. In 1999 several experienced authors submitted EB papers as well: this may indicate some lack of confidence in the review process, but we have no specific evidence for this, and know that in two cases it was related to the planning of Australian visits from overseas.

Of the seventeen EB papers submitted, eight were accepted at once, and nine were returned for some revision. It was relatively easy to match EB papers with appropriate referees, because these papers were the first to be allocated. Referees recommending revision were asked to provide more detailed comments than they might for more experienced authors. Most referees also agreed to review the revised papers, which made for more consistent reassessment.

Examples of Referees' Comments on Early Bird Papers

We have probably all had to cope with submitting a paper that has been worked on and sweated over for a long time and receiving a "not accepted" judgement. Sometimes the referees' comments cause irritation and disbelief—how could someone have said what they did; how could they have missed the paper's real intent? Such irritation is exacerbated when the review is less than objective or indicates any weaknesses in a referee's knowledge. While MERGA provides a pro forma for use of referees, not all use it, and not all address all the criteria. The loss of educational potential in such cases is obvious.

For the 1999 EB papers there were examples of both exemplary and less than useful reviews. In an extreme case just one word—"accept"—was all that was given. Interestingly, some short acceptances were given to papers where careful responses from other reviews required a subsequent re-write of the paper. But in other cases all three referees took considerable trouble, providing useful, supportive comments presenting the author with examples of common flaws, omissions or illogicalities, and indicating a useful base for modification and eventual acceptance of the paper.

Comments like "the main reason for asking you to rewrite the paper is to do with length and layout; but as you have the chance to revise there are a couple of minor points that you could fix as well … " show a concern for the author and also an understanding of the subject matter. This referee made suggestions about every aspect of the paper, pointing out superfluous comments made by the writer as well as the need for further discussion on some points.

As well as providing detailed comments within the pro forma provided, extensive comments were sometimes written on the actual papers. This strategy is probably of even more value for inexperienced authors. Comments like,

- a statement is required here that summarises the apparent benefits,
- this is the important one the others could probably be omitted,
- I suggest that these be reworded, and
- not sure how this follows or what the point is, perhaps clarify or omit;

directed the attention of an author to a specific point in the paper. Some referees checked mathematical calculations and were able to encourage authors to consider re-checking these before resubmitting. For example, "I carried out the usual *t*-test method and found slight disagreement with the ranges given." Comments like these would have helped the author not only to rewrite the paper but also to polish it so that it became a very good paper indeed. It was evident from re-submitted EB papers that most authors took the majority of the comments and suggestions seriously.

Of course, other authors had less support, particularly when referees merely ticked each of the criteria sections or provided imprecise comments like "some sentences are quite long and maybe shortening them would be valuable". Fortunately, such cases were a definite minority.

Changes Made on the Basis of the Referees' Comments

Sometimes the revisions required were technical and easy to implement. But in others referees gave detailed comments such as "the methodology could be clearer. … An example of a question/problem would help here. … I think that the text about each student [could be included in a table]. … I suggest that [the following] be reworded." (Quotations have been slightly modified to preserve anonymity.) Authors took most of these criticisms to heart, and we were amazed in some cases just how different the original and revised papers were. As one referee observed, "The author has acted on comments in my review and the revised version is now dramatically improved." Unfortunately, the requirement of anonymity prevents us from citing particular examples.

While there were examples of changes *possibly* made to please the referee and ensure acceptance, there were other examples of creative rethinking in response to criticism which is at the heart of the educational process. For example, the suggestion quoted above to incorporate a large amount of text into a table was only partially followed; other relevant details were introduced into the narrative at appropriate places. In other words the referees' comments were taken as observations about weaknesses which needed to be corrected, but the author decided his or her preferred way of correcting the weakness.

A particularly interesting case where authors did not always take up referees' comments occurred when referees pointed out the limitation or omissions in the References. Sometimes they suggested that "a wider reading list should have been included", but gave no suggestions about appropriate references. Such a comment could easily be beyond the knowledge, experience or paradigm of the author and very difficult to act upon. In other cases, "missing" references which were specified by the referee were still not included in the revised papers. This may have been because of the time required to find and incorporate new ideas or perhaps a difficulty in understanding why the referee made the suggestion. Turn round time for resubmission was tight and there is no opportunity for discussion with the referee. So frequently the resubmission contained unchanged References. Authors who decide to do this should realise that some referees do suggest that their own work should have been cited!

While we have focused on the authors' reactions to thoughtful reviews, we were also sent comments by a small number of "first time" referees saying how much their own reviewing skills had been improved as a result of having been involved in the EB process. They appreciated the opportunity to read a paper carefully, consider and write their own review and also see the modified paper.

So we may say that the EB refereeing process led to significant improvement in several cases. At its best it was of educational value for both authors and referees. But a few referees did not see their work as contributing to a rich educational activity, and even the best reviewers made some impracticable suggestions given the time constraints involved. While there is room for improvement, and especially for more reflection time, the EB process as it stands may certainly be seen as an effective educational model for refining skills.

Special Features of Reviews of Standard Papers

Three differences from EB reviews may be seen in those of standard papers. Putting aside minimalist reviews clearly done under time stress, rather more standard reviews were relatively brief, especially if they were positive judgements. Some of these came from referees who had prepared very detailed reviews of EB papers. One referee actually wrote, "because I recommend that this paper be accepted my comments are short." Referees may not be aware of how often editors must consider and weigh up quite conflicting reports. For example, three referees of one paper responded in totally different ways:

- this paper is well written and addresses an area of concern;
- very unimaginative, purely statistical analysis;
- why was this topic even investigated?

Editors do need to read reviews to ensure that no serious injustice is being done. This is not easy. Another paper received a damming negative review from one referee, with another reviewer making *the same observations* but with an "accept" decision. Carefully argued cases for positive decisions are as valuable both to editors and authors as those for negative ones.

The second difference we noticed was that many referees of standard papers tended to give authors the benefit of the doubt. EB referees could recommend revision, but standard paper referees could not, and were well aware that failure to accept a paper could have significant implications for an author's funding and attendance. While many were prepared not to accept a paper, others, perhaps those not really expert in a particular field, probably took the editors' advice not to require journal article standards and decided in an author's favour. For example, two positive reviews of one paper wrote

> The standard of English is not very good. On the other hand the academic content of this paper warrants some dissemination and discussion; so I lean towards acceptance.

> This is an interesting paper with a constructive purpose and some helpful results. ... It would benefit from a rather tighter and more general conclusion, and from being set within a wider base, but it represents a useful step in the direction of making research relevant to harassed classroom teachers.

The absoluteness of a negative decision probably also influenced responses to weaknesses of layout. In one case all three referees indicated many typographical and grammatical errors and unclear segments in the paper, but all three recommended acceptance. But in other cases, similar errors were the only basis for a negative decision. It is possible, however, that some authors may be presuming on this prejudice in their favour. One referee wrote:

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> There needs to be some insistence that writers have their work read by another before they send it in. Noting poor grammar and spelling should not be the responsibility of reviewers. It does not happen to the same extent with journal articles that I review, so why with Conference papers. Are they not meant to be of similar quality?

Papers are often prepared under time pressure, so technical issues are likely to receive minimal attention. Not only does there need to be more consistency about judging such errors, but, as we shall see below, inadequate attention to such details may prejudice an author's chances of acceptance.

The Value of Referees' Comments for Standard Papers

It is difficult to assess the value of the refereeing process for standard papers, because little feedback is available and the diversity of responses must be confusing for some authors. But some general comments can be made.

Tables

Tables are a minefield for authors and referees alike. Creation, organisation and presentation of clear and accurate tables is a more difficult task that most of us would assume. Many referees mentioned accuracy, clarity, and shading of sections in tables, or the use of a table when text would have been just as effective. Comments to justify referee' problems of interpretation were often particularly clear and can be of both special and general value to authors:

- check Table 1—difficult to read;
- the two columns for this aspect should be omitted;
- the data in Table 1 can be stated more easily in the text following it;
- I recommend that … sample sizes be indicated in Table 1;
- in the right hand column of Table 1 it would be clearer to head it … .

Value for Authors

Just as for EB papers, some reviews were written in such detail that they would have provided authors with material for thinking about their presentation at the conference and possibly even reviewing their written presentation of conference papers in the future. As one referee wrote:

> I have been fairly critical of the paper on the assumption that it is part of a large project and probably for higher-degree studies. The faults have been identified for the benefit of the author's future work.

One referee included a personal reflection at the conclusion to a review, which may have helped the author to relate better to it. On the other hand, some reviews became so detailed, that, while they might have been useful for journal submissions, as conference reviews they were rather precious.

But we are unable to assess the extent to which authors benefited from their referees' reports. Questionnaire responses were usually happy about reviews received, but we did not ask about the influence of the reviews on authors' professional skills. A longitudinal study might be of value here.

Value for Referees

We have already noted how some first-time referees valued the refereeing of EB papers. It is very difficult to surmise the benefits in general, especially for experienced researchers. In some cases quotes made by the author have been challenged, the source of these quotes located and discussed, and the intention of the original author argued. We may presume that such activity was of some value to the referee as well. But perhaps even the writing of a detailed review by an expert who can draw quickly on relevant knowledge may provide a valuable learning experience, especially if the paper is approaching old ideas in a new way. Educational theory would certainly suggest so. However, we are unable to provide hard data on this issue.

Some organisations distribute all three reviews to all three referees at the same time as the reviews and editors' decisions are sent to authors. In our personal experience this can be a valuable learning process, and we recommended its introduction to the MERGA executive.

Did Referees Understand the "Big Picture"?

Rejected authors frequently reported that referees were "not able to see the 'big picture' in my paper". Compressing complex ideas into eight pages is not easy. New or different approaches are particularly difficult to compress. If these ideas are very new a referee may consider the author does not understand the field sufficiently. This important claim needs to be considered in detail with reference to two such comments. The first was made by a senior researcher:

> I feel however that two of the reviewers, even the positive one, did not understand the point of my paper. Of course, that is probably my fault for not being more explicit but I had expected reviewers to be able to identify the main ideas in the papers they were reviewing. It could be they were looking at a straight qualitative or quantitative research study, which I do not believe is the only kind permissible at MERGA.

The second was made by a less experienced author who submitted a joint paper with a supervisor:

> However, suggestions such as, to include primary material as well as secondary material, and that focusing on one content area is too narrow, makes me wonder if the push [is] to go broad instead of looking more deeply at issues in Maths Ed.

Both sets of authors used quite complex theoretical structures. In both cases a small number of detailed case studies were used to illustrate the validity of the structures. The problem is obvious: detailed reports and detailed theoretical hypothesising do not fit easily together into an eight-page paper. But authors need published papers to get to conferences, and if their work is new or complex, this should not hamper their chances of acceptance.

But it is not surprising if the second author felt not to have been understood when comments were made like "perhaps some of this information could have been presented in tabular form for conciseness and to maintain the interest of the reader" and "the discussion fails to link the data descriptions to the theoretical perspective ... or to extend this perspective as well as it could". On the other hand, in our view, neither of these comments demanded more than might be achieved within eight pages using the same data and theory used by the author. The judgements were about balance, which is a reasonable thing for a referee to do. But the author's view was that "some suggestions indicated strong preferences to other structures and ideas that could not be covered in 8 pages". When different perspectives like these are presented via the anonymity of the refereeing process, authors are unlikely to change and develop as a result of referees' comments. Given that the author was happy to accept other criticisms from these referees, and that their reviews seem to us to be reasonable, there does seem to be a need for face to face discussion.

The first case is more complicated. It is true that only one referee gave evidence of being well read in the literature of the field, but both the other referees made comments about technical weaknesses in the presentation which any experienced reviewer might have made. One of these also commented

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I have some concerns about the paper, but in the end I have been swayed by a belief that this topic is worthy of a Conference presentation. A topic such as this is difficult to address purely in text form, and the discussion format of a presentation should provide an additional avenue for the exchange of ideas.

Here we can see the "benefit of the doubt" syndrome again. Nevertheless, in our view, while the author's claim that the ideas were not well understood by the referees may have some validity, the rejection was based much more on technical issues rather than on the difficulty of the topic or the referees' inability to see the big picture.

A more detailed investigation than this is needed to be sure whether referees are inclined or not to see the big picture. The evidence presented here in detail neither confirms nor refutes the claim, but does show, as we have argued above, that technical weaknesses in paper construction may divert referees' attention from the big picture to what they see as more pressing issues for a conference presentation.

Discussion

This paper has shown that the MERGA refereeing process clearly fulfils some criteria for being an effective teaching/learning process. The vast majority of the reviews are constructive assessments of the authors' work, and many provide useful advice for further development of the ideas. These reviews address both the process of doing research and also of reporting it. But teaching is much more than telling, and we have also been able to show that for the EB papers authors have been able to implement recommendations to the obvious betterment of their papers.

Such results might be seen as obvious, even trivial. But in the process of examining them deeper questions have also been raised and not always answered. These may be listed as:

- the relationship between the quality of conference papers and journal articles, especially given their perceived equality by funding bodies;
- the need for referees to give authors confidence in their expertise;
- the potential value of the refereeing process for referees;
- the importance of good presentation, especially of tables, for ensuring acceptance;
- the possible value of allowing authors and referees to make contact with each other at some stage in or after the assessment process.

MERGA is an educational organisation, so its practices should conform with high educational principles. The refereeing process is a rich environment for teaching and learning by both authors and referees, and we have shown that it is often used constructively. But we have also shown that some pedagogical opportunities are being lost which could easily be taken up. This reflects a point of view which sees MERGA not merely as an organisation to *examine* education, but puts more emphasis than at present on *practising* it. MERGA is already doing many things well to assist the life-long educational development of its members; when this is seen as an integral focus of its work, then some of the harder educational issues are more likely to be addressed, for the general benefit of all members and mathematics education in general.

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Current Research on MS&T Curriculum Integration in Western Australia: Examining the Knowns and the Unknowns

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This paper summarises the final report on a study first described to MERGA conference delegates in 1998. Following an extensive review of research on integration spanning the past 30 years, a number of middle school settings of integrated teaching and learning from across a range of demographic and socio-economic areas were studied over three years in order to provide data upon which a number of assertions related to curriculum integration were formulated. Using a number of approaches to data collection, the facts about curriculum integration were documented and six assertions were complied concerning what we believe we know about the process. Another five assertions about what we believe we still need to understand about this process were also compiled. Arguments for and against integration are discussed from the viewpoints of the students, teachers and administrators.

Curriculum integration has created an upsurge of interest among teachers and administrators in Australia and overseas in recent years, with discussions focused particularly at the middle school level (National Council of Teachers of Mathematics, 1995; Nixon, 1991; Rutherford & Ahlgren, 1990; National Research Council, 1989). Proponents of integration argue that subjects like mathematics, science, and technology should be integrated with each other, or with other learning areas, so that students can recognise the contextual relevance of the subject matter and identify the links that exist between these discipline areas (Kleiman, 1991). Further, integration often has been proposed as a potential cure for many of the ills associated with teaching young middle school adolescent students – ills such as their perceived alienation from traditional curriculum subjects and structures (Bean, 1991; Cormack, 1996; Cumming, 1996). And yet, research examining integration in practice is rare (Berlin, 1989; Hargreaves, Earl, & Ryan, 1996). The two central questions, whether students learn the basic mathematics, science and technology concepts when teaching and learning occurs in an integrated environment, and whether, indeed, integration addresses the fundamental problems associated with teaching adolescents, remain unanswered. Teachers and researchers simply do not know the answers to questions about the advantages and disadvantages of integrated teaching practices in mathematics, science and technology, nor do they know much about the consequences of this approach to teaching in terms of student learning.

This paper attempts to fill part of this void by reporting a synthesis of findings from a three-year study of integrated practice in several schools in Western Australia. Because of the research design which involved the investigation of many approaches to integration (in different schools and localities, through a series of case studies) a broad base of information has resulted. The paper presents a meta-analysis of these findings in order to draw together the outcomes of the study, discuss the implications of the findings and point to areas of continuing concern.

Theoretical Framework

The argument for integrated teaching presented in the literature is strong. A provocative analogy is presented by Kleiman (1991), who suggests that if we taught music the way we traditionally teach mathematics or science, students would practice scales for years without playing a song or that students of art would draw lines and shapes without ever getting to create a picture. If we view integrated teaching and learning from an humanistic perspective, Kleiman (1991) argues, integration "provides a vehicle for thinking, a medium for creating and a language for communicating" (p. 48). According to advocates of integration, mathematics, science and technology should be used in rich environments integrated with other learning areas so that students can learn to appreciate the contextual utility and elegance of the subject matter.

From a related perspective, Hargreaves et al. (1996) point out that integration promotes greater relevance, especially between high status, academic subjects (like mathematics and science) and lower status, practical ones (like technology, manual arts or home economics). However, integration threatens the advantage of people who traditionally have benefited from an academic curriculum, including teachers. This is because integration usually involves the recognition of forms of achievement and intelligence that are beyond the established methods of evaluation. Complaints that an integrated curriculum lowers standards, destroys 'real subjects' and results in a 'wishy-washy' curriculum are criticised by Hargreaves et al. (1996) as the result of political and philosophical manoeuvres to maintain structures of power and control. The structures of power and control that Hargreaves et al. refer to are maintained within high schools by practices such as formal subject-based examinations and subject departmental divisions. Critics question whether the students learn the central concepts and skills that define the separate disciplines of mathematics and science. Thus there are theoretical and ideological arguments for and against integration, and all of them were encountered in the three year study of integration reported here. The results shed some light on the arguments, in particular, from the point of view of the participants in the various integrated programs.

Methods

The research reported in this paper resulted from a collaborative study involving the authors and the Education Department of Western Australia. The authors visited 16 schools, including 12 high schools and 4 primary (elementary) schools in the metropolitan and country districts within the State of Western Australia. The schools were selected on the recommendation of several key State curriculum personnel who advised that curriculum integration had been attempted in these schools in Grades 6-9.

Government, Catholic and Independent high schools were included in the sample, and the visits involved classroom observations, interviews with teachers, students and other school personnel, and document analysis. Some of the visits focused on teaching, others on learning, some on mathematics, science and technology teaching in integrated contexts. Written reports at the intermediate and final stages of the research were fed back to the participants for comment and correction. The three-year research project has been reported in several presentations and journal articles (Venville et al., 1998, 1999a, 1999b, in press) and in a monograph casebook (Venville et al., 1999c), a copy of which has been placed in every government high school in Western Australia courtesy of the State's Education Department. In this paper, we use a cross-case analysis to report a synthesis of our findings over this three-year period, focusing on what we believe we now know about integration, and what we still need to know.

Findings

What we believe we know about Curriculum Integration

1 Integration is a curriculum ideology

Our first proposition is that integration is an idea or stance about curriculum that is associated with a particular value position about schooling for adolescents and about the importance of subject matter. While this value position has broad acceptance in the educational community, it is not without its critics. Many educators believe that in seeking common ground, integration results in a 'watering-down' of the syllabi and creates problems in other aspects of the integrated areas (such as assessment). Also, there is some evidence from our work to suggest the existence of the uncritical adoption of the integration idea by mathematics, science and technology teachers without a sound understanding of its ideological basis. An example of this was the case of the mathematics teacher in a government high school who, because of his conviction of the value of curriculum integration, believed that it was incumbent on him to carry out all aspects of the process personally – that is, demonstrate to his class the role particular mathematical concepts played in physics, music, chemistry, English and the social sciences. A worthy sentiment, but the extent to which he engrossed himself in this task created other stressful problems for the individual.

2 Integration comes in many forms

Integration is often described as if it were a single form of instruction. However, we found a variety of different forms of integration being utilised by the 16 schools in the study. Each form is described briefly below:

- Thematic approach:

This approach was observed in two schools. Theses were used to integrate the curriculum, and teachers responsible for each of the learning areas developed a program of work complementary to the theme. Examples included: the Sydney Olympics, decision making in business, family relationships and ancestry, communication nets, popular culture, Earth and people, and scarce resources. Teachers commented on the different levels to which they were able to integrate their learning topic area within the theme, with most teachers expressing the view that they were able to make a number of significant links.

- Cross-curricula approach:

Cross-curricular issues such as numeracy, computing skills and literacy served as a basis for integration in several schools. Foe example, a technology teacher pointed out that literacy was clearly integrated into the technology course through the interpretation of a design brief and written assignments, and that mathematical skills were utilised, especially in the measurement area. Computing was also integrated, with students using word processing for their research assignments, and in using programs such as CAD for the design of products.

- Technology-based approach:

Students in one school were given a project to work on for 10 to 12 weeks. It included a mathematical, a technological and a scientific research component. The project brief was to design and produce an electric powered vehicle that could climb a steeper gradient than anyone else's on a standard test track. The mathematics research component investigated the effects of changing variables on Logo model hill climbers and recording, analysing and presenting the groups' results from the time trials.

- Topic approach:

In one school, a mathematics teacher and a social science teacher initiated a project amalgamating mathematics, statistics and social science on the topic of World Environment. The project ran for three weeks and required students to research a developed and a developing country, and then perform various mathematical and statistical analyses on the data they collected. – such as extrapolating population trends, calculating means, medians, modes and standard deviations, and presenting the results in graphical form.

- Competition approach:

Competitions such as the Solar Car Challenge and Science Talent Search provided good opportunities for integration between mathematics, science and technology in several of the observed schools. In the Solar Car Challenge, the mathematics component of the integrated activity involved the students in calculating the most efficient angle for the solar panel, and conducting and documenting the statistical analysis of the car racing trials.

- Integrated assignment approach:

Three schools used integrated assignments between learning areas which took between one lesson and two weeks to complete. In one school, the mathematics teacher organised integrated mathematics and science assignments every five weeks. One of these investigations was on pendulums and involved the class in developing a hypothesis and experimental procedure, statistically evaluating the data and presenting the results in graphical form

- Common teaching approach:

One form of integration involved using the same framework for designing and writing up investigations in mathematics and science. Teachers stated that using the same approach assisted students to perceive similarities in the processes involved in the Working Mathematically and Working Scientifically strands of the Student Outcome Statements. The common approach provided a catalyst to integrate both content and skills across the learning areas.

- Synchronised content and processes approach:

Teachers in three schools explained how the content in their different learning areas was synchronised as much as possible to allow similar content and processes to be taught at similar times. This involved the teachers in each school writing down all they hoped to achieve during the year, making a map of the possible links, and then re-arranging the program to facilitate the plan wherever possible. One example of these links involved chi-square analysis in mathematics and statistics and Punnett square problem solving in genetics. The teachers were able to synchronise the teaching of these concepts and explicitly link the ideas together.

- Local community projects approach:

Two schools had been involved in local community projects which integrated skills and content from a number of learning areas. One project involved students, in liaison with a local shopping complex, providing plans for an improved arrangement of parking and pathways prior to major earthworks taking place. Another school described how students had worked with the local council to prepare an inventory of services provided to the community.

- Natural/Informal approach:

Teachers in four primary school explained how much integration happened naturally because they taught the majority of learning areas to the same students, and 'links just cropped up'. In one school, technology was promoted as a vital learning tool to extend other subjects. For example, mathematics investigations were used in technology projects, and the students explored situations such as the cultural value and profit margin of the artefacts produced. One group of students made a bird-feeder by designing a template and then constructed the final product.

It can be seen from the above approaches that integration is like an umbrella under which many views on various aspects of integration are represented.

3 Integration challenges the grammar of schooling

Our third finding is that integration challenges what Tyack and Tobin call "the grammar of schooling" (as cited in Hargreaves et al., 1996, p. 86) – that is, the 'apparatus' of schooling, including teacher training, curriculum materials, subject departments, professional associations, assessment structures, school timetables and students' university futures – and vice versa. While the teachers in our project were being urged by school authorities to integrate the curriculum, the grammar within the school often seemed to be working against the rhetoric on integration.

4 Instances of successful integration are idiosyncratic

We found that instances of successful integration – where teachers seemed to making genuine attempts to integrate and the classroom atmosphere seemed conducive to learning – were patchy. Rarely was integration a school-wide phenomenon, rather it generally relied instead on a few dedicated teachers or teaching teams. We noticed that over time, some teachers tended to drift away from integrated practice back towards discipline-based teaching. Integration also ebbed and flowed as teaching teams configured and reconfigured within a school.

5 Integrated teaching works best in inclusive contexts

According to Bean (1991), integration is about wholeness and unity rather than separation and fragmentation. We documented several examples of this kind of integration where students were able to make interdisciplinary links (between mathematics, science, technology, English, other languages, physical education, the social sciences, music and art) and transfer knowledge and skills from one context to another. These linkages were enhanced in inclusive contexts, where students and teachers were also found to be working in team environments and where students were able to call upon community and family support.

6 Integrated learning involves gains and losses

The gains we noticed in students' capacity to transfer knowledge were often offset by deficiencies in specific subject matter knowledge. We recorded several instances of students' retaining naive mathematical and scientific understandings in a situation where there was an absence of remedial teaching to address such deficiencies. There were also gains and losses for the teachers, in terms of the time and effort involved to create and implement an integrated program, and there was very little formal evidence available to measure the success or otherwise of their endeavours.

What we still need to know

1 What is the problem that integration is addressing?

This has been a continuing question for the research team. As mentioned previously, we noticed many instances of uncritical adoption of integrated teaching methods and observed the large amount of time invested by teachers to develop integrated support materials. However, we wonder if sufficient consideration has been given to the nature of the problem itself. What exactly are the problems surrounding the schooling of adolescents, and are there other ways (perhaps even more effective ways) of addressing them?

2 How is integration addressing this problem?

If we do recognise the problem as one of alienation, for example, how is integration supposed to be a solution? In our own work, we have observed several instances of integrated learning environments that appear to be addressing such problems. We have also observed other less successful learning environments. Is the key to the problem good teaching (of mathematics, science and technology) rather than integrated teaching per se?

3 What are students actually learning in integrated contexts?

While we can report on some instances of successful integrated teaching and learning in mathematics, science and technology classes, we are still short on solid evidence of what it is that students are actually learning. Part of this problem lies in the nature of the learning and the absence of measures for such learning, and part lies in the dearth of close-up research into what students know and can do as a result of integrated settings. We suggest that the move towards outcomes based teaching and learning will throw some light on these issues.

4 How can integration be scaled up and sustained over time?

We observed that integrated teaching appears idiosyncratic and erodes over time. If it transpires that integration does provide an answer to the problem of adolescent schooling (as many commentators would have us believe), we are still faced with the problem of scaling up and maintaining the momentum. How, in the face of the constraints identified earlier, can integration be envisioned as a broad-based solution to the systemic issue of adolescent schooling?

5 How is assessment best handled in integrated contexts?

If one imagines a school 'assessment continuum' with, say, mathematics at one end and science at the other, then we have a representation of non-integrated subject assessment wherein assessment procedures are separate and distinct. Integrated assessment would fall somewhere within that continuum, and assessment procedures in this case are somewhat blurred. The question of how best to assess an integrated syllabus or curriculum appears to be largely unresolved, although the assessment of cross curricular integrated projects seemed to pose no problem, with credit assigned according to the quality of the science, mathematics and technology input. Is the project the only form of assessment vehicle one can use in integrated contexts though?

Reporting on assessment in integrated teaching contexts is clearly problematic in many schools at present also. We saw almost as many different report forms in use in schools as there were schools in the sample. These ranged in appearance from simple, one-sided forms to four-page booklets providing for most detailed qualitative feedback from all teachers involved in integrated activities. Teachers expressed concern at either the over simplistic nature of the forms, or (more often) their complexity, which had resulted in confused and sometimes angry responses from parents.

Conclusion

Curriculum integration in schools is being implemented in various countries throughout the world. The responsibility for implementing integrated curricula rests with the classroom teacher who has very little time to spend on this time-consuming task. Developing activities and assessment tasks that meet integrated curricular goals is difficult, and the fact that the term is not understood by some school administrators, teachers and parents makes the job of implementing it all the more harder. Lonning and DeFranco (1997) believe that several key issues must be examined if we are to answer the question "what does the integration of science and mathematics really involve?" Their first point is that integration can be justified only when students' understanding of the mathematics and science content is enhanced. Second, integration makes sense only when it grows from the mathematics and science curricula rather than from other sources that may provide only shallow activities lacking meaning for both students and teachers. Third, it seems clear that one method of teaching in a manner conducive to integration is to focus more on how concepts can best be taught, rather than on how they can be integrated. This is because not all mathematics, science and technology concepts can or should be taught in an integrated fashion.

The authors of this paper believe that we have identified further issues that need to be resolved before we can be satisfied that we are successfully integrating the curriculum. Integration is a potentially powerful means for assisting teachers and students to understand the world from a broader perspective. Its potential for enhancing teaching and learning in mathematics, science and technology warrants our attention.

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Strategy Transfer between Computer Programming and Mathematical Problem Solving

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This paper presents part of the findings of a study into the effect of teacher intervention, designed to improve metacognitive skills, on learning transfer between MicroWorlds programming and mathematical problem solving. The study compared two classes of Year 6 students both extensively involved with programming. Students who were exposed to the teacher intervention activities in addition to programming outperformed the group who did not receive the intervention activities on a test of mathematical problem solving.

This paper presents part of the findings of a study into the cognitive and affective outcomes of programming using MicroWorlds software in upper primary classes (Walta, 1999). One aspect of the study, outlined here, sets out to examine the effect of a teacher intervention designed to improve metacognitive skills, on transfer of learning between programming in MicroWorlds (LCSI, 1993) and mathematical problem solving. MicroWorlds is a programming package for children based on the language of Logo, incorporating a range of features, such as drawing, animation, sound and text processing.

The study is underpinned by the writing of Papert (1980) and Polya (1973). Papert created Logo, a language for children to use to make individualised creations using computers. Papert suggested that, through programming, children would acquire problem-solving skills that would transfer into other areas of learning. Papert's initial publication of *Mindstorms* in 1980 led to a proliferation of studies in the 1980s which set out to demonstrate the effect of Logo use on a range of areas, including the development of mathematical problem-solving skills. Many of these studies were later found to have limitations in their research design.

Later studies (Swan, 1991; Clements, 1987), which continued to involve small groups in experimental situations, found that learning transfer did not occur unless teachers were proactive in teaching for transfer. Among such studies was that of Lehrer (1989) who highlights the need for instructional practices to enhance the likelihood that strategies acquired when children learn to program will transfer to other areas. Both instructional method and the type of transfer task (analogical or similarity based) were found to have an influence on the degree and quality of transfer. Analogical transfer occurs when strategies developed in one context are spontaneously applied to a different domain of learning. In this study transfer between programming and mathematical problem solving is one such example. Lehrer concludes that to facilitate analogical transfer, teachers need to assist children to construct and reflect on different ways of composing problems, evaluate the consequences of their choices and establish correspondence between programming and non-programming contexts.

The work of Polya (1975) provided a framework for considering the complexity of mathematical problem solving behaviour. He divided problem solving into four stages; namely (a) Understanding the problem, (b) Devising a plan, (c) Carrying out the plan and (d) Looking back. This framework provided a generic model for considering problem-solving behaviour and structuring the teacher-intervention strategy.

The Classroom Programming Context

The subjects of the research were two classes of Year 6 students at one Melbourne school, who were working extensively with MicroWorlds, with lap-top computers. The study took place over the duration of an entire school year. One class was taught by the researcher and the other by an experienced colleague.

The term programming, as it is used in this study describes such activities as problem definition, design development and organisation, code writing and debugging. It also refers to the specific behaviours associated with the use of a code or computer language to generate outcomes not predetermined by the software. These outcomes include animation of objects, movement of objects within a page to achieve special effects and the use of text boxes 'programmed' to appear at critical points and report specific text. Other techniques described as 'programming' are the use of buttons and sliders, which are devices to which code can be attached to execute a specific effect.

All students involved in the study were taught programming in the same way over a term through a problem-solving, discovery-based learning approach. Students were assisted in the mastery of basic programming syntax through a series of class-based activities, which in total occupied about an hour a week over term one of Year 6.

After basic competence in MicroWorlds was attained in term one, students worked on five Lap-T tasks during the rest of the year. Each Lap-T occupied the students for around four weeks. These tasks had evolved with the previous Year 6 class in the year before the study as a way of integrating learning outcomes across the key learning areas while developing the problem-solving strategies associated with Logo programming. Presentation of each completed task to the class enabled students to share knowledge about programming as well as the content knowledge that was an essential part of the task. The idea of teaching for transfer of these strategies was linked to the desire to improve mathematics problem-solving outcomes for Year 6 students. The topics of the five Lap-T tasks were:

- writing a program to illustrate both problems and solutions associated with a maintaining a balance in a particular ecosystem,
- writing instructional software to teach the calculation of perimeter or area,
- constructing a time-line about women in Australian history,
- representing issues associated with a particular endangered Australian animal and
- using LOTE to construct an interactive game to reinforce certain vocabulary.

Initial observations of student response to these work-tasks indicated that students increasingly showed motivation and persistence to engage in and solve associated problems as the year progressed. In addition, students appeared to demonstrate increasing skill, creativity and sophistication in their ability to interpret problems posed and to write complex MicroWorlds programs in response.

Strategy Training

Central to the research was the teacher intervention strategy adopted in one class and known as strategy training. This was a series of lessons designed to improve students' metacognitive awareness and assist them to make connections between skills and strategies developed in a programming context and those needed for mathematical problem solving and to improve the likelihood of transfer of these skills and strategies. Over a period of eighteen weeks, students in one class (called the Strategy Training Group) were involved in a weekly lesson of approximately forty-five minutes. Activities undertaken in strategy training sessions were largely computer based, involving students working on a short challenging programming task using MicroWorlds software. As part of the session, time was also spent in planning, discussion, reflection and application of skills into a far-transfer context. There were eighteen sessions, conducted in terms two and three. Each of the sessions followed the same format, which involved five different sections summarised below.

Awareness raising. Students were told that as they undertake short programming tasks they would be considering the strategies they used and their possible application to other learning contexts. This section was included to raise awareness of the types of strategies common to problem-solving contexts and to introduce a metacognitive component to the discussion.

Collective reading and planning. This section was included to highlight the value of understanding the problem and planning for a solution. Students also needed to verbalise their thinking on why their particular solutions were more likely to succeed. This step was included in acknowledgement of research which indicates that encouraging children to use 'think aloud' techniques provides them with a useful strategy to aid in reflecting, and by implication improving, their own thinking (Short & Weissberg-Benchell, 1989).

Group co-operation in the writing and debugging of the program. Students working in pairs were asked to reflect on progress at intervals. This section was included to reinforce the processes of reflecting and debugging. Rowe (1993), in discussing metacognitive outcomes of computer programming, notes the value of students working in pairs to encourage reflection through thinking aloud about their ideas and strategies.

Construction of a similar problem. Children then suggested tasks they could undertake which involved use of similar strategies for seeking solutions. This section was designed to assist in raising awareness of the idea that skills and processes used in one context could be applied in another.

Training to facilitate analogical-based transfer. At the conclusion of each lesson a sample mathematics problem was briefly presented to assist students to generalise generic problem-solving skills learned in the programming context to a mathematical problem-solving example. The class was encouraged to reflect verbally on how the strategies utilised in the programming task could be applied to it.

In total nine different programming tasks formed the subject matter of the strategy training sessions. Each was selected because it was complex enough to require careful planning and consideration and had the potential to be solved using a number of processes and strategies. The activities were designed so that one strategy seemed the most likely approach to solving the problem posed by the wording of the task. For example, the strategy "Draw a picture or diagram" was discussed through an activity of designing a car rally and programming one car to win the race. Reproducing a complex geometric design on the computer illustrated "Divide the problem into smaller related parts/work backwards from a given problem". Running a pre-written program and then rewriting the program using another approach to produce the same outcome illustrated "Relate the problem to a similar related problem".

Method

The study involved a comparison between the mathematical problem-solving performance of the two Year 6 classes. One class (the Strategy Training Group or STG), received explicit teaching for transfer of strategies acquired in programming as outlined above. The other class (the Independent Learning Group or ILG) was taught programming and mathematics in exactly the same way, but did not receive explicit training for strategy transfer. Each class had 18 students. Students in both the STG and the ILG took pre-tests and post-tests in mathematical problem-solving and general mathematical achievement (Progressive

Achievement Test, ACER, 1990). The strategy training took place in the middle terms of the year, but the post-test was not conducted until the end of the year, when students had finished all of the Lap-T tasks.

The ten items contained in the Maths Problem-solving pre and post-test had been selected after trials with students in the previous year. Some items were from *Profiles for Problem Solving* (Stacey, Groves, Bourke, & Doig 1993) and others were constructed by the researcher. These problems were considered to conform to three criteria: they readily showed evidence of use of problem-solving strategies when successful outcomes were achieved, they were complex enough to require planning, reflection and debugging skills and they could be resolved by at least two thirds of the trial group in the allotted time.

The problem-solving tests were analysed to assess student success in three main dimensions of mathematical problem-solving. These were obtaining correct answers, selecting appropriate strategies and articulating strategy choice. These dimensions were analogous to the dimensions of correct answers, method used and quality of explanation used in Profiles for Problem Solving. It was hypothesised that the STG would be better than the ILG in overall problem solving and in each of the dimensions above. Furthermore, it was hypothesised that improvement would mainly be evident in the students who were initially low That is, that students in the STG with relatively low initial mathematical skills, will show greater improvements in overall problem-solving performance than will their counterparts in the ILG. There will be no difference in the improvement of students with high initial mathematical skills. This was hypothesised because it was judged that able problem solvers probably already had knew and used useful strategies and had more developed metacognitive skills. Schoenfeld (1985) establishes that expert adult problem solvers have these characteristics: direct data on young children is not available.

Results

Table 1 shows the means and standard deviations on both pre-test (T^1) and post-test (T^2) for both groups of students. It shows that that the ILG made more improvement on the PAT Maths test than the STG and the STG made more on all three dimensions of problem solving. The pre-test differences between the STG and the ILG meant that regression analysis rather than first difference scores (i.e. pre-test minus post-test scores) is an appropriate technique for comparing improvements in the two classes.

Hypothesis1 states that the STG will improve more between the beginning and end of the year at finding correct solutions to problems than the ILG. In the regression equation which follows, the dependent variable is Obtaining correct answers at time 2 (OCA_2) and the independent (explanatory) variables are Obtaining correct answers at time 1 (OCA_1) and Group (STG = 1, ILG = 0). The independent variables were entered simultaneously, although it should be noted that stepwise entry (Obtaining correct answers 1 entered first) yielded an almost identical result. The value of R^2 is 0.45.

Equation 1

$$\begin{array}{lllll} OCA_2 = & 36.42 & + \; 0.61\,OCA_1 & + & 6.53\,\text{Group} \\ & (2.90) & \quad (5.07) & & (0.95) \end{array}$$

This equation, which is similar in form to all subsequent equations, should be read in the following way. OCA_2 is first a function of a constant (the intercept term in the regression equation). Secondly, OCA_2 is positively related to OCA_1, the metric regression coefficient being 0.61. OCA_2 is also positively related to Group where the STG was coded 1 and the ILG coded 0. The coefficient of 6.53 for Group indicates that, controlling for scores at time 1, the STG improved by 6.53 points (on a 0-100 scale) more than the ILG.

If we treat the two Groups as populations this difference in improvement may be regarded as 'real'. However, if we treat the two Groups as samples of Year 6 girls and so take notice of the t values (i.e., the standard errors of the estimates, which are shown in parentheses beneath the regression coefficients), we would infer that the difference in improvement between the two classes was not significant at the conventional 0.05 level. (It should be noted that t values larger than 2.0 are required for significance at this level).

Overall, it can be inferred that there is some confirmation of hypothesis 1, although it can also be argued that the null hypothesis of no difference between the Groups cannot be decisively rejected. This latter caveat essentially springs from having a small number of subjects in the population, as well as considerable variance of scores within both Groups.

Table 1

Means and Standard Deviation of STG and ILG on Problem Solving Dimensions and PAT Maths Test and Mean Improvement: Pretest (T^1) and Posttest (T^2)

Variable	Test T^1 (Feb) T^2 (Dec)	Means/Std Deviations STG (n = 18)	Means/Std Deviations ILG (n = 18)
OCA	T^1	45.6 (25.9)	39.1 (30.7)
OCA	T^2	58.2 (22.8)	47.8 (28.8)
Improvement		12.6	8.7
Strategy choice	T^1	47.2 (29.6)	44.1 (29.2)
Strategy choice	T^2	70.2 (18.4)	51.2 (32.8)
Improvement		23.0	7.1
Articulation	T^1	41.4 (19.5)	39.8 (24.9)
Articulation	T^2	58.9 (20.5)	51.0 (23.1)
Improvement		17.5	11.2
PAT Maths test	T^1	74.5 (18.10)	58.2 (24.3)
PAT Maths test	T^2	78.1 (21.7)	69.4 (24.1)
Improvement		3.6	11.2

Note. Abbreviations: OCA = Obtaining correct answers; Articulation = Strategy articulation

Hypothesis 2 predicts that the STG will show a greater improvement in selecting appropriate strategies than the ILG:

Equation 2

$$\text{Strategy Choice}_2 = \underset{(4.41)}{58.16} + \underset{(4.63)}{0.58\ \text{Strategy Choice}_1} + \underset{(2.27)}{16.21\ \text{Group}}$$

Equation 2 confirms the hypothesis and indicates that the difference between the two groups of 16.21 points on the 0-100 scale was significant at the 0.05 level ($t = 2.27$). The value of R^2 is 0.44. The STG was making better strategic choices at the end of the year than the ILG. This indicates possible transfer of skills, so is a promising result.

Hypothesis 3 predicts that the STG will improve more than the ILG during the year in their ability to articulate the strategies they used to solve problems.

Equation 3

$$\text{Articulation}_2 = \underset{(4.17)}{44.10} \quad \underset{(4.54)}{0.58\text{Articulation}_1} \quad \underset{(1.44)}{8.10\ \text{Group}}$$

This hypothesis is weakly confirmed, but with the coefficient for Group not being significant at the conventional 0.05 level ($t = 1.44$).

Hypothesis 4 also compares the two groups of students and is based on an *overall performance measure* (Performance$_1$ = Performance at time 1, and Performance$_2$ = Performance at time 2). This scores was obtained by giving a 50% weight to Obtaining correct answers, and 25% each to Strategy choice and Strategy articulation.

Equation 4

$$\text{Performance}_2 = \underset{(3.35)}{41.59} + \underset{(5.42)}{0.62}\ \text{Performance}_1 + \underset{(1.59)}{10.65}\ \text{Group}$$

Equation 4 shows that the overall performance of the STG (controlling for performance at time 1) improved by 10.65 points on the 0-100 scale more than the performance of the ILG. Again, the result was not quite significant at the 0.05 level (t = 1.59). The value of R^2 is 0.50.

Table 2 shows the results of students with high and low PAT Maths scores at the pre-test. The low group contains students at stanines four or below and the high group contains students at stanines 8 and 9. The low students from the STG recorded a high average improvement in problem solving performance, while students in the ILG class improved only minimally. To assess statistical significance, Equation 4 was re-run just for students in the low initial maths scoring group. The metric regression coefficient of 25.3 for the variable 'Group' had a significance level of p = 0.029 so the hypothesis that initially low students improved more in the STG group was accepted.

Among students in the high group, the ILG students actually registered a greater improvement in problem-solving but a t-test indicated that this difference was not statistically significant at the 0.05 level. It is however possible to speculate that the strategy intervention was not useful in improving problem-solving skills for students with established mathematical skills. Students in the ILG actually made greater gains.

Table 2

Improvement in Overall Performance During the Year for High and Low Performers in STG and ILG)

STG groups	STG Performance			ILG groups	ILG Performance		
	T^1	T^2	Improv.		T^1	T^2	Improv.
Low (n = 3)	26.6	53.1	26.5	Low (n = 7)	30.0	30.4	0.4
High (n = 5)	63.6	67.3	3.7	High (n = 4)	62.7	71.4	8.7

The results are of course based on small numbers of students who comprised the 'low' and 'high' achievers in both classes. However the results do indicate the strong possibility that students who are overall weak performers in mathematics might be helped to better problem solving performance with strategic training.

Discussion

Considering the results obtained for Hypothesis 5 in conjunction with the overall positive findings for the effects of strategy training on students' mathematical problem solving, it is reasonable to say that the greatest benefit was obtained by lower maths performers. From a teaching standpoint it is significant that students of low mathematical ability registered much greater improvements in problem-solving as a result of strategy training than did their counterparts in the ILG, who did not receive strategy training. It appears that explicit strategy training in a variety of contexts in the area of problem-solving may well be a promising method of overcoming deficits in mathematical problem-solving performance, especially for students with a low level of general maths skills.

However, in general terms, the differences found in the greater ability of the STG to choose and articulate strategies are particularly encouraging, because it is reasonable to suppose that students who are conscious of the strategies they are using will, in general, be better able to transfer their skills to a wide range of analogous tasks. This may well be the most important factor in achieving far-transfer to mathematical problem solving for the STG.

More broadly the research addressed in a small way the issue of how teachers can utilise time effectively by ensuring that learning is transferred and students are challenged. Programming tasks, in the form that they were presented to the students, were successful in engaging, motivating and challenging a range of students. Their subject matter ensured that students engaged in higher-order thinking processes as they researched, classified, processed and presented the solutions to their programming tasks. As such they were universally successful as one way of reinforcing content knowledge associated with a subject area or areas and contributing to the development of a range of important learning skills. Such outcomes are minimal expectations for teachers wishing to model learning with Logo-based software around the Lap-T(asks).

Programming has been shown in this study to improve mathematical problem-solving processes in some upper primary students with specific teacher mediation to systematically reinforce cognitive and metacognitive skills and strategies associated with programming and problem-solving. Teacher mediation which encourages application to knowing the problem, persistence with the task and constant editing and reflection seems to make a contribution to bridging the gap for students, resulting in transfer of useful problem-solving aspects of programming into a maths problem-solving context. This research also found that some levels of metacognitive awareness, namely being able to verbalise relational features of analogous contexts was not evident despite the apparent strategy transfer between two contexts (Walta, 1999). Further research into the relationship between the application of useful strategies to mathematical problem solving tasks and a student's ability to verbalise these understandings would be interesting.

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Primary School Children's Knowledge of Arithmetic Structure

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This paper is a report on the beginning of a much larger study that examines students' understanding and knowledge about the structures of arithmetic and generalising problems. Ninety-four students in their final years of primary school responded to a series of tasks designed to probe their knowledge of associativity and commutativity. The responses indicated that these students had failed to abstract from the experiences in arithmetic some of the mathematical structures believed necessary for the successful transition from arithmetic to algebra.

Introduction

Recent studies, such as Australian Year 12 students' performance in the Third International Mathematics and Science Study (1998), have reported the misconceptions many students hold not only with understanding the concept of a variable, but also in solving algebraic equations, and in translating word problems into algebraic symbols. While many students are experiencing difficulties, algebra still holds a pivotal role in the school curriculum in the 1990's (*Principles and Standards for School Mathematics*, Draft, 1998,). School algebra may be seen as a focus on understanding variables and their operations, and formulating and manipulating general statements about numbers (Kieran, 1996). The goals of algebra typically include: generalising arithmetic; studying the procedures used for solving certain kinds of problems; representing the relationships between quantities; and studying algebraic structures (Thorpe, 1989). Even with the rise of technology in the algebraic domain, students still need to have a fundamental understanding of the concept of a variable and basic algebraic structures (Kieran, 1996).

Recently in the literature the distinction has been made between algebra, algebraic thinking, and pre-algebraic thinking (Kieran, 1992). Beginning algebra students are required to move from arithmetic thinking to algebraic thinking. Pre-algebra is defined in the literature as the transition between arithmetic and algebra transition (Boulton-Lewis, Cooper, Atweh, Pillay, & Lewis, 1998). It is when students think about the numerical relations of a situation, discuss them explicitly in simple everyday language and eventually learn to represent them with letters (Herscovics & Linchevski, 1994). This transition is believed to involve a move from knowledge required to solve arithmetic equations (operating on or with numbers) to knowledge required to solve algebraic equations (operating on or with the unknown or variable). This transition is referred to as breaching the cognitive gap or didactic cut between arithmetic and algebra.

Two aspects are considered to be crucial in this transition stage. These are, first, the use of letters to represent numbers and, second, explicit awareness of the mathematical method that is being symbolised by the use of both numbers and letters. This involves a shift from purely numerical solutions to a consideration of method and process. Yet many students experience difficulties in achieving this transition (Boulton-Lewis, Cooper, Atweh, Pillay, & Lewis, 1998). Kieran and Chalouh (1992) suggest that most students are not given the opportunity to make explicit connections between arithmetic and algebra.

When operating in an algebraic world students are frequently required to manipulate symbols. This is often achieved by simply using transformational rules such as, "when you move something from one side of the equation to the other side change the sign". Students typically have little understanding of the relationship among quantities or the structural properties of the mathematical operations (Kieran, 1992). Typically these students cannot provide a rationale for the transformation. Kieran (1992) believes that a predominant reason given for this is that students have limited knowledge about mathematical structure.

An understanding of algebraic structure is typically derived from knowledge of the structure of arithmetic. In this instance, knowledge of mathematical structure is knowledge about the sets of mathematical objects, relationship between the objects and properties of these objects (Morris, 1999). The knowledge is considered to be about relationships between quantities (e.g. equivalence and inequality), properties of quantitative relationships (e.g., transitivity of equality), properties of operations (e.g., associativity and commutativity), and relationships between the operations (e.g., distributivity). In a beginning algebra course it is implicitly assumed that students are familiar with these concepts from their work with arithmetic. From repeated classroom experiences in arithmetic it is assumed that by inductive generalisation students arrive at an understanding of the structure of arithmetic. Thus, knowledge of structure is considered to be at a meta-level, derived from experiences in arithmetic. How do classroom experiences impact on students' ability to derive structure?

Previous research has documented ways in which students' arithmetic experiences constitute obstacles for the learning of algebra. Most of this research has focussed on the differences between the two systems, for example, differing syntax (Lodholz, 1993), closure (Kieran, 1992), use of letters as shorthand (Booth, 1989), manipulations (Booth, 1989), and equality (Wagner & Parker, 1993). Little research has focussed on students' ability to induce mathematical structure from their experiences in arithmetic.

The current Queensland primary curriculum number strand consists of 4 main components, namely, numeration, basic facts, an understanding of the four operations (addition, subtraction, multiplication, and division) and the algorithms (Department of Education, Queensland, 1987). The emphasis appears to be on arithmetic as a computational tool, that is, counting and calculation. Calculation typically involves the use of prescriptive algorithms (Department of Education, Mathematics Sourcebooks, 1988). Students' experience with number in the primary school appears to be limited to numeration, operations, and the implementation of the algorithms. We argue that the over riding emphasis on computational procedures at the expense of exploring relationships is largely responsible for children's limited understanding of arithmetic structure.

This study reports on the pilot stage of a larger study designed to investigate students' pre-algebraic thinking. In particular, it explores students' ability to understand mathematical structure and extract general relationships and principles from problem situations. The focus of this paper is on children's knowledge of the properties of the operations, that is, associativity and commutativity. The sample was drawn from the upper primary school. At this stage students have completed all formal experiences with the four operations.

The particular aims relating to this paper were to ascertain students' ability to

1. recognise the commutative and associative law;
2. explain these laws in every day language; and
3. represent these laws in symbols.

Method

Instrument

A written test consisting of five tasks was developed. This paper reports on the results of the first two tasks. These two tasks focussed on identifying children's knowledge of associativity and commutativity in arithmetic. Each task consisted of four components. In the first segment students were given examples of the four operations and asked to indicate which were true or false (see Tables 1 and 5). The numbers chosen in the examples were believed to be small enough to allow students to complete the calculations without the assistance of a calculator. In the second segment, students were asked to create two more examples for the operations that they believed were true. They were then asked to explain the patterns they had discovered and asked to express the pattern using symbols such as ♥ and ∞ instead of using numbers. The instrument was administered under test conditions. Students were allowed ample time to complete each of the tasks and the test was calculator supported.

Participants

The test was administered to ninety-four children aged from 10 years to 12 years. Half the sample had just completed the second last year of primary school and the other half had just completed the final year of primary school, ready to make the transition to secondary school. The children attended three different primary schools in Brisbane. Each school was located in a middle to high socio-economic area. The participating schools were considered to be typical in their approach to teaching mathematics, that is, textbook supported and an emphasis on the algorithms.

Results and Discussion

Initially, the Year 6 and Year 7 students' results were analysed separately. χ^2 tests were used to identify the differences between the two groups. On the whole, the groups were not significantly different. Thus for reporting purposes, the two groups were combined (n = 94).

The first task related to gauging students' understanding of the commutative law. Students were initially asked to indicate for which operations the commutative law was true. These results are summarised in Table 1.

Table 1
Percentage Distribution of Responses for the Commutative Law Question (n = 94)

Operation	Example	True	False
Addition	$1 + 3 = 3 + 1$	94.7	5.3
Subtraction	$1 - 3 = 3 - 1$	11.7	88.3
Multiplication	1 x 3 = 3 x 1	95.7	4.3
Division	$1 \div 3 = 3 \div 1$	16.0	83.0

Most of the students recognised that the number sentences were valid for addition and multiplication. Of concern was the number of students who believed that the statements for subtraction and division sentences were true, especially as the students were in the final stages of their primary school experience.

Students were asked to create two more examples for the operations they believed were true. The results are summarised in Table 2.

Table 2

Percentage Distribution of Responses to "Create two more examples" (Commutative Law)

Operation	One example	Two examples	Three examples
Addition	67.0	21.3	1.1
Subtraction	0.0	0.0	0.0
Multiplication	60.6	13.8	1.1
Division	0.0	2.1	0.0

As can be seen in Table 2, there was a marked difference between being able to recognise that the commutative law held for the addition and multiplication examples and being able to create two more examples.

The next segment required students to explain in their own words the patterns they had discovered in the commutative law. Students' responses fell into six broad categories delineated from the data. The categories appeared to represent increasingly more sophisticated levels of response. Table 3 lists the categories and summarises the percentage of responses in each category.

Table 3

Responses to "Explain to a friend the patterns you have discovered" (Commutative Law)

Category	% frequency
1. No explanation	20.2
2. Offered a statement of the form "it goes in patterns"	46.8
3. Used only numbers in the explanation (e.g., 2 times 3 = 3 times 2)	13.8
4. Attempted to generalise but incorrect	2.2
5. Gave a valid explanation without the inclusion of the particular operation (e.g. turned around each time)	10.6
6. Gave a valid explanation with the inclusion of the particular operation (e.g., When you add and multiply it would be the same if you switch them around)	6.4

A typical category 6 response was, *the reason that 1 and 4 are true is because that when you multiply or add something then turn it around it will always equal the same thing, and when you subtract and divide the numbers and then turn them around it won't.* One student responded by focussing on the number sentences that were not true: *Because 1–3 will give you an answer of –2. While 3–1 will equal 2. And 1÷3 will give you an answer of 0.33 while 3÷1 will give you 3. While the other two operations are true.*

Two-thirds of the responses fell into the first two categories; that is, no response, or simply gave a statement such as "it goes up in patterns". Only 17 percent of the sample supplied a valid explanation. Six percent of the sample included the specific operations in their response.

This explain segment for the commutative law was the only component of the test where there was a significant difference between the Year 6 and Year 7 responses ($\chi^2 = 20.1$, $p = 0.003$). A significantly greater number of Year 6 responses fell into the first two categories and no Year 6 student offered a category 6 response.

Table 4 summarises the percentage of students who were able to express their patterns in symbols.

Table 4

Percentage Distribution of Responses to Using Symbols to Represent the Commutative Law

Operation	Correct	Incorrect
Addition	67.0	3.2
Subtraction	0.0	2.1
Multiplication	45.7	2.1
Division	0.0	3.2

The students experienced greater difficulty in expressing the commutative law symbolically for multiplication in symbols than the addition law.

In summary, approximately 95 percent of students indicated that the examples for addition and multiplication were correct. Twenty one percent of students were able to create two more examples for addition and 13.8 percent were able to create two more examples for multiplication. Two thirds of the students were unable to explain the pattern in words, with the Year 6 students experiencing greater difficulty than the Year 7 students. Two thirds of the sample could express the addition pattern in symbols and under half could express the multiplication pattern in symbols.

The second task related to ascertaining students' understanding of the associative law. The format of the associative law task mirrored the format of the commutative law task (refer to Table 5). The results for each of the four components are summarised in Tables 5, 6, 7 & 8.

Table 5

Percentage Distribution of Responses to the Associative Law

Operation	True	False
(2 + 5) + 8 = 2 + (5 + 8)	88.3	11.7
(2 – 5) – 8 = 2 – (5 – 8)	12.8	87.2
(2 x 5) x 8 = 2 x (5 x 8)	89.4	10.6
(2 ÷ 5) ÷ 8 = 2 ÷ (5 ÷ 8)	19.1	79.8

Just less than 90 percent of the sample believed the addition and multiplication number sentences were true. As for the commutative law, of concern were the number of students who believed the number sentences were true for subtraction and division, with up to 20 percent of the sample indicating that the division number sentence was valid.

Table 6

Percentage Distribution of Responses to "Create two more examples"(Associative Law)

Operation	One example	Two examples	Three examples
Addition	54.3	19.1	0.0
Subtraction	1.1	0.0	0.0
Multiplication	50.0	12.8	0.0
Division	0.0	2.1	0.0

A small percentage of the sample could create two more examples for the addition pattern and up the multiplication pattern. Half the sample created only one more example for each. Up to a quarter of the sample could not create any more of the examples of the associative law.

Table 7 summarises the distribution of responses relating to asking children to explain the patterns they had discovered.

Table 7

Responses to "Explain to a friend the patterns you have discovered" (Associative Law)

	Category	% frequency
1.	No explanation	30.9
2.	Offered a statement of the form "it goes in patterns"	39.4
3.	Used only numbers in the explanation [e.g., (2+3)+4=2+(3+4)]	11.7
4.	Attempted to generalise but incorrect	2.1
5.	Gave a valid explanation without the inclusion of the particular operation (e.g. move the brackets)	13.8
6.	Gave a valid explanation with the inclusion of the particular operation (e.g. When you add and multiply you can move the brackets)	2.1

Compared with the commutative law, even more students experienced difficulties in explaining the patterns in language with 70 percent of the sample proffering no explanation or "goes up in patterns". Unlike the commutative law for this component of the task, there was no significant difference between the Year 6 and Year 7 students.

Table 8

Percentage distribution of responses to using symbols to represent the associative law.

Operation	Correct	Incorrect
Addition	51.1	2.1
Subtraction	0.0	2.1
Multiplication	42.6	5.3
Division	0.00	5.3

In summary, approximately 88 percent of students indicated that the examples for addition and multiplication were correct. Nineteen percent of students were able to create two more examples for addition and 12.8 percent were able to create two more examples for multiplication. Seventy percent of the students were unable to explain the pattern in words. Approximately half of the sample could express the addition pattern in symbols and under half could express the multiplication pattern in symbols.

In order to compare students' facility with the associative and commutative laws, each student was allocated a score for the first segment of each task accordingly to whether their response was correct or incorrect. The results of a paired t test indicated that there was no significant difference between students' facility to recognise the commutative law and the associative law ($t_{93}=1.704$, $p = 0.092$). There was also no significant difference between the level of response when students were asked to describe the pattern in their own words for the associative law and commutative law. In fact, most students performed poorly on both of these segments of the two tasks.

Implications and Conclusions

The results of this initial study highlight a number of concerns and implications for teaching and research in early algebra education.

There were a significant number of students who believed that the number sentences for subtraction and division were correct, with more students experiencing difficulties with division than with subtraction. This could occur for a number of reasons, for example, (i) a focus in early mathematics education on discovering relationships rather than also exploring non-relationships, (ii) little opportunities for students to explore their own conjectures and inductions, or (iii) teaching mathematics in a non-calculator supported environment making it difficult to explore the non relationships that exist in subtraction and division.

Many students experienced difficulties in finding more examples. Most could only find one more example, even though they were asked to find two. One reason for this could be that they made decisions in the first segment of the tasks purely on computational grounds without recognising the mathematical structure being represented by the number sentences. This could also reflect limited experience with the use of "=" as equality. Falkner, Levi and Carpenter (1999) reported that young children have enduring misconceptions about the equal sign. Even children as young as Grade 1 had already formed misconceptions. The reason for this, they suggest, is the equal sign is typically used in equations where one number comes after it.

The segment of the tasks relating to expressing patterns in everyday language was where almost all students experienced difficulties, with most proffering no answer or a basic response, such as, "it goes in patterns". The importance of natural language to the algebraic domain has been acknowledged in previous research ((Herscovics & Linchevski, 1994; Redden, 1996). Herscovics, Linchevski and Redden's research indicated that natural language description of number patterns seems to be a necessary prerequisite for representing the patterns in algebraic notation. While the present students were not strictly using algebraic notation to represent the patterns they had found, up to 67 percent of the sample could represent at least one of the patterns using symbolic notation. This raises a number of questions needing further research. How important is expressing patterns in everyday language for the successful transition from arithmetic to algebra? What is the relationship between language development and being able to describe mathematical relationships? MacGregor and Price (in press) indicate that this relationship is not straightforward. If expressing patterns in everyday language were important for the successful transition into algebra, what activities would assist students in this area of language development?

The results of this study point to a number of concerns with regard to the present curriculum. From this initial study, it seems that the majority of students are leaving primary schools with little awareness of the notion of mathematical structure. It seems that from the instances they are experiencing in arithmetic, they have failed to abstract the relationships and principles needed for algebra. More specifically, they failed to abstract the associative and commutative laws.

This has implications for the primary school curriculum. There needs to be more of a balance between calculations and searching for the implicit patterns in the operations. Students not only need many instances of relationships, they also need to explicitly discuss these relationships in everyday language. Present experiences do not appear to be reaching this balance. Students also need to explore non-relationships, such as $1 \div 3 = 3 \div 1$. They also need broader experiences in arithmetic encompassing activities where the equals symbol is used in equivalent situations (e.g., $2 + 3 = \square + 2$).

This paper reports on the beginning of a much larger study which includes interviews with selected students. These interviews will help clarify and illuminate many of the conjectures made from the written responses. Conjecturing from pen and paper responses about what children know or don't know is always fraught with difficulties.

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Factors in Primary School Teachers' Beliefs about Mathematics and Teaching and Learning Mathematics

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This paper is a report on the development of an instrument to identify and measure factors associated with primary teachers' purported beliefs about mathematics and teaching and assessing mathematics. Current literature relating to teachers' beliefs provided a theoretical framework for the development of the questionnaire. The sample consisted of 387 primary teachers. The results of a factor analysis identified seven factors that appear to represent teachers' purported beliefs – three relating to beliefs about assessment, two relating to beliefs about mathematics, and two relating to beliefs about teaching mathematics.

Introduction

This paper is a report on the ongoing development of an instrument to identify and measure factors associated with primary teachers' purported beliefs about mathematics and teaching and assessing mathematics. In a previous paper, the authors reported on the beliefs of primary teachers with respect to the uses of assessment in mathematics and the types of assessment they use. Those results indicated that primary teachers use a wide variety of assessment methods and that they believe that the predominant purpose of assessment is to provide feedback for the teacher rather than for the learner or parents. This paper is an analysis of the data on teachers' beliefs not only about assessment of mathematics but also about mathematics as a discipline, and approaches to the teaching of mathematics.

The study of teachers' beliefs is important because of the influences beliefs have on (a) teachers' classroom practice, and (b) their likelihood of implementing changes of practice in the classroom (Ball, 1990; Thompson, 1992). The relationship between teachers' beliefs and classroom practice is quite a complex one. On one hand, there is much research indicating that teachers' practices are shaped by their beliefs about the subject mathematics and the nature of teaching and learning (Hoyles, 1992; Fernandez, 1997; Putnam, 1992). In a study of primary teachers' beliefs about problem solving in mathematics, Ford (1994) concluded that although there existed a relationship between teachers' beliefs and classroom practices, the extent of the relationship was not found.

On the other hand, there is evidence to suggest that changes in teachers' beliefs about teaching and learning are derived largely from classroom practice (Clarke, 1994; Brosnan, 1994). Guskey's (1986) model of professional development proposes that teachers' beliefs and attitudes change only when they see changes in students' learning. The Clarke-Peter model of professional growth (Clarke & Peter, 1993) recognises that teachers' beliefs are influenced by production of valued outcomes from classroom experimentation. Foley (1993) claims that even traditional teachers are willing to change their teaching methods and beliefs when empowered to make decisions that impact on their classrooms.

The close association between beliefs and practices is supported by Borko (1997) who claims that when teachers' beliefs are compatible with the ideas which underlie a professional development program, these beliefs support the change. Hence it is important to attend to both beliefs and practices in staff development. Borko and Putnam (1996) note that meaningful change in one requires change in the other. However, the order in which beliefs and practices are addressed in professional development may not be important.

Further evidence of the complexity of the relationship between beliefs and practice is research indicating that some teachers carry on with certain classroom practices in contradiction of their stated beliefs and intentions (Renne, 1992; Sullivan, 1989).

Many studies of mathematics teachers' beliefs have been conducted in specific contexts, namely the teaching of particular mathematics topics, the implementation of certain methodologies, special groups of students, or various professional development models. Hence, the instruments designed to identify and measure teachers' beliefs are themselves related to these contexts for example, implementing the NCTM Standards (Furner, 1996), teaching using the cognitively guided instruction approach (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996), teaching in a socio-constructivist environment (Van Zoest, Jones, & Thornton, 1994), teaching problem solving (Erickson, 1993; Ford, 1994; Lubinsky, 1993), low achievers in mathematics (Dwyer, 1993), and teaching mathematics with manipulatives (Sherman & Richardson, 1995).

The instrument devised for the current project is also related to a specific area, namely, assessment methods in mathematics, but it was thought that the instrument should be developed with a solid theoretical basis, to see whether the data generated from the instrument reflected the theoretical constructs. Therefore, in line with the research on beliefs relevant to classroom practice learning (Fernandez, 1997; Ford, 1994; Hoyles, 1992; Putnam, 1992), the authors decided to include beliefs about the nature of mathematics, and the teaching of mathematics, along with beliefs about the nature and purposes of assessment, and to construct items congruent with the literature in each of those areas.

In relation to the first area of beliefs, the nature of mathematics, Ernest (1989) identified three differing conceptions of mathematics, namely, (i) a dynamic problem-driven view of mathematics where mathematics is a continually expanding field of human creation and invention, (ii) a static unified body of knowledge where mathematics is viewed as interconnecting structures bound together by logic and meaning, and (iii) a bag of tools where mathematics is made up of an accumulation of facts, rules and skills. The items designed for the instrument reflected these three categories.

The literature on teaching mathematics delineates essentially two main classification schemes for classifying beliefs on how to teach mathematics. One scheme (Burton, 1993) identifies two basic approaches to teaching mathematics - a transmission approach where the teacher simply transmits information and rules to the students who are expected to absorb and regurgitate it, and a constructivist approach in which teachers are seen as facilitators of learning and students construct their own mathematical knowledge through interaction with the physical and social environment. Perry, Howard, and Tracey (1999), who called these categories transmission and child-centredness respectively, reflect this second dichotomy. Another classification scheme (Kuhs & Ball, 1986) posits four dominant views on how mathematics can be taught: content-focused with an emphasis on performance (procedural); content-focused with an emphasis on understanding (conceptual); classroom-focused where the focus is on mathematical content through classroom activity; and learner-focused where mathematics teaching focuses on the learner's personal construction of knowledge. The items written for this questionnaire reflected the classification scheme suggested by Kuhs and Ball (1986).

There is a paucity of research on teacher's beliefs about the uses of assessment. Clarke, Clarke and Lovitt (1990) claim that the major uses of assessment focus on three areas, the teacher, the student, and the parent. First, the teacher uses assessment to improve instruction by using students' responses to help identify (i) instructional strategies that are most successful; and, (ii) student learning behaviours that need to be "encouraged and developed or discouraged and replaced". Second, assessment informs students of their identified strengths and weaknesses and informs subsequent teachers of students' competencies. Finally, parents are informed of their child's behaviour so that they can give more effective support. The NCTM Assessment Standards (1995) delineates four purposes for assessment. These are promoting student growth, improving instruction, recognising accomplishments, and modifying programs. These purposes are teacher-learner focused. The items developed for the questionnaire classified assessment as informing teaching, assessment as informing learning, and assessment as a tool for reporting to external parties such as parents, other teachers and the general public.

The specific aim of this study was to identify factors in the beliefs of mathematics teachers that affect the teaching and assessment of mathematics.

Method

Many investigations of the relationship between teachers' beliefs and practices have been qualitative studies of a small number of teachers, however a number have been conducted using quantitative methods with large samples of teachers. This study falls into the latter category.

Instrument

A 56-item Likert-scale survey was designed for a large comprehensive study. The questionnaire consisted of five sections relating to beliefs about mathematics, beliefs about teaching mathematics, beliefs about the purposes of assessment, beliefs about the nature of assessment, and assessment practices. The questions used in the study were based on current literature on beliefs about mathematics, teaching mathematics, the purposes of assessment and the nature of assessment. This paper reports on the first four sections of the survey (46 items). Teacher were also asked to indicate their gender, year level at which they taught, level of qualification in mathematics (Year 10, Year 12, teacher education course in mathematics, University mathematics) and years of experience in teaching (0-5 years, 6-10 years, 11-15 years and >15 years).

Participants

Fifteen hundred forms were sent to a random selection of primary teachers representing different school year levels (one through to seven), school systems (government, catholic), socio-economic areas (high and low), and geographic locations (metropolitan, provincial, and rural). Although the return rate was low, (27%, n = 398) the resulting sample was representative of the different systems, socio-economic areas, and geographic locations. An analysis of the sample indicated that the schools served a wide range of communities. The two groups of teachers (State schools, n = 179; Catholic Schools, n = 195) did not differ significantly on the level of mathematics training they reported (Chi-square$_{(3)}$ = 7.19, N = 374; p = 0.07). Nor did the sub-samples differ in their reported years of teaching (Chi-square$_{(3)}$ = 0.63, N = 387; p = 0.89) or gender (Chi-square$_{(1)}$ = 0.05, N=383; p = 0.83). Sample sizes in the State school teachers ranged from 17 (Year 6) to 35 (Year7) and in the Catholic school group the range was from 26 (Year 6) to 31 (Year 7). No statistically significant difference was found between the two School system subgroups in the representation of Grades taught. The returned surveys also reflected a well-balanced distribution of school year level as summarised in Table 1.

Table 1

Percentage Distribution of sample by school year level

	School Year Level						
	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7
Percentage of teachers	13.6	14.7	15.7	12.3	12.1	11.1	17.0

Results and Discussion

The responses to items relating to beliefs about mathematics, beliefs about teaching mathematics, and beliefs about the purposes of assessment (Sections 1, 2, & 3 of the survey) were subjected to a factor analysis. Initially, there were 15 factors with eigenvalues > 1. Given the exploratory nature of the study and guided by the interpretability of the factors, a seven-factor orthogonal solution after the extraction of principal components and a Varimax rotation was accepted. The seven-factor solution accounted for forty-one percent of the variance. Thirty-one of the 46 items were used to delineate the factors. The solution is presented in Table 1. The naming of factors was guided by the content of the items identified with the factor. The factors were tentatively named as follows:

- Factor 1: Assessment is used to evaluate teaching
- Factor 2: Assessment is used to evaluate learning
- Factor 3: Assessment is used for accountability purposes
- Factor 4: A static view of mathematics
- Factor 5: Mechanistic view of mathematics
- Factor 6: A traditional view of teaching mathematics
- Factor 7: A contemporary view of teaching mathematics

The results for Factor 1, Factor 2 and Factor 3 (the three factors relating to beliefs about the uses of assessment) are summarised in Table 2.

Table 2

Factors related to assessment

Item	Item description	Loading
Factor 1: Assessment is used to evaluate teaching		
	I use assessment to:	
36j	help me evaluate how effective my teaching has been	0.75
36f	judge how well the class is progressing	0.72
36e	help me identify students with problems	0.69
36g	help me plan the next phase in teaching	0.68
36i	inform me about the ability levels of the students	0.65
Factor 2: Assessment is used to evaluate learning		

	I use assessment to:	
36a	inform the students about what they do not know	0.75
36c	encourage students to learn their work	0.75
36d	provide information for successful students	0.70
36h	to inform me about which students are working	0.56
36b	give students feedback on their strengths and abilities	0.55
Factor 3: Assessment is used for accountability purposes		
28	The main reason why I assess student performances is to inform parents	0.76
35	The main reason why I assess is to meet the school's requirements	0.68
37	I tend more to test rules and facts than have students solve problems	0.35

The uses of assessment seemed to fall into three main categories, assessment used to evaluate teaching, assessment used to evaluate learning, and assessment for accountability. The first factor seems to reflect a feedback component and a planning component, supporting two of the purposes of assessment delineated in the NCTM Standards, namely, improving instruction and modifying programs.

The second factor seems to support the notion of assessment promoting student growth and recognising accomplishment. There also seems to be a component that views learning as closely linked to working. "I use assessment to inform me about which students are working".

Of interest is the inclusion of the item 'I tend more to test rules and facts than have students solve problems" in the third factor. Does this imply that if teachers are assessing for an outside audience it is easier to quantify and justify rules and facts than it is to report students' problem-solving ability? What influence does parental expectations have on teachers' choice of assessment techniques? These points are worthy of further investigation.

Table 3 summarises the results for Factors 4, 5, 6, 7, the factors pertaining to beliefs about mathematics and the teaching of mathematics.

Table 3
Factors related to views of mathematics and teaching mathematics

Item	Item description	Loading
Factor 4: A static view of mathematics		
2	The good thing about mathematics is that it is an unchanging subject	0.83
3	The advantage of mathematics is that things are either right or wrong	0.70
15	Today's mathematics is no different from mathematics of long ago	0.41
1*	Mathematics is dynamic. It is a searching for patterns in the environment	-0.35
26	Students learn best by doing lots of exercises and practice	0.34
Factor 5: Mechanistic view of mathematics		
19	Mathematics is essentially about computation	0.62
20	The problem with low achieving students is that they don't learn the rules	0.57
11	I tend more to test rules and facts than have students solve problems	0.50

Factor 6: A traditional view of teaching mathematics		
14	The main tools I use for gathering data are timed tests rather than projects and investigations	0.75
34	It is difficult to plan for hands-on experiences in mathematics lessons	0.68
9*	In mathematics I teach interesting things about the world outside school	-0.56
Factor 7: A contemporary view of teaching mathematics		
17	It is important for student to use concrete materials in mathematics	0.69
16*	What I teach in maths should have little to do with students' out of school life	-0.67
5*	It is not worth spending time on collecting observational data on students	-0.55
10	Students learn better through solving problems in real world contexts	0.52
6	Students who do well on standard exercises don't necessarily do well on tests	0.45
7*	Students are encouraged to explain their ideas and teach each other	-0.36
33*	It is very important that children get the right answers to exercises	-0.32

Note. *Items with negative loadings

Primary teachers' beliefs about mathematics seemed to fall into two broad categories, a static view of mathematics and a mechanistic view of mathematics. These factors reflect Ernest's (1989) categories of mathematics as a static unified body of knowledge and mathematics as a bag of tools made up of an accumulation of facts, rules and skills. Missing is the category reflecting a dynamic problem-driven view of mathematics even though there were items in the questionnaire that were believed to reflect this view. This is a concern for all mathematics educators as this is the view strongly reflected in many curricular documents and *The National Statement on Mathematics for Australian Schools* (AEC, 1990).

The two factors related to beliefs about teaching mathematics reflect a traditional view and a contemporary view of teaching mathematics. The traditional view of teaching mathematics implies not using hands-on materials, not teaching about interesting things about the world outside schools and using timed tests to assess teaching and learning. It appears that for some teachers, teaching mathematics occurs in a fairly sterile environment. This style of teaching could be related to a transmission model of teaching, although these particular items do not necessary reflect a teacher-centred classroom. Of interest would be what these particular teachers do rather than what they do not do. It is difficult to imagine how such teachers could believe in a constructivist approach to teaching mathematics. But do they focus their teaching on developing procedural understanding (an emphasis on performance) or are they concerned with developing conceptual understanding (an emphasis on mathematical content through classroom activity); are they 'show-and-tell' teachers or 'explaining' teachers? They certainly do not seem to be too concerned about making mathematics real-world related

The contemporary view of teaching mathematics seems to reflect some of the elements of a constructivist approach to mathematics (Burton, 1993) or a child-centredness approach to mathematics (Perry, Howard, & Tracey, 1999). Of interest is the negative loading on 'Students are encouraged to explain their ideas and teach each other'. This may simply reflect the wording of the question and the fact that it consists of two components, explaining ideas and teaching each other. Teachers may believe that students need to explain their ideas but do not believe that students should teach each other. This issue needs further investigation.

Conclusion

This research begins to delineate some of the beliefs held by primary school teachers with regard to mathematics, teaching mathematics and the purposes of assessment. It was found that the factors identified with these data did reflect the theoretical constructs proposed initially. The groups of factors were associated with beliefs about (i) mathematics, (ii) teaching mathematics and (iii) purposes of assessment. Furthermore, within these groups, the separate factors were closely aligned with the theoretical components.

First, in relation to the nature of mathematics, two of the views proposed by Ernest (1989) – a static unified body of knowledge and a bag of tools - were evident in Factors 4 and 5 – a static view and a mechanistic view, respectively. It is of concern however that the third view – a dynamic problem-driven view – was not evident in the results of the factor analysis. It seems that primary teachers hold a fairly limited view of what mathematics is. This result has implications for professional development. Mathematics as a dynamic problem-driven ever-expanding field of human creation and invention seems more aligned with the constructivist model of learning, and if more teachers are to adopt teaching in the light of a constructivist view of learning, then their views of mathematics will need to be addressed along with issues of pedagogy, as noted by Borko (1997).

Second, the factors relating to teaching mathematics closely reflect the teaching styles defined by Burton (1993) and Perry, Howard and Tracey (1999), but only two categories defined by Kuhs and Ball (1986). These factors seem to reflect the types of environments that mathematics teaching tends to occur in - a traditional environment with few materials and unrelated to the real world, and a contemporary environment where hands on materials are valued, mathematics is related to out-of-school experiences, and problem solving is encouraged. The beliefs that teachers hold about teaching in these two differing environments are not clearly delineated in the responses. Perhaps further refinement of the items in the survey may be worthwhile in teasing out the different styles of teaching. If not, there may be reason to suspect that a two-category system is more appropriate than a four-category system.

Third, with regard to assessment, the three factors identified were closely aligned with the three purposes proposed initially – to evaluate teaching, to evaluate learning, and for accountability purposes. One question which arises from the results relates to the last factor: What role do outside bodies such as parents play in beliefs about assessment practices and the choices teachers make about how to assess learning? This warrants further study. Given the close relationship between beliefs and effective professional development (Borko & Putman, 1996), such issues need to be addressed if one wishes to change classroom practice.

This paper reports on the ongoing development of an instrument to identify and measure factors associated with primary teachers' purported beliefs about mathematics and teaching and assessing mathematics. While the questionnaire has resulted in clearly defined and interesting factors, as indicated in the results, the items need further refinement. The limitations of this type of research are well documented in the literature, and include the fact that what teachers believe does not necessarily reflect their classroom practice. Past research has indicated that this tension usually occurs between more liberal beliefs and more conservative practice rather than vice versa. Thus the factors delineated in this paper possibly reflect a "best scenario" situation.

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Intuition versus Mathematics: The Case of the Hospital Problem

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This paper considers 33 preservice secondary mathematics teachers' solutions to a famous sampling problem, well known for confounding educated adults. Of particular interest is the Bachelor of Teaching students' use of intuition and/or formal mathematics in reaching a conclusion. The relationships of solution strategy to students' background in formal mathematics and to gender are also considered. Implications for teaching statistics at both the secondary and preservice teacher education levels are discussed briefly.

> A town has two hospitals. On the average, there are 45 babies delivered each day in the larger hospital. The smaller hospital has about 15 births each day. Fifty percent of all babies born in the town are boys. In one year each hospital recorded those days in which the number of boys born was 60% or more of the total deliveries for that day in that hospital.
>
> Do you think that it's more likely that the larger hospital recorded more such days, that the smaller hospital did, or that the two recorded roughly the same number of such days?

The *hospital* problem, as the above problem is usually named, began its life in the work of Kahneman and Tversky (1972), in their ground-breaking work on people's understanding of representativeness. Although there have been criticisms of the wording used by Kahneman and Tversky (e.g., Well, Pollatsek, & Boyce, 1990), this problem has been used repeatedly over the years to explore understanding of sampling and the variation associated with different sample sizes (cf. Well et al., 1990). Tversky and Kahneman (1971) found that university students were susceptible to the "law of small numbers", that is, the belief that a small sample represents a population just as well as a large one. For the problem above this leads to the conclusion that the two hospitals recorded about the same number of days with 60% or more boys born. Although school students have not often been asked this question, Fischbein and Schnarch (1997) found only one student of 60 in grades 5, 7, and 9 who suggested that the smaller hospital would have the more extreme result. Watson, Collis, and Moritz (1995), in a study of students' understanding of sampling in grades 3, 5, 7, and 9, found that of 12 students asked a similar question, none could argue from a correct basis. Watson and Moritz (2000) asked 59 students the same problem and of these, only six recognised the smaller sample as more likely to have the extreme result and gave adequate justification. With younger children the choice of the large or small sample appeared random, the reasons given for choosing the small sample could also be given for choosing the large one, and story-telling often dominated answers. Reasons given for saying the large and small samples had the same chance of having the extreme result included "because it's random" or "because percentages aren't definite numbers and so are the same for each sample".

Researchers studying people's appreciation of sample size in the context of the hospital problem, however, have not explored the understanding of people with a formal background in mathematics and the potential to call upon formal statistics to answer the hospital question in terms of the actual probabilities associated with the two samples having 60% or more boys. Anecdotal evidence (N. Windsor, personal communication, November 9, 1998) indicates that many senior secondary students, when given the hospital problem immediately after a unit on

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the normal approximation to the binomial distribution, do not recognise its applicability. Of those who do, many still have difficulty carrying out the process correctly to achieve the appropriate conclusion.

Whereas perhaps not all preservice mathematics teachers would have been exposed to the theoretical mathematics necessary to calculate the probabilities in each hospital, at least it would be expected that they would have access to the basic mathematics to calculate percentages, fractions, and basic probabilities based on equal likelihood (of births of boys and girls). For these people, it is hence of interest to observe what general mathematical knowledge is used to support an intuitive suggestion. It is also of interest to see which mathematical skills are chosen by those with more formal training and whether this is backed up with any reference to intuition. Some people may have experienced sampling in their personal or professional lives that helps them appreciate the likelihood of greater variation in the smaller sample and feel no need for further justification except for a statement. Others may have the ability to calculate a normal approximation to the binomial distribution without any appreciation for the meaning of the result.

The opportunity to explore further the use of formal mathematics as a solution strategy for the hospital problem arose in the context of a case study used in a preservice program for secondary mathematics teachers. The case study, *Chances Are*, was one of a series published by Harvard University to be used with high school mathematics teachers (Merseth & Karp, 1997). The pre-case worksheet for the case study presented the problem as given at the beginning of this paper and asked participants to provide a solution and list any assumptions made. This case study was used with three groups of Bachelor of Teaching (BTeach) students.

The objective in this small study is to add to the information available on strategies adults, in particular people who are planning to be high school mathematics teachers, will choose to answer the hospital problem. The aims are (a) to observe whether mathematics, intuition, or a combination of the two is most successful in achieving an acceptable solution, (b) to document the types of mathematics employed to support intuition or as the basis for a mathematical justification, (c) to suggest propensities for solution strategies of students with more or less formal mathematics training or by gender, (d) to report students' reactions to the problem, and (e) to suggest action to improve both intuitions and confirmatory skills in this area.

Method

The 33 preservice secondary mathematics teachers in this study were aged from 21 years, having just finished a university degree, to middle age, having worked in other careers before deciding to become mathematics teachers. The older students came from careers in geology, biology, engineering, meteorology, statistics, and taxi driving. The group varied widely in its previous exposure to formal mathematics courses. One was on leave from enrolment in a PhD in mathematics, whereas others had done no formal mathematics at university but had convinced the head of school (not the author) that they had a mathematics background adequate for the course. Twenty-three BTeach students were classified as having at least a minor in mathematics and 10 were classified as "other". There were 14 females and 19 males in the sample. Although this was not a random sample of preservice secondary mathematics teachers across Australia, it is thought to be representative of the types of students in preservice mathematics courses.

The BTeach students were given the pre-case worksheet to complete overnight before the case study was handed out. They were told to answer the question by themselves and that, although the solutions would be collected and read by the lecturer, no marks would be awarded, except for completing the exercise. The students knew that they would be participating in a case study based on the problem and appeared to be motivated to think seriously about it. Only one solution was very short and appeared to be by a student who had taken the problem less seriously or had little idea of how to solve it.

The method of analysis employed was a clustering technique (Miles & Huberman, 1994), which looked at similarities in solutions to the problem. Of particular interest were those judged to be based on intuition in relation to sample size with no support in the way of mathematical calculations, those that displayed mathematical calculations but displayed no indication of intuition influencing the answer, and those that appeared to mix intuition with mathematical calculations. Further, when mathematics was employed in the solution, it was of interest whether it might be termed basic mathematics, such as percentages and fractions, or theoretical mathematics based on the binomial and/or normal distributions. Finally it was of interest whether the conclusion reached was correct or incorrect. The analysis was based on "observed" outcomes, that is, only what was written on paper. If a student held an intuition that was not stated, obviously no credit was given. After the clustering analysis had taken place, note was made of gender and whether students were considered to have a strong background in mathematics.

Results

The results are summarised in Table 1 by whether the solution to the hospital problem was based on intuition as stated by the student, whether the presentation was based solely on a mathematical argument, or whether a mixture of approaches was stated. The results are also split by the correctness of the conclusion stated, except in one case. In that case a student stated a proposition of equal outcomes, attempted to calculate probabilities, made an error that indicated the small hospital had a very slightly higher probability, and chose that hospital. Given the numerical result the student should have retained the decision of "roughly equal" and was allocated as if that were the conclusion. Examples that illustrate the categories will be given according to the method of approach.

Table 1
Clusters of Responses to the Hospital Problem

	Strategy		
Conclusion	Intuition	Mathematics	Intuition and Mathematics
Correct (n = 18)	2 males 5 females	*Formal maths* 1 male, 1 female *Basic maths* 6 males	*Formal maths* 2 males, 1 female
Incorrect (n = 15)	4 males 4 females	*Formal maths* 3 males, 1 female	*Basic maths* 1 male, 2 females
	Total = 15	Total = 12	Total = 6

Intuition Only

Fifteen BTeach students gave responses deemed to be based on intuition. Although some of these solutions contained numbers, these were not used with calculations to support the conclusion. Of these 15 solutions, 7 chose the smaller hospital based on the correct reasoning of sample size. These responses appeared to reflect previous experience of some sort involving variation in samples. A typical response is the following, which although it included numbers had no calculations.

> I can't work this out mathematically, but intuitively I would say that the smaller hospital is more likely to have more variation from the average of 50% boys. This is because even though the probability of a head (or tail) when tossing a coin is 1/2, this is only the case the more times we toss the coin (ie the head won't necessarily turn up once in every 2 tosses, but the more times the coin is tossed, the closer the heads get to a frequency of a 1/2). Similarly, the more babies there are, the more likely the frequency of boys is to converge to 50% of births.

One person in this group mentioned that data would approximate a normal distribution but expanded only to say, "the degree of fit to the normal curve can be expected to be better for a larger sample size". The statement appears to reflect previous experience but is not backed up with any further mathematical argument.

Of the eight solutions that did not choose the small hospital, two did not reach a discernible conclusion, either stating two possibilities (larger hospital closer to the norm but both hospitals roughly the same over a year) or only eliminating one (larger would not have more). One student stated the following as a final comment: "If more births occur at the larger hospital, there is a greater likelihood that more boys will be delivered at the larger hospital." Although this statement is technically correct and would not preclude a correct comment on the hospitals, no other comment was made. The other five students indicated that the hospitals would have roughly the same number of days with 60% or more boys born. Typical of these is the following.

> Percentages are proportions of original sizes. 60% is the same for both hospitals – only depends on original number to work it out. There is no reason why the larger hospital should have 60% boys more often than the smaller hospital. It's not asking which hospital turns out more boys because then the bigger hospital would do that.

Of the 15 BTeach students who used intuition only as a basis for their solutions, 8 were among the group with less theoretical mathematics in their undergraduate degrees. Of these 8, half reached the correct conclusion. Of the 15, 6 were male and 9 were female with the females being more successful (56%) than the males (33%).

Mathematics Only

Twelve students provided solutions based on mathematical arguments with no indication of previous experience or intuition that would lead to belief in *that* solution at the start of the exercise. Eight of these twelve students reached the correct conclusion. Two of these students recalled from previous experience that the problem involved the binomial distribution and went to their old texts to find out how to carry out the calculations. One used the normal approximation to the binomial for both hospitals, whereas the other used binomial tables for the small hospital and the normal approximation for the large one. Neither person commented on their surprise or otherwise with the answer, although the person providing the second response indicated an initial belief that the hospitals would have roughly the same number of days.

One other student attempted to calculate binomial probabilities but only calculated the exact probabilities for 9 boys out of 15 and 27 boys out of 45. Although incorrect to answer the question based on "60% or more boys", these values led to the correct conclusion that the smaller hospital was more likely to have exactly "60% boys" born.

The other four students with the correct conclusion used simpler mathematical arguments based on how likely it would be to achieve deviations from the expected number of births to reach the 60% value. After stating that for the total of 60 births per day, 30 on average would be boys, one student argued as follows.

> Given that, for the larger hospital to record 60% or greater boys in a given day would require at least (27/30) 90% of the average number of boys born per day, as compared to only 30% (9/30) or more for the smaller hospital, one would suggest that it is more likely for the smaller hospital to record a greater number of such days.

After displaying a few percentage calculations, another student argued as follows.

> Given that 53.3% of the boys are already present (8 for the small hospital and 24 for the large hospital). It only requires 1 more boy for the small hospital to give 60% but requires 3 boys to give 60% for the large hospital. The chances of 1 more boy is [sic] higher than the chances of 3 more boys.

Variations on this theme were expressed with probabilities, the percentages represented by each birth, and the days required by the large hospital to redress an imbalance.

Four of the students who used formal mathematics in their solutions made errors that led them to the wrong conclusion. One made an error in the formula when applying the normal approximation to the binomial and one applied a hypothesis test incorrectly to the two hospitals. Both concluded that the hospitals would be equally likely to have 60% or more boys born. A third student did not think there was enough information available to use the binomial distribution, did some calculations to estimate total births with "unknown" probabilities for "number of days greater than 60%". This led to the conclusion that the large hospital would have more "60% or greater" days. The final solution in this group was one mentioned earlier where an incorrect calculation of probabilities (0.21 and 0.23) should have led to the conclusion that the hospitals would have roughly the same number of days with 60% or more boys born.

Only one of the students in this group of 12 was considered to have a weak formal mathematics background. This was the student who used the normal approximation correctly for both hospitals. His university major was in zoology and although he had done no formal mathematics courses at university, he had worked for 10 years in positions involving scientific research. Of this group of 12 using only mathematics for the problem, two were female, one providing a correct solution. Of the 10 males, 7 were correct.

Intuition and Mathematics

Only six students provided solutions that appeared to attempt to justify intuition or previous experience with a mathematical argument, of which half were correct. The three BTeach students providing correct conclusions used very different approaches. One provided a graph of a normal distribution curve centred at 50, with the horizontal axis labelled "percentages of boys" and the vertical axis labelled "Num. of days". Under the graph was a paragraph, edited extracts of which are as follows.

> If all things are considered equal both should produce the above distribution. However with the smaller hospital I would imagine there is more chance of (looking at extremes) all babies being boys than in the other hospital with an average of 45. Hence I would envisage that the curve for the smaller hospital would be a bit flatter resulting in ... a lesser value for the 60% mark ... so although it would be flatter it would be longer hence we may find that the larger hospital with its larger sample will result in more

> days closer to 50% resulting in a more pointy distribution while the other is more flat - at this point I would imagine that the smaller hospital may lean towards a greater number.

The second student reported an a priori belief in equality of the hospitals, but then argued that as *n* approached infinity the ratio of boys to all births would approach 1/2, hence "the smaller hospital will be more likely to have 60% or more boys a day over a year". To confirm this belief the student went to a senior secondary mathematics book, found the formula for the binomial distribution and correctly calculated the probabilities for both hospitals using a calculator! The third student expressed the correct intuition based on sample size and attempted to follow the approach of the previous student. Without a reference for the appropriate formula, the student guessed and achieved the correct relationship of the two probabilities but realised that the actual values were incorrect.

The other three students who mixed intuition and mathematics used basic mathematics. All concluded that the hospitals would have about the same number of days with 60% or more boys born. One argued sequentially and erroneously, that no matter how many children are born, the probability of all of them being boys is 50%. The claim was made individually for up to four babies and then extrapolated to 15, 45, and 60. Another worked with fractions, concluding the number of ways to get 60% or more boys was 6/15 = 2/5 in the smaller hospital and 18/45 = 2/5 in the larger hospital, and hence "the two hospitals record roughly the same number of days". His final comments, however, indicate that he may have become confused by the added information about the 365 days on which data were recorded. He stated the following.

> Initially I believed that the smaller hospital has a greater likelihood of having ≥60% of births being boys because of the greater variation due to the small sample size.
>
> However because the births are independent and at each one the P(B)=1/2, I believe that over the period of 365 days (which is a fair amount of trials) the variation will average out so that the two hospitals will record roughly equal number of days where the percentage of boys born was ≥60%.

Finally, after translating percentages into numbers for the two hospitals and doing comparisons, a student argued, "For the small hospital the fluctuation between 50-60% is only 1-2 children, and 4-5 for the large hospital but they [large] have more children so I think it would be about the same for both."

Only one of the six students in this group was considered to have a weak formal mathematics background. This was the one who made the erroneous claims about all boys occurring 50% of the time independent of sample size. In this group half were male.

Discussion and Student Reaction

Several observations can be made about the strategies and success of preservice mathematics teachers in solving the hospital problem. Their success rate (55%) is not surprising considering other results for tertiary students. This did not differ greatly for females (50%) or males (58%). What may be considered surprising is the apparent lack of attempt to engage both intuition (or previous experience) and mathematical skills. Only six students (18%) appeared to be working with both. Those who used intuition only may not have encountered the binomial or normal distributions before but all appeared to understand subsequent discussions of these when presented by other members of the class. They also may not have had immediate access to sources that could supply needed information or felt that the problem was not intended to be *that complicated.* That those who used intuition only were only successful about half of the time points to the need to make students aware of the pitfalls of not looking for mathematics to back up intuitions.

The students who presented mathematical arguments with no hint of using intuition or previous experience were more successful (67%) but those who made errors that led to the wrong conclusion illustrate the difficulty when there is no intuition or previous experience to test against one's mathematical calculations.

Of the six who employed both intuition and mathematics, half reached the correct conclusion. These used formal mathematics related to the model appropriate for the hospital problem. The others based arguments on basic mathematics that was misinterpreted in the complex context. The response that indicated the chances of all-boy samples were 50%, was similar to responses about four coin tosses being tails observed by Moritz and Watson (2000), for between 28% and 49% of students in grades 6 to 11. That a preservice teacher has the same difficulty as many students points to this as an area for research based on instructional intervention.

Of the 10 students considered to have weak formal mathematics backgrounds, eight used intuition or previous experience alone to solve the problem, one used formal mathematics only, and one appeared to mix intuition and basic mathematics (unsuccessfully). Overall half of these students were successful, a result not very different from those considered to have a strong mathematics background (57%).

Student reactions to the correct response during class discussion were mixed with some who had used intuition at first seemingly overwhelmed by the theoretical solution. Several mathematics majors lamented their lack of practical experience in statistics that would have assisted in making judgments about sample size. This was summed up by the PhD student who said the following in his evaluation of the case study.

> ...there is no substitute for exposure to practical problems when it comes to understanding statistics. Even though I have completed a few courses in statistics at university level I still found that my intuition let me down when it came to working on this problem.

He further claimed in class discussion that he had never carried out any sampling during his study of statistics.

Several students made comments suggesting they had difficulty because the problem stated the extreme result in terms of boys' births only. One claimed that the problem should be reworded to include girls but then went on to solve it correctly using intuition only. Not appreciating the symmetry of the problem in terms of boys and girls was one of the features included in the Harvard case study (Merseth & Karp, 1997) as a difficulty for high school students. This point was well taken by the preservice teachers and their own difficulties enhanced the relevance of the case study. The fact that a few of the BTeach students experienced the same difficulties with the relationship of percentages and actual numbers as the students in the case study, reinforced the need to concentrate on percentages in the early years of secondary school.

During the class discussion one of the students who had reached the wrong conclusion would not consider other students' correct intuitions until his original mathematics was proved wrong. Having stated "$\mathrm{Bi}(n,\theta) \approx \mathrm{N}(n\theta, n\theta(1-\theta))$", he forgot the square root for the standard deviation. He thus concluded that $z = 0.2$ for all n and hence both hospitals have roughly the same number of days. Once convinced of his error, however, he quickly produced the following argument for the class.

> As $\mathrm{Bi}(n,\theta) \approx \mathrm{N}(n\theta, \sqrt{(n\theta(1-\theta))})$, for a value of $\theta = 0.5$ and an x-value of .6 or more boys, $z = (0.6\mathrm{n} - 0.5n)/\sqrt{(n(0.5)(0.5))} = 0.2\sqrt{n}$. Hence as n increases the z-value increases. Since we are interested in $\mathrm{P}(z \geq 0.2\sqrt{n})$, when $n = 15$, the probability is greater than when $n = 45$.

A suggestion made in the Harvard case study (Merseth & Karp, 1997) was to use 15 coins and 45 coins to simulate the births in the two hospitals. The preservice teachers immediately saw the value of this analogy but also realised that it might be difficult to implement in a classroom. Less noisy ways to provide simulations involved using computer software or a graphics calculator. The probability simulation software for Macintosh, *ProbSim* (Konold & Miller, 1993), provided the opportunity to see samples being created. The outcomes were plotted by the class to document the relative occurrence of extreme results for different sized hospitals to observe the trend with increasing n. Windsor (1998) provides the instructions necessary to carry out the simulation on a TI-83 and discusses using it with senior secondary students. BTeach students were given the opportunity to trial this method as well. One of the students who had used theoretical mathematics incorrectly produced a spreadsheet simulation in *Excel* that for one year found extreme results occurring for a 15-baby hospital on 118 days, a 30-baby hospital on 68 days, and a 45-baby hospital on 36 days. The PhD student noted above wrote a *Mathematica* script to simulate a year's daily births at each hospital. Running this program for 1000 years produced average values of 42.7 days for the large hospital and 110.6 days for the small hospital for the extreme results. In the end, based on their own experiences and the case study, all students agreed to the importance of using some kind of simulation in conjunction with the theoretical mathematics for the hospital problem.

For people preparing to be mathematics teachers it is disappointing that so few naturally mixed intuition with an attempted mathematical justification. The case study setting of a school class provided an excellent opportunity to discuss this point, as well as the mathematics teachers' role in modelling the relationship between intuition and justification. In the area of chance and data in particular it is essential that preservice teachers be exposed to situations where natural intuition without previous experience may lead to erroneous conclusions. This should reinforce the importance of both types of experiences: intuitive and mathematical. It is only when one has both that one can be sure that the mathematics makes sense.

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The Use of Teacher Action Theories in the Articulation of Practice: The Use of Stencils in Upper NSW Primary Classrooms

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Research studies have reported upon the difficulties teachers' display in their articulation of classroom practice and teaching values. Other studies have reported inconsistencies between a teacher's espoused theory and their theory of action. Argyris and Schon (1974) claimed that behaviour was driven by individual action theories. This study used a modification of the Theory of Planned Behaviour (Ajzen, 1988) to uncover the teacher action theories regarding the use of stencils in primary mathematics classrooms from a selected sample of NSW upper primary teachers ($N = 115$). It will discuss these theories and their use in assisting teachers in their critical reflection and articulation of current practice.

The substantial body of research literature upon effective schools (Sammons, Hillman, & Mortimore, 1995) has moved to factors other than structures and resources to account for differences in student performance (Owens, 1998). One of these factors was the influence of the teacher. There have been attempts to quantify the importance of teachers towards student performance. Wyatt (1996) using a meta-analysis of over 80 British and Dutch school effectiveness studies suggested that only approximately 9 per cent of the total variance in student performance could be explained by the effects of attending different schools. Most of the variance was explained by differences attributable to membership of individual classes, which implied that it was teachers and not schools who made the difference in student learning. Other studies support this implication. For example, Hill and Rowe's (1996) study into school and teacher effectiveness and public examinations revealed that differences between classes and faculties within schools were larger than differences among schools. An Australian study also reported that the order of importance of factors in school effectiveness began with the staff, followed by ethos, curriculum and resources (Piper and McGaw, in McGaw, Piper, Banks, & Evans, 1992, p. 167). Thus it was not surprising that Ayres, Dinham and Sawyer (1998) in their summary of school effectiveness research stated "it has been seen that teachers and individual classrooms within schools are where attention should be focussed if effective teaching and student success is to be better understood" (p. 9).

While there was considerable research concerning novice and expert teachers (Berliner, 1994), there was also a problem with asking expert teachers to describe their classroom behaviour as:

> the expert might be characterised as often arational. Experts have an intuitive grasp of the situation and seem to sense in nonanalytic and nondeliberative ways the appropriate response to be made. They show fluid performance, as most people do when they no longer have to choose their words when speaking, or think about where to place their feet when walking ... When things are going smoothly, however, experts rarely appear to be reflective about their performance (p. 6022).

Perhaps as a result of a lack of reflection, experts and other teachers find it difficult to articulate what they do in the classroom. The issue of articulation of practice is not the only problem faced by research into teacher behaviour. There is a considerable body of research concerned with the linking of teacher beliefs with action (Pajares, 1992) Research studies have described inconsistencies between the two (Thompson, 1992). Does this mismatch arise only from the difficulties experienced by teachers in articulating their behaviour or are there other issues that deserve consideration?

Another problem is associated with a lack of research that focused upon either values or valuing (Bishop, 1999). It was suggested that one reason for this lack was that teachers were rarely aware of their teaching values and this had implications for teacher change. Values have been described as the deeply held beliefs that dispose a person to act in a certain way (Hill, 1991). Others regard values as deeper and more stable than beliefs or attitudes (Seah, 1999). In either case, there was agreement that values or value systems influenced both beliefs and attitudes, and that actions helped to communicate these values.

The difficulty for studies reported above concerned the link between beliefs and behaviour which was claimed to be a complex mix of intentions, attitudes, social influences, evaluations and motivations (Ajzen & Fishbein, 1980). Argyris and Schon (1974) stated that behaviour was driven by individual action theories. In their early research they identified and described espoused theories and theories-in-use and expected that individuals would not design and implement a theory-in-use which was significantly different from their espoused theory. What they discovered was quite the opposite. Not only were there significant differences between the two types of theories but "individuals develop designs to keep them unaware of the mismatch" (Argyris, 1993, p. 51). Thus, change could not occur until these theories were uncovered and made available for examination and reflection.

It has been noted that the aim of conventional mathematics instruction was the presentation of content in a linear fashion of carefully sequenced topics. This was then reinforced by the use of stencils and the mathematics textbook, which followed a similar format of skill building (Clarke and Sullivan, 1990, p. 165). While stencils are used extensively in NSW primary schools, there was little research available into the intentions, attitudes, beliefs of teachers and the use of stencils. A recent study (White, 1996) defined a prepared stencil as a teacher or commercially produced sheet containing a set of questions with spaces provided for working and/or answers. The results of this phenomenographical study identified and described a range of teacher beliefs for using and for not using stencils in an upper primary classroom. The results are contained in tables 1 and 2 below.

Table 1
Teacher Beliefs for Using Stencils

My decision to use stencils is because they:
1. would save time in preparation and marking.
2. would assist assessment and evaluation of whole class.
3. would assist the ranking of individual students.
4. would allow targeting of specific student needs.
5. would provide enrichment and extension activities.

However the study was unable to sort between beliefs that were the basis of teacher behaviour and beliefs that were the basis of teachers' espoused theories. The aim of this current paper is to discuss the use of teacher action theories (White, 1999) to assist the sorting needed by teachers in their articulation of current practice. By making explicit the complex mix of beliefs, attitudes, intentions and motivations that underpin classroom behaviour, teacher action theories provide a vehicle for uncovering and making available the basis of teacher behaviour for examination and reflection.

Table 2

Teacher Beliefs for Not Using Stencils

My decision to not use stencils is because they:

1. would waste paper and time - easier to use the blackboard
2. would not be relevant to some activities such as measurement
3. would be too narrow a medium - just the use of pen and paper
4. would be boring and repetitive
5. would not be suitable for students with weak reading and language skills
6. would make it easier for students to cheat

Theoretical Issues

Teacher Action Theories are based upon the Theory of Planned Behaviour (TPB; Ajzen, 1985, 1988) which is an extension of the Theory of Reasoned Action (TRA; Ajzen & Fishbein, 1980). Both have been very successful in using a small number of beliefs to predict behaviour across a wide range of contexts and both come from an objectivist (positivist) stance, which uses inferential statistical procedures. This stance has been the target of a great deal of criticism (see for example Lather, 1991) and it is beyond the scope of this paper to address this criticism.

The models TRA, TPB and Teacher Action Theory are described in detail in an earlier paper (White, 1999). In brief the immediate determinant of behaviour was intention and the three major factors that determined a teacher's behavioural intention were a personal or attitudinal component, a perceived control component and a social or normative component. The attitudinal component measured the teacher's attitude towards performing the behaviour and was "simply a person's general feeling of favourableness or unfavourableness for that concept" (Ajzen & Fishbein, 1980, p. 54). The normative component of the theory dealt with the influence of the social environment upon intention and behaviour. This was a measure of the teacher's perception of whether most important people supported or didn't support the performance of the behaviour. The third component was a measure of the degree of perceived control a teacher had over the performance of the behaviour. This implied that a teacher would usually intend to perform a behaviour that was positively evaluated, had the support of significant others, and was under control.

The three components could be investigated further. Attitude towards behaviour was determined by the product of the teacher's beliefs about the consequences of performing the action (behavioural beliefs) with an evaluation of the consequences (outcome evaluation). Subjective norm was determined by the product of the teacher's perceptions of social expectations to perform the behaviour (normative beliefs) and the motivation to comply with the perceived expectations (motivation to comply). The control component included either self efficacy beliefs that dealt with confidence in the performance of a behaviour or perceived behavioural control beliefs that dealt with the perception of control, and it was here that the model differed with TPB. Correlation data between the two items were very weak and suggested that the two items were not related linearly and were measuring two distinct concepts. As a result, the item that made the greatest contribution to explained variance was included.

Research into the domain of values, shared similar concerns with the desire to understand behaviour. Values are considered in the literature as deeply held beliefs that influence the individual to act in certain ways. Hill (1991, p. 4) proposed that there were three key elements to valuing:

1. A value could be described by a statement expressing a person's belief in an idea but judging its worth (value judgment).
2. The intensity of belief was considered to be the person's attitude.
3. The deeply held beliefs disposed a person to act in certain ways. These acts were consistent with the person's value system.

This set of propositions divided beliefs into two sets. Those beliefs that direct behaviour, are regarded as values, and which Argyris and Schön regard as contributing to an individual's theory-in-use. Those other beliefs that are not values, but are known by the individual, Argyris and Schön would regard as possibly contributing to an individual's espoused theory. Certainly, teacher action theories have used these ideas within its structure. For example, one construct of attitude was constituted from a product of both belief strength and outcome evaluation. Thus values proposed by Hill and others (Bishop, 1999; Seah, 1999) are beliefs reflected in action, and as such are a part of the basis of teacher action theories.

Method

A questionnaire was constructed according to TPB using six beliefs chosen from the positive and negative beliefs about the use of stencils. This was given to a sample chosen randomly from across the population of primary teachers who worked at a NSW Department of School Education primary school and who taught mathematics to years 5 and/or 6. A modification involved the inclusion of items to measure both self efficacy and perceived behavioural control of the teachers. The modification arose as a response to perceived confusion within the research literature (Fishbein & Stasson, 1990, p. 196). A total of 119 teachers returned the questionnaire and the data were entered into a spreadsheet and the software program Statview 4.0 was used to analyse the data. A decision was made not to include four returns because of the amount of missing data. The questionnaire construction is described in detail elsewhere (White, 1999). An attempt was made to collect data concerning performance of behaviour but it was possible for only 51 subjects of the sample.

Results

The demographic results of the sample if summarised using modal characteristics indicated that the teacher was most likely to be female, aged between 36 and 45, with 11 to 15 years teaching experience and with a class consisting of only year 6 students. The descriptive statistics of the constructs of the teacher action theory are listed in Table 3 and show that the majority of teachers intended to use stencils within the classroom.

The teacher action theory for the use of stencils is illustrated pictorially in Figure 1 and shows the correlation scores between the components of the model. They indicate a strong linear association between intention and attitude (r = 0.76), no linear association with subjective norm (r = 0.00) and a weak negative one with perceived behavioural control (r = -0.22). The correlation results involving the belief based measures showed a moderate linear relationship between normative beliefs and subjective norm (r = 0.45) and a weak one between behavioural beliefs and attitude (r = 0.26).

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Table 3

Constructs of the Teacher Action Theory: Mean Values and Standard Deviations

	Mean	Standard deviation
Attitude toward behaviour		
Direct. DAI	1.8	3.9
Belief product. BBAI	11.7	13.0
Subjective norm.		
Direct. SND	- 0.1	2.9
Belief product. SNB	0.4	8.3
Control belief. PBC	2.5	0.8
Behavioural intention	1.0	1.5

Note. The table contains both the direct measures and the summed product measures of attitude and subjective norm. For attitudes the direct measure range is -9 to 9 and the product measure is -54 to 54. The subjective norm direct measure range is -3 to 3 and the product measure range is -36 to 36. The behavioural intention range is -3 to 3. The control belief range is -3 to 3.

Multiple regression analysis of the sample (N=115) revealed that taken together, attitudes, subjective norm and PBC accounted for 62.9% (adj. R^2) of the variance in intention to use stencils in the classroom. The partial standardised regression coefficients show a moderate influence between intention and DAI ($\beta = 0.78$, $p < 0.0001$), a weak negative influence by PBC ($\beta = - 0.22$, $p < 0.001$) and a very weak negative influence by SND ($\beta = - 0.14$, $p < 0.05$).

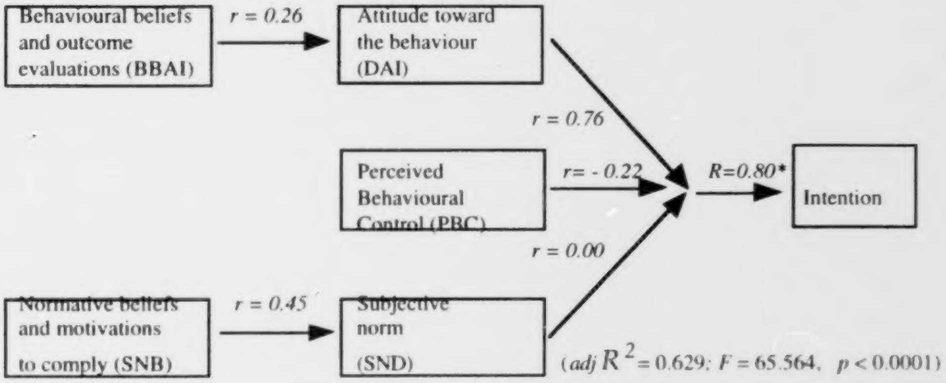
Behavioural beliefs and outcome evaluations (BBAI)
r = 0.26
Attitude toward the behaviour (DAI)
r = 0.76
Perceived Behavioural Control (PBC)
r= - 0.22
R=0.80*
Intention
r = 0.00
Normative beliefs and motivations to comply (SNB)
r = 0.45
Subjective norm (SND)
(adj R² = 0.629; F = 65.564, p < 0.0001)

Figure 1. Teacher action theory for the use of stencils: Relations among beliefs, attitude, subjective norm, control and intention.

Attitude was the strongest contributor to intentions for the teacher action theories. In an examination of both components of attitude the high intent group revealed that they believed the use of stencils was:

1. likely to save time in preparation and marking which was regarded as a good outcome;
2. unlikely to be not relevant to some activities such as measurement and if they weren't relevant then this was a very bad outcome;

3. likely to assist the ranking of students which was a good outcome;
4. unlikely to be boring and repetitive and if they were then this was a very bad outcome;
5. quite likely to provide enrichment and extension activities – a very good outcome; and
6. not related to encouraging or discouraging cheating.

The high intent group reported that they received support from all four significant referents. They believed that the use of stencils in class was encouraged by the Principal, the Parents, the Students and the Other Teachers and that they were inclined to comply with this influence. Their strongest perception of encouragement was from the Parents. The teachers believed they had almost full control over the decision to use stencils, which they regarded as an extremely easy behaviour to perform.

The low intent group indicated they were:

1. undecided whether stencils would save time in preparation and marking which if it did happen was a good outcome;
2. undecided whether stencils were relevant to all activities and it would be an extremely bad outcome if stencils were used that were not relevant;
3. likely to agree that stencils wouldn't assist the ranking of students and if they did then it was a slightly bad outcome;
4. unlikely to agree that stencils would be boring and repetitive and if they were then it was an extremely bad outcome;
5. likely to agree that stencils provided enrichment and extension activities which would be a good outcome; and
6. unlikely to agree that stencils make it easier for students to cheat and if they did then it was a bad outcome.

The low intent group believed the use of stencils in class was discouraged by the Principal, the Parents, the Students and the Other Teachers. They indicated they were likely to comply with the influences of Parents and Students but were ambivalent about the other two. They also believed they had almost full control over the decision to use stencils and yet they regarded it as only slightly easy to perform.

Discussion

Teacher action theories were designed to provide a framework for understanding the basis of teacher behaviour. In general, the high intent group displayed greater belief in the positive beliefs of using stencils. Thus they were stronger in their beliefs that the use of stencils was likely to deliver the positive outcomes and they evaluated the outcomes of these positive beliefs much higher. The negative beliefs, they indicated, were unlikely to happen and they evaluated the negative belief outcomes less strongly. In contrast, the low intent group rated the positive belief outcomes as less likely to happen and the negative belief outcomes as more likely to happen. Their evaluations for the positive belief outcomes were weaker and the negative belief outcomes were stronger. They also indicated that their significant others discouraged stencil use in the classroom. For this group, the difficulties contained in the negative beliefs were too strong to make the effort to achieve the positive outcomes. Thus there are two distinct action theories and as a consequence, it is possible that the teacher action theories may be more effective if they included the full range of positive and negative beliefs rather than a sample. A greater degree of explained variance should be achieved. Thus examination of the quality of teacher action theories emerging from presentation of the full range of beliefs would be a future direction.

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The low influence of subjective norm requires further investigation and could indicate that the volitional control of the teachers was high resulting in the construct of perceived behavioural control being small, which it was. There are a number of further research questions that arise. Are primary teachers existing in a professional isolation where there was little opportunity to watch other teachers at work or to be observed teaching themselves? Do teachers prefer isolation, regarding themselves as the expert in their classroom and ignore input from other sources thus confirming the 'closed door' syndrome (Haberman 1995 as reported in Lumpe, Czerniak, & Haney, 1998)?

In comparison to teacher action theories for other teacher classroom behaviours (White, 1999), for the use of stencils they were moderately strong in relation to the amount of explained variance. While the results have been a little disappointing, they have provided some directions for further development and reflection.

Conclusion

The inability to articulate successful teaching practice has serious educational implications.

> It follows that if such teachers experience difficulty in identifying what they do, and how they acquired their craft knowledge and skills, their chances of passing on this tacit knowledge must obviously be problematic, particularly due to the professional isolation that seems to characterise the operations of so many teachers who rarely have the opportunity to either watch other teachers at work or to be observed teaching themselves (Ayres, et al, 1998, p. 12).

Teacher action theories have the potential to provide a framework for the articulation of current classroom practice. The need for such a framework has been discussed in terms of the difficulties that teachers have exhibited in their attempts at articulating their practice. The educational implications of teacher action theories, however, go much wider than this need for description.

> problems of teaching and learning are not likely to be solved by attempting to prescribe how teachers and learners should approach their teaching and learning, but by developing techniques and ideas which will help teachers and learners critically reflect upon their present teaching and learning practices (Prosser, 1994, p. 39).

Thus in helping current and future teachers work towards practices that are consistent with the wider goals of the educational system, teacher action theories could play an important part. Teachers would benefit from an examination of the relationships between their beliefs, attitudes, intentions and behaviours. Teacher action theories provide a framework for understanding and discussing teacher behaviour. Early and continued reflection of teacher action theories and practice in both pre-service and inservice programs may provide a further strategy in the process of improving the quality of teacher classroom practice in the teaching of mathematics.

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Collaborative Problem Solving and Discovered Complexity

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Two collaborative groups of senior secondary mathematics students with similar ability to solve unfamiliar challenging problems demonstrated different levels of engagement and different levels of conceptual development when they worked with the same task. Study of the nature of task complexity led to the formulation of *discovered complexity* as a useful tool to analyse student response to tasks. It was found the task undertaken provided the opportunity for discovered complexities that possessed the potential to enhance student learning. Use of such a task was found insufficient to ensure discovered complexity resulted.

Introduction

As a teacher of senior secondary mathematics I gradually recognised my original teacher-centred approach catered inadequately for students in a mixed-ability mathematics classroom. Some students did not have access to the curriculum because they lacked the required background. In my opinion, too many students felt they were failures at mathematics and too few students were challenged and motivated by the mathematics presented. Over a period of years I developed my own collaborative approach (Class Collaboration explained in Table 2) and found students often demonstrated a high level of engagement with the task and articulated an increased understanding of mathematics. This collaborative approach to mathematics learning excited students of both genders:

> I just love maths now. It makes such a difference when you can explain what you understand to each other. I get so excited when we solve problems. I know I can now solve them by myself.
> (Year 11 female student)

> It is so exciting when you can report something you know no one else [no other group] has found.
> (Year 12 male student)

M. Csikszentmihalyi's (1992) concept of flow is consistent with the engagement I observed as students in some collaborative groups became so involved in the task that they lost all consciousness of time, self and the world around them. Naidra, a Year 11 student studying mathematics through Class Collaboration (Barnes, in Press), described breakthroughs or insights in mathematics as "magical". He claimed that such moments often occurred for him and that when they did it was "great". M. Csikszentmihalyi and I. Csikszentmihalyi (1992) found people in flow described such feelings of pleasure and exhilaration and that optimal learning conditions existed. Either individual or group flow could occur when people worked just above their present skill level with a challenge almost out of reach. The optimal learning conditions that accompany group flow are consistent with the increased conceptual development that can result when students collaborate in socio-cognitive settings (Bell, 1993; Brown, 1994; Cobb, Wood, Yackel, & McNeal, 1992).

As "No activity can sustain it [flow] for long unless both the challenges and skills become more complex" (Csikszentmihalyi and Csikszentmihalyi, 1992, p. 30), the nature of task complexity and how the complexity of the task might change are integral to studies of sustained student engagement. In an attempt to understand this link between task complexity and student engagement, I undertook research in my own classroom posing the question: What is the nature of task complexity and how do students respond to this complexity when problem solving is undertaken in a collaborative setting?

The term *complex* is frequently used to describe tasks (Schoenfeld, 1985; Tang, 1993) without explicit definition of *complex*. Where an implicit or explicit explanation of the nature of task complexity exists, task complexity is most frequently equated with intellectual complexity (Smith and Stein, 1998; Cohen, 1994; Tannenbaum, 1983). Advocates of the use of complex mathematical tasks for either instruction or assessment presently lack a structure (and a vocabulary) by which to identify and evaluate the different characteristics of such tasks and their relation to the sort of student activity they promote. An understanding of those aspects of task complexity that facilitate student engagement and enhance conceptual development will be of assistance in task construction, task selection and the implementation of curriculum related to such tasks. The present scarcity of definition in the literature raises questions about the usefulness of the term 'complexity' as utilised in such documentation as VCE Mathematics 2000 (Victorian Board of Studies, 1999).

Theoretical Constructs and Methodology

In this study, the term *unfamiliar challenging problems* refers to tasks that: (a) are presented before the relevant mathematical concepts have been 'taught'; (b) cannot be solved by the application of algorithmic procedures assumed known to the students; and (c) require students to analyse mathematical representations to connect mathematical ideas and to build concepts new to the them.

A distinction is made between a student's capacity to solve unfamiliar challenging problems (*Student Ability*) (Krutetskii, 1976) and the demonstrated problem solving achievements of students in a collaborative setting as they solve a problem together (Tang, 1993). Whether Student Ability is innate, dependent on previous problem solving experiences or a combination of such factors is immaterial to this study.

Table 1.
Theoretical Hierarchical Construct of Ability Formulated using Bloom's Taxonomy (1956) and Krutetskii (1976)

Categories of Ability	Hierarchy of mental activity activated (in descending order)
Evaluative-synthesis	Recognise inconsistent information
Synthesis	Combine two or more concepts to create an original concept
Analytic-synthesis	Explain the need for extra information Use more than one pathway
Analysis	Recognise the need for extra information Build on a concept to answer a question with a slight change
Comprehension	Understand a concept
Knowledge	Repeat a taught idea

To ascertain the relative Ability of the students in this study, a theoretical construct of Ability was formulated using a hierarchy of cognitive activities based on Bloom's Taxonomy (1956) and Krutetskii's (1976) empirical data. These cognitive activities are displayed in descending order in Table 1. Student capability with respect to this hierarchy was measured using a pen and paper test designed to measure Student Ability. These results informed my selection of collaborative groups with similar Student Ability distributions for case study. The construct of cognitive abilities developed to design this test focused my analysis of student response to the task.

Collaborative instruction operated at two levels within the classrooms studied. Each small group collaborated as they worked with the problem but the class as a whole also collaborated to share ideas at regular intervals. The main characteristics of the Class Collaboration approach are detailed in Table 2 and Williams (1997).

Table 2.
Features of the Class Collaboration Approach and Elaboration of these Features

Feature	Elaboration of this Feature
The learning culture	Build from each student's present understanding Any justified solution pathway is acceptable
Type of task	Open ended task undertaken over several lessons A maximum of ten minutes teaching prior to task
Classroom management	Small collaborative groups Regular feedback from each group to whole class
Questioning technique	As groups develop ideas, the teacher: Visits each group regularly for 30 seconds to 2 minutes; Listens to and sometimes participates in discussions; Demonstrates an interest in the students' thoughts; Asks questions to assist groups to clarify, refocus upon a productive pathway, experience an insight, analyse and evaluate progress; Shares the excitement when groups find new ideas; Does not provide hints nor indicate correctness of pathway.
Reporting process	Class members may ask the reporter for clarification. A group may discard, use or build upon ideas presented.

The nature of task complexity was explored through interviews with experts from the fields of Mathematics, Mathematics Education and Gifted Education (*Experts*). These Experts provided opinions about the relative complexity of two unfamiliar challenging problems. The purpose of this comparison was not to draw conclusions as to which task was more complex in any absolute sense, but rather to use both tasks as a mechanism for eliciting the opinions of these Experts about the nature of task complexity. Experts found the six dimensions of complexity within the Williams / Clarke Framework (linguistic, contextual, operational, conceptual, intellectual, and representational complexity) sufficient to identify the dimensions of complexity within the tasks (Williams & Clarke, 1997).

Although Experts differed in their opinions about the nature of task complexity all Experts included some aspect of Intellectual Complexity in their description (Williams & Clarke, 1997). *Discovered Complexity* emerged as Expert opinion was analysed. It is a major premise of this study that an analysis in terms of discovered complexities advances our understanding of both student collaborative activities and the function of mathematical tasks in promoting such activity. *Discovered complexities* are not apparent at the beginning of the task but become evident during task performance. These discovered complexities possess two key features that meet Csikszentmihalyi's (1992) stated conditions for flow; (a) students focus on a search to answer a question implicitly or explicitly formulated by the group (intellectual challenge); and (b) this search encompasses mathematical ideas and concepts new to all group members.

Prior to this research, I devised the task undertaken by the students in this study, 'Understanding the Double Derivative', to allow students to develop their own concept of why the second derivative in combination with the derivative gives information about the shape of the curve. A prerequisite for this task is the knowledge that the derivative of a function is the gradient of its graph. The task also assumes that students have done no prior work on second derivatives (Williams & Clarke, 1997).

The research subjects were students in two classes studying high level final year secondary mathematics (Specialist Mathematics) in a large Victorian metropolitan government school. These students had undertaken a Gradient Investigation prior to the research period and the Ability test at the commencement of the research period. The Gradient Investigation was intended to provide all students with a common background in the mathematical ideas prerequisite for both the Ability test and the second derivative task. Without such common background, valid inferences could not be drawn about students' cognitive activity as they responded to the Ability test.

Group 1 (Talei, William and Gerard) from Class 1 and Group 2 (Dean, Alistair, Tony and Rez) from Class 2 were selected for case study because these two groups had similar Ability distributions, but demonstrated different learning outcomes. On the Ability test, the majority of the members in each group demonstrated the ability to 'combine concepts to create an original concept'. William (Group 1) also demonstrated the higher level ability to 'recognise inconsistent information' and the highest cognitive activity demonstrated by Rez (Group 2) was the ability to 'use more than one pathway' (see Table 1).

Both groups undertook collaborative work and individual work, but in the opposite order. Changes in each student's level of mathematical understanding were monitored by tests of mathematical understanding administered before and after each instructional period. Students were video taped as they responded to the task during Class Collaboration. The mathematical focus of each discussion was identified and the time devoted to this topic recorded (Tables 3 and 4). I concluded that a student had improved his or her development of a concept if the understanding displayed in the video and the post-test of mathematical understanding, given after the collaborative work, was deeper than the understanding displayed on the pre-test of mathematical understanding (Skemp, 1976, 1979). To draw inferences about student engagement, relevant aspects of body language were considered (Quilliam, 1995) and the validity of my interpretations of body language was checked through discussion with four colleagues undertaking their PhD in Education.

Results

Group 1 (Talei, William and Gerard) undertook Class Collaboration before individual work. Table 3 summarises the nature and number of complexities discovered by Group 1 in the first thirteen minutes of collaborative group work on Task A and during the extra minute that student discourse continued after the prescribed time interval had ended.

Group 1 worked for short time intervals to develop each of many mathematical concepts as they resolved the focus questions related to a variety of discovered complexities. My explicit formulation of the spontaneous question associated with each discovered complexity, in Table 3, demonstrates the progressive development of conceptual understanding. As all three students participated in the initial discussions related to each discovered complexity, it can be assumed conceptual development was realised by each of the three students.

Table 3.
Group 1, Mathematical Content of First Collaborative Session (Talei, William, and Gerard)

Amount of Time	Mathematical Content (emergent in student dialogue)	Complexities and * Discovered Complexities
4 mins	Construction of gradient graph. Sign Diagram: meaning and construction.	Intellectual (analytic-synthesis), Conceptual, Representational.
2 mins	Return to problem to clarify. Analyse graphs and diagrams searching for patterns.	Linguistic, Intellectual (understanding, analysis).
1 min	What are the relative benefits of a sign diagram and a graph?	**Discovered Complexity*
	What characteristics of *f(x)* are evident (shape) and are not evident (position) through analysis of $f''(x)$? Analysis through algebra of extra information required.	**Discovered Complexity*
2 mins	What do the critical features of $f''(x)$ indicate (inflections) and not indicate (turning points) about *f(x)*?	**Discovered Complexity*
1 min	Continue searching for patterns. $f''(x)$ shows inflection but not turning points. Why?	Intellectual(analysis) **Discovered Complexity*
2 mins	Further analysis of relative benefits of sign diagram and graph	**Discovered Complexity*
1 min	How much information is required to generate *f(x)* from $f''(x)$?	**Discovered Complexity*
1 min	Why isn't the turning point of *f(x)* a critical feature of the graph of $f''(x)$?	**Discovered Complexity*

Note. *All Discovered Complexities included Conceptual Complexities, Representational Complexities and higher level Intellectual Complexities.

In the minute of collaborative group work, after the buzzer, Talei and William (and initially Gerard) displayed a high level of engagement as they investigated the discovered complexity precipitated by William's statement that there "must be something more". William, Talei and Gerard did not respond to the buzzer but instead continued to lean forward over the page and build upon each other's comments. The three students did not appear to hear the teacher providing instructions about the reporting session. William and Talei did not appear to hear Gerard who had turned eventually to listen to the teacher and then turned to ask Talei and William a question about his forthcoming report for the group. After further brainstorming with Talei, William exclaimed—with the accompanying hand movement tracing a minimum point in the air—"it is always turning the same way". The video record captured the instant where William suddenly appeared to grasp the overall concept. His utterance "That's it!" and the accompanying hand movement together with the apparent expression of enlightenment on his face indicated something had suddenly become clear. Talei's hand movement at almost the same instant indicated she had also grasped the concept. These students were in flow as the concept was constructed and a "jolt of thrills" was evident in William's exclamation (Sato, 1992, p. 92).

Table 4.
Group 2, Mathematical Content of First Collaborative Session (Rez, Alistair, Dean and Tony)

Amount of Time	Mathematical Content (emergent in student dialogue)	Complexities and * Discovered Complexities
4 mins	Derivatives investigated and generalised algebraically .	Numerical , Intellectual (comprehension, analysis)
5 mins	Comparison of $f(x)$ and $f'(x)$ through interpretation of graph with focus on gradients. Explanation of the change in gradient, disagreement with Rez's $f'(x)$ graph.	Representational, Intellectual (comprehension, analysis, evaluation)
2 mins	Exploration of the possibility of a hybrid graph.	Representational, Intellectual (analysis, synthesis)
	Discussion of relative magnitude of the gradient at two points.	Intellectual (analysis), Conceptual, Representational
2 mins	Require links between $f(x)$ and $f''(x)$. Gradient of the gradient discussed. Can a negative gradient be increasing?	Intellectual (comprehension) Linguistic, Intellectual **Discovered Complexity*
	Tony: '$f''(x)$ gives curve of graph'. No response from other students.	Conceptual, Intellectual (synthesis)

Students in Group 2 appeared to develop misconceptions during the individual work they undertook before collaborative work and retain these during collaborative work. These misconceptions were evident in student responses to questions, student explanations of diagrams, dialogue in the subsequent collaborative session and responses to tests. The development of these misconceptions affected the commonality of background achieved by the students through participation in the Gradient Investigation.

Group 1 discovered more complexities, maintained a higher level of engagement in the task, and developed an understanding of a greater number of concepts. This group continually worked just above their present level of mathematical understanding, moving through their individual *Zones of Proximal Development* (Vygotsky in Wertsch, 1984) as they brainstormed to create new mathematical ideas together. Group 2 discovered only one complexity (Table 4) and generally worked at a level below the present mathematical understanding of one or more group members. Rather than building new ideas together, there were frequent demands for further explanation and challenges of other student's ideas.

Discussion and Conclusions

It is a major proposition to emerge from this research that the process of recognising and exploring *discovered complexities* in a task is a fundamentally constructive and connective activity that inevitably promotes student awareness of the interrelationship of task components and maximises a form of conceptual connectedness. The discovery of complexities is idiosyncratic to the particular group; students do not necessarily discover the same complexities as they solve a problem. A task that provides opportunities for discovered complexities has the potential to optimise student learning but the use of such a task is not sufficient to ensure complexities are discovered.

The use of a task that provides the opportunity for students to discover complexities in a learning environment like Class Collaboration supports and encourages student autonomy and promotes mathematical exploration. This autonomy makes the actual direction of the groups' exploratory activities more difficult to predict. The uncertainty in the direction a group might explore and the differences in the mathematical experiences of each collaborative group create a new set of challenges. Educators could support the implementation of such an approach by addressing issues such as: (a) how to plan a curriculum; (b) how to deliver this curriculum within the time constraints of the school; and (c) how to improve some teacher's perceived lack of capacity to keep pace with the developing ideas within the different collaborative groups.

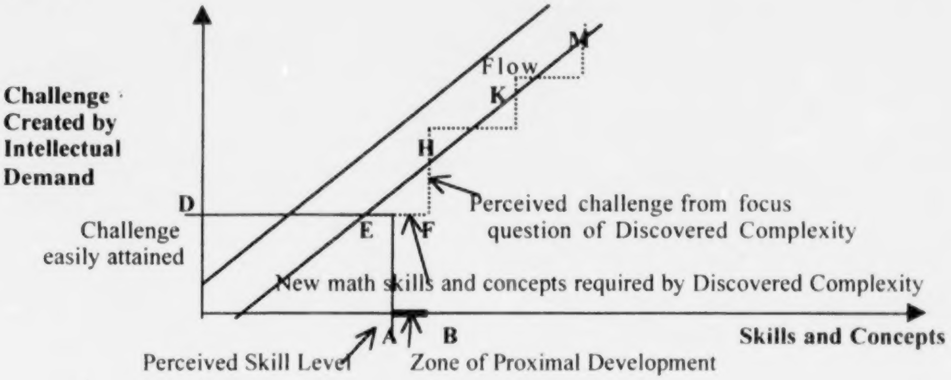
Challenge Created by Intellectual Demand
D
Challenge easily attained
Flow
M
K
H
E
F
Perceived challenge from focus question of Discovered Complexity
New math skills and concepts required by Discovered Complexity
A
B
Skills and Concepts
Perceived Skill Level
Zone of Proximal Development

Figure 1. Explanation of sustained engagement integrating Discovered Complexities and Zone of Proximal Development into Csikszentmihalyi's representation of optimal conditions (1992, p. 259).

Tasks providing the opportunity for discovered complexities contain a dynamic aspect that could explain the sustained engagement of members of a collaborative group. An extension of M. Csikszentmihalyi and I. Csikszentmihalyi's (1992, p. 259) schematic representation integrates flow, the zone of proximal development and discovered complexity (Figure 1). The horizontal axis indicates the student's perceived level of skills and concept development and displays the student's present potential for new learning. The vertical axis represents the intellectual challenge the student presently perceives. A student, positioned at **E**, who perceives their skills and concepts level is **A** and the challenge they can easily attain is **D,** is not in flow (between the parallel lines) because this student is working at their present skill level and the challenge is easily within reach. The discovery of a new complexity would position this student in flow at **H** as new mathematical skills and concepts are required (horizontal distance **EF**) and an appropriate challenge is present (vertical distance **EH**). The progressive discovery and familiarisation with complexities sustains flow by moving the student from **H** to **K** to **M**.

The small number and the specificity of the groups in this study would suggest the results should not be generalised without further investigation. Research by Brown (1994) and Tang (1993) with Grade 2 science students and first year tertiary physiotherapy students, respectively, supports the association between student autonomy, student engagement and learning gains. Further research could utilise *discovered complexity* as an analytic tool to explore student response to unfamiliar challenging problems within a broader curriculum context using a variety of tasks, students representative of a wider range of Abilities, and collaborative groups composed of a range of gender proportions. Such a study could extend our understanding of how to create student engagement whilst maximising conceptual development.[1]

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Changing Immigration Patterns and Teacher Perceptions of Responses in Mathematics Classroom Education in the Last Fifty Years

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Research indicates that appropriate mathematics teaching is connected to an understanding of and an interactive relationship with the cultural environment in which it takes place. The cultural environment of Australia's mathematics classrooms has changed dramatically in the last fifty years due largely to an influx of migrant students of diverse origins. This paper investigates the changing patterns of migration over this period and their links with mathematics classroom environments through documentation of the perceptions of a group of Victorian mathematics teachers. Also investigated were perceptions of any responses through the provision of migrant related resources by mathematics educators and of their own roles in generating responses to the migrant presence in their classrooms.

As part of a wider study of the response of Australian mathematics educators to the arrival of students of diverse ethnic origins in mathematics classrooms, this paper provides some details of the patterns of migration to Australia over the last fifty years. The consequent effects of these patterns on the composition of the mathematics classroom together with the nature of any responses related to classroom mathematics teaching emanating from mathematics educators and from mathematics teachers themselves as perceived by classroom mathematics teachers, has been investigated.

Theoretical Framework

A national overview of the ethnic and birthplace composition of Australia's population as revealed by the 1991 Census and published in the Atlas of Australian people (Hugo and Maher, 1995) indicated that, out of the population, current at that time, which numbered 17.3 million, some 26% of Australia's population were born outside Australia , placing it second only to Israel in the significance of immigration to the population. A further 30% were the first generation Australian born of one or both immigrant parents and many more had varying degrees of lineage traceable to ethnic backgrounds.

Recent research has negated the notion that mathematics education is a universal and culture free entity and its findings have moved to the notion that appropriate mathematics education requires an interactive relationship with its cultural environment (Abraham & Bibby, 1998; Bishop, 1994; Bishop, Brew, Leder, & Pearn, 1997; Brent, 1978; Clements, 1989;Ernest, 1991; Tanner & Trown, 1979, Thomas, 1997). In Australia, as a result of significant immigration to its shores beginning directly after the end of World War 2 , the cultural environment of the mathematics classroom changed substantially; initially through the changed ethnic origins of its students, and later through the changed ethnic origins of a significant number of its teachers. This paper is part of a study of the extent to which mathematics educators recognized and responded to these changes in the classroom.

Immigration Statistics

Where did these migrants settle in Australia? According to the results of studies of regional population growth in Australia from 1986 to 1991 during which time the national population grew from just over 16 million to 17.3 million the locations of migrants were unevenly distributed. Settlement took place in just a few locations, predominantly within two states, Victoria and New South Wales. In 1993 these two states accommodated 73 per cent of all immigrants, considerably more than the 60 per cent of the national population who lived in them. The Australian Bureau of Statistics (ABS) estimated that more than 80 per cent of immigrants to Australia have settled in the five largest cities and that 60 per cent have located in Sydney or Melbourne. Migrant students therefore are concentrated largely in relatively small surface areas which are urban and densely populated (Maher & Stimson, 1994). Access to migrant students could not have presented difficulties in terms of educational resources.

Post World War 2 Australia accepted in excess of five million immigrants but this has varied considerably on an annual basis as is demonstrated in Table 1 (Easson, 1990). It can be seen that there were peaks in intake in the early 1950s, around 1970 and to a lesser extent in the early 1990s. These peaks were followed by troughs, most markedly in the mid-1970s, and 1992, 1993 (the latter years not recorded in Table 1) Were there also fluctuations in the origins of migrants?

Table 1

*Immigration Trends 1948/49 – 1988/89**

Year of intake	Troughs	Peaks
1947/48	40 000–60 000	
1948/49		180 000–200 000
1953/54	80 000–100 000	
1955 /56		120 000–140 000
1961/62	60 000-80 000	
1969/70		160 000–180 000
1975/76	40 000–60 000	
1981/2		100 000–120 000
1983/84	60 000–80 000	
1989/90		120 000–140 000

*Australian Immigration Consolidated Statistics.

The peak between 1948 and 1952 was made up primarily of European born immigrants the largest contingents coming from Great Britain, Greece, Italy and Yugoslavia. While the first Immigration Minister, Mr Arthur Calwell, pledged his support of the white homogeneity of the White Australia policy, immigration became less linked with population politics and was seen as a way of removing labour market shortfalls. The next big peak between 1963 and 1974 was connected with the liberalisation of entry requirements by the then Minister Mr Harold Holt and with it the last vestiges of the White Australia policy began to disappear in favour of an explicit policy aimed at the provision of a reservoir of labour of a more technological nature in manufacturing.

During later immigrant expansions, the composition of intake changed considerably. In 1966 there were few Asian countries listed in the top ten sourced countries of new settlers in Australia. Ten years later Malaysia and the Philippines ranked fifth and sixth among the top ten sourced countries. By 1986 the majority of the top ten sourced countries were Asian with the highest numbers coming from the Philippines, Vietnam, Malaysia, Hong Kong and China. (Easson, 1990)

Table 2

Top Ten Sourced Countries of Birth Listed in Order from Highest to Lowest Intake

196–67	1976-77	1988-89
UK and Ireland, Italy, Greece, Yugoslavia, Germany, New Zealand, USA, Netherlands, Lebanon, India, other countries combined.	Other countries combined, UK and Ireland, Lebanon, New Zealand, Cyprus, Malaysia, Philippines, Yugoslavia, Greece, Italy, USA	Other countries combined, UK and Ireland, New Zealand, Philippines, Vietnam, Malaysia, Hong Kong, China, India, South Africa, Sri Lanka

Teacher Perceptions

Twent-eight Victorian teachers completed questionnaires relating to their experiences during the period studied. These questionnaires, and an advertisement, were included in the June, 1998 edition of the Mathematics Association of Victoria publication "The Common Denominator". Questionnaires with their responses were returned by interested teachers to the researcher.

Definitions and Perceptions of the Descriptor "Migrant"

The wording of the advertisement and questionnaire was constrained by considerations of space. The complex question of the meaning of the term "migrant" was therefore not defined on the questionnaire document. In the personal interview extension of the project, however, migrant students were defined by the researcher as those students born outside Australia or born in Australia of at least one overseas-born parent. To qualify, these students must have attended Australian schools or were in the process of being educated in Australian schools for a period of at least one year. It became apparent that the definition did not resonate with the mathematics teachers taking part in the project. Their concept of the term "migrant student" was more elusive and concerned ethnic and cultural connections. The use of the term "migrant students" used by most teachers in the project had more to do with a hegemonic viewpoint and could be described more accurately as those students who qualified in terms of the definition but who were representatives of those ethnic minorities in the population who came from countries outside Great Britain and Ireland.

Teacher Experiences and Backgrounds

Twenty-four secondary teachers and four primary teachers responded to the questionnaire and their combined mathematics teaching experiences covered all levels of these divisions. Twenty-two teachers had worked in Government schools, four in Catholic schools and five in Independent schools. Of these schools 27 were in the metropolitan area of Melbourne and three were in country Victoria (totals were higher than 28 because some teachers had experiences in different sectors and locations).

Teachers were asked to signify their periods of teaching in 10-year intervals. No teacher in the period 1940-1949 responded. Responses were received from two retired teachers with experience in the period 1950-1959; seven teachers with experience in the period 1960-69 responded; and for each of the periods, 1970-1979, 1980-1989 and 1990-2000, an average of 24 teachers responded.

Questions relating to first school education and language background revealed that 20 teachers first attended school in Australia and of these, 17 listed English as their first language. Of these 17 cases, 10 had no other language and seven listed varying levels of second languages which included French(6), Italian(3), German (1), Swedish (1), Mandarin (1)and Latin (1). Three teachers whose first schooling was in Australia listed languages other than English as their first language and these included Italian (2) and Maltese (1). In all three cases English was listed as a second language.

Eight teachers first attended school overseas and the countries of their education included Malta, New Guinea, Fiji, Greece, UK (2), Hong Kong and Ireland. First languages listed by this group included English (4), Maltese (1), Hindi (1), Greek (1) and Chinese (1). Second languages listed by this group included English (4), Gujrati, Fijian, Greek, Vietnamese, French(2), Gaelic (1) and Hindi (1).

Migrant Student Numbers and Origins

The migrant student numbers and their origins in the various schools are shown in Table 3 below.

Table 3

Percentage of Migrant Students in the Mathematics Classroom, Nationalities and Sectors

Perceived percentage of migrants in the classrooms, with sector and school locations	Most common nationality
<10 per cent Independent/Metropolitan 4	Russian, Israeli, Hong Kong, Malaysian, Singaporean, Chinese, Indonesian, Japanese
10-30 per cent Government/Metropolitan 3	Vietnamese, British, Greek
31-50 per cent Government/Metropolitan 6 Catholic/Metropolitan 1	Greek, Italian, Asian, Lithuanian, Yugoslavian, Polish, Seychelloise, Mauritian, Indian, Sri Lankan, Lebanese, Chilean, Columbian, German, Dutch, Vietnamese, Asian, Arabic, Somalian, Croatian, Philippino, South American, Iraqi, Iranian
51-70 per cent Government/Metropolitan 4 Independent/Metropolitan 1 Catholic/Country 1 Catholic/Metropolitan 2	Italian, Greek, Russian, Chinese, Lebanese, Sri Lankan, Ethiopian, Indian, Indonesian, Slovenian, Yugoslavian, Fijian, Turkish, Cambodian, Croatian, Macedonian, Albanian, Vietnamese, South American
>70 per cent Government/metropolitan 6 Independent/Metropolitan 1	Greek, Italian, Turkish, Lebanese, Vietnamese, Chinese, Macedonian, Maltese, Albanian, Korean

Migrant-Directed Resources

These included resources contributed by the school or through agencies outside the school which were provided specifically for the needs of migrant and those strategies devised by the teachers themselves to assist migrants in their mathematics learning.

Table 4
*Migrant-Directed Resources Contributed by Agencies outside the Classroom**

1. Opportunities to attend courses or information sessions outside the school	11
2. Provision of documentation from expert sources	6
3. Withdrawal of students from the classroom for specific help	12
4. Employment of migrant language aides in the classroom	7
5. Employment of English speaking aides in the classroom	4
6. Meetings/information nights arranged with parents	8
Other comments:	
Generally little or no help was available.	5
Students attended intensive language school before they attended secondary school	1
Years seven and eight students of non-English speaking backgrounds (NESB) were grouped together for teaching mathematics.	1
Although some migrant students are withdrawn from Maths for specific help, the school tends to leave migrant students in Maths, withdrawing them in other subjects.	1
Diagnostic tests administered by school places students. Sometimes students move between levels in Maths as English ability changes.	1

Note. * Numbers 1-6 were tick box responses, other comments were interpreted and grouped by the researcher.

Personal Strategies Generated by Teachers to Assist Migrant Students

The perceptions of most teachers were that migrant students needed mathematical help although five teachers perceived that migrant students did not need help in mathematics. Most teachers described at least one personal strategy which they had devised to address migrant disadvantage and their comments were collated and listed in Table 5.

Table 5

Perceptions of Teachers about their Migrant-Related Resources they Generated

No special provisions were made at all. Migrant students did not need extra assistance.	5
Care was taken with the wording of examination questions	1
Newer migrant students were seated together with migrant students more conversant with English	1
Very tight specific formal instructions focussed on a few points,	2
Migrant students were closely supervised and constant feedback was used	1
Students worked together in groups speaking their own language after teacher explained work to be done. Students permitted to use language translation calculators (electronic pocket dictionaries which translate words) or to translate for each other. Maths material was taped in seven languages.	4
Simple language patterns and vocabulary were used when possible and emphasis was given to any new topic-specific vocabulary. Simplified notes and textbooks were used.	6
Extra assistance was given by the teacher in and out of classroom time including help with language and expression	2
1 to 1 instruction with teacher or student peer-mentor was used .	7
Revision sheets were provided that went back to basics and cumulative sheets added skills on a weekly basis	1
Survey was done, where possible, of books used and work covered in previous countries of migrant students, when introducing new topics.	2
Repeat explanations were sometimes given	1
Migrants were excused from tests in mathematics until their grasp of English was reasonable. Test results were not counted until teacher was sure they reflected maths performance rather than language inability. Reporting was modified.	5
More questioning was used to establish that understanding has been achieved and students were asked to write their own maths questions to improve language skills.	2
Greater use of hands-on, concrete aids and activity based approaches. School's Task Centre used for years seven and 8. RIME materials were used.	4
Several mathematics lessons were taught by a visiting French maths teacher in French, and maths posters were made in Junior secondary classes were displayed in French, German and Japanese .	1
Liaison was formed with English/ESL department for language difficulties experienced in maths problems – used particularly for prepositions in VCE maths problems.	1
Mathematical games and individual learning material and programs were used	2
Co-operative group work with reporting back of students to the class as a whole gave ideal opportunities for students to pick up maths language, display their expertise and develop support networks within the classroom.	2
Students made aware of language at all the times through discussion.	1
Students were made aware that teacher came from overseas and spoke other languages.	1
Teacher undertook professional development eg ESL in the Mainstream to develop strategies to help not only ESL students but all students in the maths classroom.	1

Summary of Results

Over 90 percent of responses came from teachers working in metropolitan schools which was probably a reflection of the settlement patterns described in the immigration statistics section of this paper. Most responding teachers came from the secondary division of the government sector but this may have been a reflection of the circulation and readership of the MAV publication. While 20 out of the 28 teachers first attended school in Australia, three of this number did not state English as their first language. Of the eight teachers not attending their first school in Australia, four stated English as their first language. The total therefore using English as a first language was 21 out of 28 (75%) and this total did not reflect their countries of origin. The complexity therefore of defining migrant status applied in both student and teacher contexts. The range of languages used by teachers was very wide and included examples of European, Asian and Pacific Island languages thus conforming to the elements of both major booms of immigration described in the migration statistics.

The migrant numbers and origins of students covered an astonishing number of nationalities. This was most apparent in the classrooms of teachers where at least 30% of students were listed as migrants. The precision with which teachers noted these in the small space provided on the questionnaire sheet indicated the importance which they attached to these different migrant origins. Words such as Asian and European were rarely used. Teacher awareness of and sensitivity to the national identities of their students perhaps reflected the individualism of Australian educational methodology. The number of migrant students in the classrooms of the majority of responding teachers were very high and did not reflect the one in four ratio of migrants to national population as described in the migrant statistics.

A minority of teachers made use of or had access to each of the migrant related resources provided by sources outside the classroom as described in Table 5. Apart from single comments about school structures no other use of outside resources was noted. The highest number of participants (12) used availability in their schools for withdrawal of students from their mathematics classrooms for specific help. Only one teacher took part in professional development to assist in language aspects of mathematics teaching However most teachers had developed personal strategies to assist migrant students which they perceived as necessary and these were diverse. Few teachers used the same strategies but most perceived the need for them and had put their particular preferences into practice in their teaching. A small minority of teachers (5) perceived that migrant students did not need assistance. No teacher perceived advantage in migrant abilities and devised strategies to enhance this or to use these abilities to assist non-migrant students although different mathematical methods in other countries were shared by an even smaller minority (2) of teachers.

Conclusion

Most of the 28 Victorian teachers who responded to the questionnaire teachers taught mathematics in classrooms where the ratio of migrant students to non-migrant students of students within the mathematics classroom exceeded that of the national population and the majority of them described teaching experiences which occurred in the last three decades of the twentieth century. Two major peaks of immigration had occurred before to this time so that elements of both the European labour force wave and the Asian manufacturing force wave had combined to create highly diverse classrooms and some diversity in the origins of teachers. While a formal definition of migrant status was problematic most teachers did not seem to have difficulty in identifying migrant students in their classroom and they had detailed knowledge of migrant student origins. An implicit belief in the connections between mathematics learning and culture perhaps stimulated most teachers to devise personal strategies which they considered appropriate for the migrant students in their own classrooms. Under 50% of teachers made use of or were aware of each of the systemic responses contributed by mathematics educators outside the classroom. Migrant disadvantage was the main perception which drove most teachers to use personal strategies directed to the teaching of migrant students. A minority of teachers perceived that migrant students did not need help. The perception that migrants may have been advantaged and that they could have been considered as one of the resources available in the design of personal strategies did not emerge.

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How Children's Understanding of the Number System varies as a Function of Ethnicity and Socio-Economic Status

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This study looked at Year 3 children's understanding of the number system at four schools with varying socio-economic status. Just over one third of the children were Maori (the indigenous people of New Zealand). In total, 221 children were interviewed using a task-based interview. Children's understanding of the number system varied as a function of ethnicity and socio-economic status. Performance on several key tasks was used to assign children to a level on a developmental framework for numeracy.

A little over a year ago, a new literacy and numeracy goal for young New Zealanders was announced by the Minister of Education. The goal stated that "by 2005, every child turning 9 will be able to read, write and do maths for success" (Ministry of Education, 1999a). It is not clear just what "doing maths for success" meant, or how it was to be measured. However, it is unlikely that this goal could be met without having a good understanding the number system and the ability to apply this knowledge to problems within a variety of contexts (including measurement, statistics, algebra, and geometry). In view of the enormous variation in children's understanding about numbers when they enter school on their fifth birthday (see Young-Loveridge, 1989, 1991), it seems clear that some children will require a great deal of extra assistance if they are to met the numeracy goal by their ninth birthday. In order to identify which children are in particular need of such assistance, appropriate assessment tools will need to be developed.

Assessment has been a particularly controversial issue in New Zealand over the last few years. In May 1998, the Government published a Green Paper: *Assessment for Success in Primary Schools*, proposing the introduction of national externally-referenced tests (Ministry of Education, 1999b). There was a three month period of consultation during which the public could make written submissions to the Ministry of Education (Gilmore, 1999). Responses to the Green paper showed overwhelming opposition to the idea of introducing national externally-referenced tests, with more than 14,000 reasons given for this position. However, there was strong support for the development of additional diagnostic tests for use in primary schools. A call was made for these diagnostic tests be closely related to the curriculum, valid, reliable, culturally appropriate, easy to administer and not too demanding of time. Mathematics was identified as one of the curriculum areas in which the development of diagnostic tests was most urgently required (Gilmore, 1999).

Informal discussions with teachers over the past several years has consistently included concern about the need for a model of developmental progression in mathematics. Several researchers have developed models to explain how children's understanding about numbers develop as they progress from beginning to competent thinking (eg, Bergeron & Herscovics, 1990; Boulton-Lewis, 1996; Fuson, Smith & Cicero, 1997a; Fuson et al, 1997b; Hiebert, 1988; Jones et al, 1996; Resnick, 1983; Ross, 1989; Thomas, 1996; Wright, 1996). However, each of these models has problems or limitations, which include being focused on a very narrow domain of understanding, being overly complicated and difficult to apply quickly in a classroom context, or lacking a clear rationale for progress to more advanced stages. Recently a framework was developed to overcome some of those problems and to provide teachers with a tool which could assist them in identifying strengths and weaknesses in children's understanding about the number system and meet the learning needs of these students more effectively (see Young-Loveridge, 1999a).The framework consists of four stages, each characterised by a major shift in ways of thinking about numbers (see Figure 1). The framework shows how children's understanding of the number system becomes increasingly sophisticated as their thinking develops. The framework begins with a unitary (by ones) concept of numbers, moves through a transitional stage of ten-structured thinking, leading to a multi-unit (by tens and ones) concept of numbers, and finishing with an extended multi-unit stage where multiple units can include any power of ten.

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1. **Unitary concept**
   Knowledge of number word sequences, processes for constructing quantities, ways of working with part-whole relationships, and names for numerals and number patterns

2. **Ten-structured concept**
   Partitioning of numbers into a whole decade and extra ones, and partitioning of the whole decade into units of 10 ones

3. **Multi-unit concept**
   Units of tens and ones are counted separately, and can be traded and exchanged for units of different value (eg, 10 ones for one 10, or vice versa).

4. **Extended multi-unit concept**
   Units can be any power of ten.

Figure 1. Developmental framework for the acquisition of numeracy.

This study was designed to explore children's understanding about the number system during their third year at school in order to identify those with limited understanding about the number system who might benefit from intervention. The tasks used to assess children's understanding of the number system were developed also with the aim of providing diagnostic tools which might be useful for teachers in primary schools. As part of the study, children were assigned to one of the first three levels of the number framework (Young-Loveridge, 1999a).

Method

Participants

The children who participated in this study came from Year 3 classes in four schools in a large urban centre of New Zealand. There were 221 children in total, with approximately one third of them Maori (the indigenous people of Aotearoa, New Zealand), half of them European, and the remainder from other ethnic groups (see Table 1). The socio-economic status of one of the four schools was average (i.e., School A was at level 5 of a 10-point scale ranging from 1 for low decile to 10 for high decile), while the SES of the other three schools was low (Schools B & C were at decile 1 and School D was at decile 2). The group of children from School A were the youngest in their year group (average age = 7.6 years), whereas the groups of children from Schools B, C, and D included all of the children in Year 3 attending those schools (average age = 7.9, 8.0 and 8.0 years, respectively). Because of School C's small size and its similarity to School B, the two decile 1 schools were subsequently combined for the purposes of statistical analysis.

Table 1

Percentages of Children in each Group

	School			
	A	B	C	D
School's Decile Level	5	1	1	2
Group Size	73	52	19	77
Ethnicity				
European/Pakeha	62	10	26	56
NZ Maori	22	65	63	33
Other	16	25	11	11
Gender				
Boys	41	50	58	49
Girls	59	50	32	51
Age in years				
Range	7.1 to 8.1	7.3 to 8.3	7.5 to 8.5	7.4 to 8.6
Average	7.6	7.9	8.0	8.0
SD	0.28	0.29	0.34	0.28
Level				
Unitary	30	62	47	38
Ten-Structured	34	30	53	43
Multi-unit	36	8	0	19

Procedure

An individual task-based interview was used to assess the number knowledge of each child. Tasks included reading and writing numerals, constructing quantities using grouped materials and money of different denominations ($1, $10, $100), part-whole understanding including mental operations, written computation, and word problems, demonstration of place-value understanding, and knowledge of number-word sequences. The strategies children used to solve the problems were noted, in addition to the accuracy of their responses. Children's performance on several key tasks was used to assign them to a level within a framework showing developmental progression in children's understanding of the number system (see Young-Loveridge, 1999a). The task which was used to differentiate children at the Unitary level from those at the higher levels was one in which children were asked to imagine that they had 20 lollies and were given 8 more lollies. Children assigned to the Ten-structured level or higher could say immediately that there were 28 lollies in total, whereas those at the Unitary level had to count by ones to determine the sum of 20 and 8. A collection of tasks used to differentiate children at the Ten-structured level from those at the Multi-unit level involved demonstrating the link between individual digits in a multi-digit numeral and the quantities these digits represented. Sharon Ross's (1989) task with buttons was modified to create a pencil-and-paper version in which the objects (small boxes printed on paper) were presented in a standard arrangement with boxes organised in groups of ten. Children were asked first to determine how many boxes were presented in a particular display, then to write that number on the line above the display, and finally to circle with a pencil the collection of boxes to which each digit in the multi-digit numeral referred. An example of this three-step process was given to the children using the number "12". Children who responded correctly for each digit in 14, 21, 31 and 125 were assigned to the Multi-unit level of the number framework.

Results and Discussion

The majority of children were at the Unitary Level (42%), or the Ten-structured Level (38%), with only 20% at the Multi-unit Level. Table 2 shows the percentages of children at each level of the framework as a function of ethnicity and socio-economic status.

Table 2

Percentages of Year 3 Children at each Level of the Framework as a Function of Ethnicity and Socio-economic Status

		Ethnicity			
	(Overall)	European	Maori	Other	
School A (decile 5)	(73)	(45)	(9)	(19)	
Unitary Level	(22)	22	78	26	
Ten-Structured Level	(25)	34	15	67	
Multi-unit Level	(26)	44	7	7	
Schools B & C (decile 1)	(71)	(10)	(46)	(15)	
Unitary Level	(41)	20	63	67	
Ten-Structured Level	(26)	60	35	15	
Multi-unit Level	(4)	20	2	7	
School D (decile 2)	(77)	(43)	(25)	(9)	
Unitary Level	(29)	40	28	56	
Ten-Structured Level	(33)	41	52	22	
Multi-unit Level	(15)	19	20	22	
Total		(221)	(98)	(80)	(43)

Note. Numbers are shown in brackets.

In general, children at the average decile school had better understanding than those at the three low decile schools, and Non-Maori children had better understanding than Maori children. However, several notable exceptions to this general pattern were found for School D, a decile 2 school. At School D, there were substantially more children at the Multi-unit level, compared with Schools B and C (15% compared to 4%). At School D, the proportion of Maori children at the Multi-unit level was very similar to that of the Non-Maori children

(20% compared to 19% & 22%), whereas for the other cohorts, the proportion of Maori children was substantially less than for Non-Maori children at that level. At School D (decile 2), only 28% of the Maori children were at the Unitary Level, compared to 63% of Maori children at Schools B and C (decile 1), and 78% at School A, (decile 5).

A comparison of Maori children across the four different schools showed that Maori children at School D did substantially better than those at Schools A, B, or C, with 72% at the Ten-Structured Level or better (compared to 37% and 22% at the decile 1 and 5 schools, respectively). There are many possible reasons for the success of Maori children at School D. School D seems to have a particularly strong commitment to bi-culturalism, with 5 of its 21 classes offering education in Te Reo Maori (the Maori language). Researchers such as Bishop and Glynn (1999) have written about the importance for Maori children of feeling positive about their language and culture. In addition to promoting strong cultural identity, School D also placed a high priority on numeracy and improving achievement levels in mathematics. This could be seen in one of its learning goals for the year, which was "to improve basic facts knowledge and mathematics results." These two factors may have contributed to Maori children at School D having better knowledge of number facts than same-age peers at the other three schools, including the decile 5 school. Superior knowledge of number facts was very evident at School D, where children were consistently better at recalling single-digit doubles (4+4 to 9+9, and 10+10) than children at the other schools. Children at School D also did better than the children in Decile 1 schools on recalling number combinations for ten (e.g., 10 – 1 = ?). (Note: Direct comparison with the decile 5 school was not possible because the tasks were presented as missing addend problems rather than as subtraction problems). Not only did School D put a strong emphasis on learning number facts, but it also provided incentives for children to master these facts by creating a special "club" for children who had gained a perfect score on the test of number facts, or made a substantial improvement in test score from one term to the next.

The disappointing performance of Maori at School A raises some important questions which need addressing. At School A, Maori children were in a minority and many may have been isolated and marginalised. Until quite recently, the option to learn in Te Reo Maori was not available to children at School A. Another possible explanation is that the decile level does not capture the heterogeneity in socio-economic status of the families served by School A. If Maori at School A includes a disproportionately large number of children from disadvantaged families, while constituting only a small proportion of the school roll, then this may no be reflected in the decile level. It is possible that in schools where Maori are in a small minority, their need for additional resources is equal or higher than in schools with a high Maori population where cultural identity is strong. It might be all too easy to overlook those small numbers of Maori children in a large school such as School A with more than 600 pupils.

Children's performance and use of particular strategies was examined according to the level of the framework to which they had been assigned. There was remarkable consistency in the use of strategies within each level. Few, if any, of the children at the Unitary level could use anything other than a counting-by-ones strategy to add or subtract, either mentally or with written problems (presented horizontally), or to construct quantities using grouped materials or different denominations of money. Few, if any, could demonstrate place-value understanding using the boxes task, which required them to demonstrate the link between each digit in a multi-digit numeral and the number of boxes (arranged in rows of ten) to which the digit refers. On the other hand, most children at the Multi-unit level could add or subtract, either mentally or with written problems (presented horizontally), and construct quantities using grouped materials or different denominations of money, using units larger than one. Most were able to solve 2- and 3-digit addition problems with regrouping, recall all single-digit combinations for ten, recall number facts for all doubles with single-digit addends, and count by tens to at least 100. All these children, by definition, had demonstrated place-value understanding with the boxes tasks for all of the problems presented, including the 3-digit number. At the Ten-structured level, many children were able to add or subtract using units larger than one and construct quantities using grouped materials or different denominations of money, recall number facts for doubles with addends up to 6, recall all single-digit combinations for ten except 7 and 3, and count by tens to at least 100. Some of the children at the Ten-structured level were able to demonstrate place-value understanding for some of the multi-digit numbers presented, and a few were able to solve 2- and 3-digit addition problems with regrouping.

The tasks which were used to assign children to one of the three levels of the framework were remarkably effective. The task which involved the addition of imaginary objects, with 20 lollies plus 8 more lollies differentiated the children at the Unitary level from those at higher levels. This task was very simple and quick to give, and the use of a counting-by-ones strategy was easy to identify because of the extra time needed to count 8 or more imaginary lollies. Only one child of the 221 interviewed was able to demonstrate place-value understanding without succeeding on the "20 plus 8" task. That kind of consistency supports the idea that there is a Ten-structured stage that is transitional between Unitary understanding and Multi-unit or Place-value Understanding (Fuson et al, 1997a, 1997b). Helping children to combine and partition quantities without counting by ones, and to appreciate the way that the number system is constructed around groupings of ten may be a vital step towards their eventual understanding of place value. The one child who used counting-by-ones for the "20 plus 8" task, even though she could demonstrate place-value understanding with the all four of the boxes tasks, was able to add and subtract using units larger than one as well as construct quantities using grouped objects and different denominations of money, providing strong support for the idea that she was operating beyond the Unitary level.

It was interesting to note that being able to demonstrate place-value understanding with the boxes task did not necessarily mean that the children were able to use that knowledge to solve problems with regrouping. For example, for the word problems involving multi-digit addition (18 + 15, 29 + 23, and 125 + 85), only about half to two thirds of the children at the Multi-unit level were able to calculate the correct answers. It is possible that some of them decided not to attempt the more difficult "sums" because these were presented at the end of the interview and fatigue had set in. A few of the children at the Ten-structured stage could solve those addition equations with regrouping, but not one of the children at the Unitary level was able to do them.

The tasks which used money of different denominations ($1, $10, $100) suggest a possible direction for teachers who want to use real-life contexts to aid place-value understanding and other related concepts. Most children at the Ten-structured stage were able to assign different values to different money items as part of determining how many dollars there were in a mixed collection of $10 notes and $1 coins. This fits with Baroody's (1990) idea that a different-looking ten like a $10 note, which has no physical relationship to 10 $1 coins, may have advantages over the use of a pre-structured ten like the place-value "long," which is virtually identical to 10 small cubes interlocked. The different-looking appearance of

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the $10 note compared to the $1 coin means that it is impossible to "see" the 10 ones which are equivalent in value to the $10 note. However, this did not stop one seven-year-old from tapping the $10 note ten times with her finger as she counted from one to ten, before counting on the additional four $1 coins. A substantial number of children at the Unitary level simply added up the number of money items present without regard to the differential value of the notes and coins. It would be interesting to explore the usefulness of money as an aid to solving multi-digit addition and subtraction with regrouping.

There are a number of important implications of these findings for teachers who are trying to help their students understand the number system. This includes the importance of building a strong foundation in Ten-structured understanding prior to expecting children to learn about the significance of position for the value of a particular digit within a multi-digit numeral. Using a range of materials with groupings by ten invites the child to move away from a "by ones" (Unitary) strategy towards combining and partitioning larger units. There are many different resources which can be used to strengthen Ten-structured understanding (see Young-Loveridge, 1999a). For example, embodiments of the empty number line in the form of 100-bead strings composed of groups of ten beads in alternating colours provide a linear representation of numbers. Grouped objects such as plastic beans stored in tens in clear plastic bags can provide a different model for numbers. Money has the added advantage of being highly motivating for children, many of whom have played board games with money. Through activities such as these, children may come to understand place value, not as an isolated piece of knowledge, but as a particular instance of part-whole relationships involving groupings of tens and ones (Fuson, 1992).

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Using Study Groups to Support Mathematics Learning in Preservice Teacher Education

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Primary preservice teachers usually do not have strong knowledge nor positive attitudes towards the study of mathematics. In a context where educators need to explore effective and cost-effective modes of delivery, forms of pedagogy that support learning and affect are needed. This project investigated the use of study groups within a primary preservice program in supporting students learn aspects of mathematics. Guidelines and future areas of concern are provided.

Researchers have noted that the mathematical understandings of primary school teachers are often poor (Kaminski, 1997; Kanes & Nisbet, 1994) and that many students enter their study of mathematics and mathematics education with a sense of reservation (Carroll, 1994). Often these attitudes towards mathematics can be a result of the students' experiences in secondary school mathematics (Biddulph, 1999). The powerful effect of their earlier experiences in mathematics often means that when students enter teacher education, they hold strong (negative) views of mathematics. Such views are quite resistant to change and as Schuck (1996) has shown, the earlier experiences of students have the potential to frame their professional lives if they go unchallenged. Employing agencies and government authorities are demanding teachers have strong levels of competency in the areas of literacy and numeracy. Within this context, it is critical that preservice teachers exit their programs with high levels of numeracy and are not fearful of the subject.

In concert with these hurdles for mathematics education, the higher education context has been severely cut economically. This means that many of the [supposedly] fat pedagogies of teacher education are no longer viable in today's anorexic institutions. This is in a context where greater numbers of students from wider range of backgrounds and mathematical experiences are entering preservice courses. In these new times, effective practices are needed that must be economically viable and sustainable and cater for the diversity in the student population. In this context, this project investigated the use of study groups as a means to address these concerns. Students formed small groups where they could support each other cognitively and affectively in the learning of the course material. In these small groups, they were able to attend sessions where they could work in their groups, or seek advice from teaching staff when they were unable to resolve questions within their study group. In this context, the research problem posed was to evaluate the effectiveness of study groups as a pedagogical tool within the context of preservice mathematics education. Arising from this problem, two research questions are addressed in this paper.

- How were study groups used by the students for learning mathematics?
- What benefits do students see from participating in study groups?

Literature Review

While there are difficulties with the use of constructivism as a discourse (Zevenbergen, 1996a), its strength lies in the recognition that students come to construct their understandings of mathematics through interaction – with both the physical world and the social world. Central to constructivist positions of learning is that the learner enters the learning context with a set of assumptions, beliefs, and prior knowledge that provides the lens for subsequent construction of meaning (Cobb, 1991; Cobb, 1994; Cobb, Yackel, & Wood, 1992). These factors impact on how and what students will construct from a given learning episode. This approach is central to the theoretical experiences that preservice mathematics teachers have in their learning about young students as learners of mathematics but is also central to their learning of concepts within the course of study.

For students coming to learn mathematics, prior knowledge, experiences and beliefs impact on their subsequent learning. For students who have had positive experiences of mathematics, their capacity and willingness to learn mathematics is likely to be substantially different from the students who have had less positive experiences in learning mathematics. This difference must impact on how and what students will learn as a consequence of their encounters with mathematics learning. In related studies of students learning mathematics (Hagedorn, Saidat, Fogel, Nora, & Pascarella, 1999) it was found that there were substantial differences in mature age students and school leavers. In these studies school leavers were defined as those students who undertake tertiary studies within 3 years of completing post-compulsory secondary schooling. Such studies have shown that mature age students enter the learning contexts willing and eager to learn but with low skills and confidence. However, they are more likely to exceed their school-leaving peers insofar as achievement levels when they exit from their studies. However, support was needed to help this cohort of students realise their goals.

Preservice Teachers Understandings of Mathematics

It is widely recognised that preservice teachers have limited understandings of mathematical concepts and processes (Cooney, Shealy, & Arvold, 1998; Taplin, 1992). Such studies have demonstrated that preservice teachers often hold very broad misconceptions about mathematical concepts. For example, a common misconceptions is that the addition of two common fractions is simply a case of adding the numerators and then adding the denominators eg _ + _ = 2/6 rather than finding a common denominator and then adding the "same" fraction ie $^{2}/_{4}$ + _ = _. The difficulty for teacher education is to reteach these basic concepts in order that exiting teachers have an understanding of why processes and procedures are adopted in order that they do not misteach such concepts to their students. In most cases, what is proposed is a shift from doing mathematics to understanding mathematics.

Preservice teachers also enter their initial training courses with poor attitudes towards mathematics and often full of self doubts as to their capacity as a mathematics learner (Cooney et al., 1998; Philippou & Christou, 1998). In most instances, the negative attitudes they hold towards mathematics as a discipline and towards their capacity to learn mathematics, become stumbling blocks to their learning. Indeed, if students believe that they do not like mathematics and their capacity to do mathematics is restricted, then their subsequent capacity to participate and learn mathematics is restricted and so engenders a self-fulfilling prophecy. In order to alleviate the effect of poor perceptions – of self and mathematics – considerable support for the students is needed. In considering the current context of higher education, such support is difficult to provide within the traditional forms of pedagogy offered with the sector.

Study Groups as a Medium for Learning

The use of study groups for learning has been used across a number of contexts and for a range of purposes. It has been widely explored as a tool for professional development among practising teachers (Charles, 1995; Crowther, 1998; Murphy, 1995; Powell, 1992) and school administrators (Mohr, 1998); and in the higher education sector (LaBonte, 1995; Woods, 1996). Within the area of content, it has been used across a range of curriculum areas including literacy (George, 1997; McCutchen, 1993); social studies (Grimes, 1996); chemical engineering (Woods, 1996); and Spanish classes (Llyod, 1996). In these studies, the use of study groups has been found to help students gain confidence in the nominated curriculum areas. Learning outcomes were reported to increase with the gains in self confidence facilitated by the use of study groups.

The use of study group to aid among at-risk students (McCutchen, 1993; Mercure, 1993) has been found to be highly successful. It has provided a forum through which students are able to work with students at similar levels of understanding and are less intimidated by knowing others. In such environments, students were found to be more active in their learning and gain greater confidence in the areas being covered. It has also been used successfully to support mentor programs where the beginning teacher is supported by a practising teachers (Devlin-Scherer, 1993) for similar reasons cited in other studies.

These studies indicate the breadth of use of study groups as a tool for learning. In most cases, the effectiveness of the use of study groups as a tool for improving learning outcomes (Downs, 1995); confidence in the subject area and/or as effective professional development (Charles, 1995) has been documented.

In summary, preservice teachers are likely to enter their study of mathematics with very different experiences of mathematics and very different levels of mathematical understandings. From constructivist standpoint, these different learning experiences will impact on students' capacity and confidence to learn mathematics. The economically constrained and highly accountable context of higher education poses a big challenge for teacher education to deliver content to students. Study groups have been shown to be highly effective across a number of different studies. As such, this project explored the use of study groups within a teacher education program within the narrow context of the students learning the mathematics component of their course of study.

Method

Students undertake a core subject in their preservice degree (the first of three mathematics education subjects). The subject integrates subject knowledge (mathematics) and pedagogic knowledge (mathematics education). Study groups were implemented as a way in which students could undertake supported learning. The form of the study groups progressively evolved over the semester as a consequence of an action learning model. Using focus groups and random surveys, the needs of the students formed the basis of the evolving study groups.

The subject had an enrolment of 150 students, with approximately 45% of the students being mature age, 80% of the total students being women. The course of study involved students undertaking the study of mathematics up to a Year 10 level but with most content focused on the primary school mathematics content. Student had to gain an 80% pass rate on both of 2 quizzes in order to pass the subject.

Students were given the option to form study groups and 4 extra sessions were provided to support the learning of the students (on a needs basis only), but also to provide a physical space for students to meet whether or not they may have needed support from teaching staff. The project evolved whereby three of the workshops where used in the manner intended by the project, and one group becoming its own study group.

Data Collection

The project cited here is from a much larger project where a number of tools have been used to collect data. The data cited in this paper comes from focus group discussions. These were used to compliment other forms of data collected throughout the project.

Focus Groups – At the completion of the subject, two focus groups were held. A series of semi-structured questions were asked and students were able to respond. Focus group meetings were tape recorded and transcribed. Students were also asked to complete individual responses to the questions that were posed. The questions posed focused on the strategies they used in their groups, identifying key aspects of the approach that they felt were critical to effective learning; and the effectiveness of study groups in their learning of mathematics.

Discussion

The discussion is centred on the three questions related to the research questions:

1. What strategies do preservice teachers employ when working in study groups to negotiate and enhance learning outcomes?
2. Does the use of study groups facilitate a growth in the affective domain of mathematics learning?
3. Does the use of study groups facilitate change in self-reported learning?

A range of strategies were used in and across the study groups but central to all their meetings and discussions was the use of worksheets. A series of worksheets were made available through the library website as the text book questions were very unclear and many redundant for the subject. The questions were highly focused on what was essential knowledge for primary school teachers and were based on the particular content strands being covered. Answers were provided with the sheets. In all groups, the worksheets formed the basis for discussions and meetings. These were seen by all groups as useful tools to focus their work and discussions rather than as the focus of their work.

Strategies Used by Student Study Groups

From the focus groups discussions, it appears that there were a number of distinct strategies used within the study groups. The strategies were adopted to more or less degrees across the groups, depending on the activities being undertaken and group dynamics. These included reliance on a particular group member who was seen to be somewhat more knowledgeable than the others; using the study groups as a form of support – both moral and mathematical; and a group that had a special mix of needs and hence relied more on teaching staff.

Strategy 1: The Collective Experts

This strategy tended to have a experts who were relied on more heavily than other members of the group, depending on the work being undertaken. This person/s would offer some advice when the group would get stuck. After input, the group would then work on their tasks again.

> S: We just worked through the worksheets and the people that could… Sandy got everything – she'd sort of go at how one hit and we'd say "Sandy, what does this mean" and we'd sort of help each other and when we got stuck on Question 11 or something, and then we'd all come back and meet again and work it all out together, say the different ways that go to the conclusions.

As a group, they relied on a range of resources to help them through the content, including their peers. This strategy was often inclusive of most group members as at different times, different group members would have the necessary knowledge.

> S1: We worked as a group, especially for the tests, together, going through the worksheets and doing the problems as they arose, comparing answers and referring

> S2: to [the lecture notes] if we needed to as we went along. If we found there was a discrepancy in our answers or one of us wasn't quite sure, the others would model it for them and try to explain it. If we weren't sure, we'd go to [the textbook] and we found that worked really well. We just methodically worked through the chapters that way.

Many of the older students commented that they could work out answers but would not understand why they undertook the methods that they did. For example, the new method of subtraction with decomposition is very different from 10 years ago so many of the older students had difficulty with this method whereas the younger students had grown up with the method so were very familiar with it. The younger students were able to tutor their older peers in this task.

Strategy 2: The Supporters

A second strategy tended to use the group for morale so that the group took on a role where they would support and motivate each other as well as providing a forum in which they could discuss the mathematics. The more affective component of the learning was seen to be important in this strategy. This is evident in the comment:

> S: I think that is where working in a group helps because you don't give up. There is someone there to support you and someone there that perhaps has grasped the system [the procedure used to calculate and answer] and whether we yelled at each other or were frustrated with each other, we eventually got through it and everybody understood. If you were on your own you would tend to give up.

Strategy 3: The Directors

This strategy was not the type of self-directed study group that was envisaged for the project. In contrast, this strategy was used by students when they could not work out particular problem. It was a dominant strategy used by a particular group, which comprised "very" mature students (some with adult children) and who felt that they needed more teacher-directed input. One woman commented that she did not feel comfortable working in a group as her ability and confidence were so poor as it had been so long since she had done maths that the study group format tended to confuse her more. Another member had a hearing impairment so had many gaps in her learning and she felt somewhat different from her peers and preferred to work with her teachers. This group met during one of the workshop sessions and would work through the work sheets and would raise areas of difficulty. They sought greater input from the staff than the other group. In many ways, this was the most disempowered group of students as they lacked the confidence to work through problems in small groups of peers. There was a strong perception that the teacher was the expert and that this was their best way to meet their needs as they could get too confused with diversions.

> S: I found the small group sessions with you [the lecturer], where it was the small group, that we were able to ask questions and if we didn't understand it, you'd go through specifically for us. I guess that's the way our brain functions, we're all a bit different and you eventually got some method that we would understand.

Interim Conclusion

The value of the study group as forum for learning was unequivocal. Students saw the value of the study groups as providing a forum in which they could negotiate tasks and answers in a manner that was non-threatening and supportive. The study group environment allowed students to work together in ways that they were able to negotiate and develop. The different strategies indicate the range of techniques used by the students to negotiate and learn the mathematics of the subject. Students consistently stated that the study groups gave them a chance to work together and have mutual gains. This was often done within the context of a very social gathering.

Study Groups and Learning Outcomes

Two means for documenting learning outcomes can be used. In the first instance, there were just over one third of the students who failed the first quiz and needed to resit the quiz. These students were predominantly the students who made use of the study groups and extra

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support provided. When students sat the second quiz, only 10% of the total cohort needed to resit the quiz. This suggests that the learning of content had improved potentially as a consequence of the use of study groups. On it own, such statistics suggest only correlation, not causation, so at best, it can only be claimed that the study may have been useful in improving learning.

At a more subjective level, students reported that they learnt a lot more through the use of the study groups. They reported that when they did not understand particular aspects of the content, the study groups provided a forum where they could talk with their peers and say, without fear of being labelled as incompetent, that they could not understand something. Through discussions with their peers, they learned how to work through problems and why they undertook some procedures and not others. In many cases, the explanations reinforced what they had heard and read, but became far more contextualised and hence meaningful. The study groups provided a non-threatening forum in which they could discuss their lack of understanding without fear of ridicule, which enabled them to understand the work more clearly. This is summed up in the comment:

> S: I think when you have a study group, you work together as a group. I think it is a lot better because not everybody thinks the same and so you're getting ideas of how to maybe cut out a step, do it a bit different and of a sudden you go "oh yeah, OK".

The value of study groups can be aptly summed up with the following comment in which a student is discussing her transition from working solitary for most of her life and the value she experienced through participating in study groups where the students talked about ideas and problems.

> S: Oh the realisation that I was just going to learn it so much better and it was going to be so much easier if we shared it and sat down and talked about it, rather than trying to sit there [alone]. At school I remember sitting and writing down and learning stuff for history- rote and writing it out, writing it out, and I've just realised at Uni now that I don't learn by writing it out. Why did I ever do that? I don't know anymore. I actually learn by talking about it.

Implications for Teaching Practice

Within the new contexts of higher education, new forms of effective pedagogy are needed. This project has investigated the value of a particular strategy - study groups - that offers potential for teacher education. The format of the study group is very flexible and can be adapted to cater for the needs of the participating students, something not possible within the normally rigid structures of large scale lectures and tutorials. What was clear in this project was that mature age students predominantly used the study groups. Some younger students participated within some groups, but this was not common. Insofar as further research, this aspect of focus groups could be explored more. Many of the younger students said that they did not need study groups as they had only recently left school so their knowledge of mathematics was still fresh and hence did not need extra support in learning. The break between the value of study groups as a medium for cognitive, affective and social learning has not been recognised by the younger students whose focus is still utilitarian and focused on passing the subject.

What was clear from the discussions with students was the need for focus within the study groups. It would appear that it is essential for the group to have some tasks that give the group purpose and structure. The use of the work sheets proved to be an invaluable catalyst for discussions about the mathematical concepts and processes. In this study, the value of the focussing activity was not recognised until later in the project when students identified their value. In subsequent implementations of study groups, greater attention can be directed at the types of activities that can be offered as there appears to be significant value in this focused activity. In all cases, the work sheets were foundational to the activities and discussions that occurred within the groups.

The use of study groups cannot be seen to replace teaching but rather should be a support for learning. The results of this study indicate that some groups of students need extra support in learning mathematics. Study groups have provided a very useful forum in which students have greater control over their learning and hence gain more confidence in mathematics. Through discussions with peers, they are able to offer support and be supported by their peers in the learning of mathematics. While such learning is invaluable, it is necessary to provide contact with teaching staff during periods so that students are able to access staff when they encounter brick walls and problems cannot be resolved at the group level. This input is left until the students identify that they need some extra external support.

The results of this project are indicative of the potential of study groups in higher education, however, more work needs to be undertaken in order to identify the most effective strategies for improving learning and affect while maintaining an economic and pedagogic viability. This project has identified the value of study groups within teacher education along with a number of principles that have been useful in organising the same.

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Short Communications: Abstracts

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Mathematics Anxiety among South African Preservice Teachers: The Development of a Program to Minimise Anxiety through Improving the Classroom Environment

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Research findings enumerate different causes of mathematics anxiety (fear through panic, helplessness and mental disorganisation). This program seeks to find an association between these causes and the classroom environment of the preservice teachers' learning experiences. A pilot project was conducted at one of the preservice institutions by involving all students specialising in mathematics. Data were collected via the belief, attitude and anxiety scales with the purpose of building up a multiple data source that will help in deciding which direction a major research initiative should be undertaken. The What is Happening in this Class (WIHIC) questionnaire coupled with the Test of Mathematics-Related Attitudes (TOMRA)(adopted from TOSRA – Test on Science-Related Attitudes) were used to determine attitudes of the sample, whilst for belief and anxiety scales the South African and Australian versions were respectively utilised. The purpose at the end was to develop a program that will try to attend to the improvement of the classroom environment in order to minimise anxiety.

Mathematics Education in the Netherlands: Some Comparisons and Implications

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This paper describes some of the main features of primary mathematics education programmes in the Netherlands. The development of Realistic Mathematics Education is briefly traced, and comparisons are made with the present-day prescriptive mathematics education in England. Some implications for mathematics educators and teachers in Australasia are discussed.

Year 8 Students' Progression on an Integrated Learning System

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This paper explores seven Year 8 students' progress on the core mathematics course of an integrated learning system (ILS) where individual students are presented with electronic worksheets in random form. It shows the students' frustrations, confusion and stress with the inflexible marking system of the ILS, the repetitive formats of the ILS worksheets and the unusual setting-out procedures in some worksheets. It presents a model that relates student, teacher, classroom and operational characteristics to performance and progression on the ILS.

Improving Children's Thinking in Solving Mathematical Problems

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This paper concerns a constructivism model for improving children's thinking in solving mathematical problems. There are many factors that affect children's ability to solve mathematical problems and these had been listed by the Newman's model in the analysis of model. The ability to construct mathematical knowledge could be a great obstacle to children in solving mathematical problems. A model has been proposed by the author to explain how children could construct and retain mathematical knowledge more effectively. The model comprises four stages: evaluation, transmission, construction and reinforcement of mathematical knowledge. This paper describes in details how this model works and case studies have been conducted using this model and have shown significant results.

Emergent Properties of Teaching-Learning in Technology-Enriched Classrooms

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The introduction of technological devices into mathematics classrooms has created opportunities for new pedagogies, as well as changes in requirements associated with mathematical content. We develop the notion of emergence, whereby interaction between human and technological agencies results in outcomes not envisaged for the technologies according to their primary purpose. Emergent properties are illustrated from activities within four classrooms. The contexts involve alternative uses of a viewscreen in concert with graphical calculators, to enhance the quality of interactive learning.

Processing Mathematical Word Problems in Chinese: A Study in two Chinese Institutions

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A study of two groups of Mandarin speaking college students, resident in PR China, was undertaken, to investigate students' ability to solve mathematics word problems. Ability was measured by the correctness of the answer and the times taken to understand solve and complete these problems. The results show a significant difference between the 2 groups in terms of time taken to undertake mathematical word problems but no significant difference between the number of answers correct. Possible reasons are considered.

Learning to Teach Mathematics for Understanding: The Role of Reflection

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In this session I explore how a group of eleven pre-service secondary mathematics teachers developed their understanding of what it means to teach for understanding. These teachers were all concurrently enrolled in a field experience and secondary mathematics teaching methods course. The students were required to attend their allocated field placement every day for approximately two-and-a-half hours each morning for fifteen weeks. The methods course was conducted once each week for four hours in the evening. A major thread tying the course together was that of connected teaching and teaching for understanding. Recent work by Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, and Human (1997), Carpenter and Lehrer (1999), Fennema, Sowder, and Carpenter (1999), and Goodell and Parker (In press) informed my understanding of what connected teaching and teaching for understanding really meant, and all of the activities I implemented were designed in order to deconstruct these concepts in ways that would be meaningful to pre-service teachers. The work of Schön (1987) and Loughran (1996) guided my thoughts about the value of reflective practice, and many of the course requirements were structured around reflective journal writing. Students were required to make two journals entries each week. The first was a reflection on each methods class. The second was a reflection on a critical incident that had occurred during that week in the classrooms in which they were teaching. At the beginning of each methods course session, the class shared their critical incidents with one another. Other more traditional methods-course activities are also discussed. At the end of the course, students had to write a paper outlining what they had learned throughout the semester in the practicum and methods course. Evidence suggests that the majority of students had developed their understanding of what teaching for understanding really meant, and that certainly for some, the critical incidents discussions and reflective writing were key elements of that development.

Questioning as a Prompt to Help Scaffold the Learning of Mathematical Processes

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A teaching experiment was designed to assess the usefulness of some strategies designed to help tertiary students completing a bridging mathematics course improve their mathematical competencies. The strategies were: stressing mathematical applications, making links to the students prior mathematics learning, and using questioning as a means of guiding students through the process of problem-solving. In this paper I look at some results in terms of improved confidence and success in the course and at some students reaction to the questioning technique as opposed to the telling or showing technique.

Using a Computer-Based Interactive Arithmetic Test as an Adaptation of Vygotsky's Method of Interactive Assessment

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This presentation describes a recent study in which Vygotsky's method of interactive assessment was adapted to produce a computer based interactive arithmetic test (CBIAT). The test was applied to two groups of primary school children: non-Aboriginal children from the Perth metropolitan region, and Aboriginal children from the remote Kimberley region of Western Australia. The results of the test were compared with those from the written arithmetic subtests of the Key Math Test (KMT). In a one way analysis of covariance (with age as covariate) it was found that the Aboriginal children performed significantly ($p < 0.001$) less well in the KMT, but not in the CBIAT. This presentation uses charts to display data from multiple sessions of the Addition subtest of the CBIAT. It shows the later sessions reveal better performance capabilities for both sets of children, but that the improvement was greater for the Aboriginal students. In the early sessions the non-Aboriginal children performed relatively better than the Aboriginal students, but after four sessions at the CBIAT there was no significant difference between the two groups.

The Development, Implementation and Evaluation of an Instructional Model to Enhance Students' Understanding of Primary School Geometry

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This paper reports ongoing research that is meant for completion by September 2000. The purpose of the research is to determine, through classroom intervention, whether primary school students' geometric thought develops according to a specific pattern. The research pays particular attention to the possible development that occurs in students' geometric understanding during a six-month intervention programme. The study involves the implementation of an instructional programme in a grade 6/7 South African classroom that covers the prescribed syllabus for that particular grade for the academic year. The research focuses predominantly on the students' reaction to the instructional programme in a normal classroom setting. A pre- and post-test will be administered to ascertain whether a shift in students' understanding has occurred. Additionally the research method includes the collection of quantitative data by means of a questionnaire for students to establish their perceptions of the classroom environment. A variety of other data collection methods will be introduced including observations, video-taping of classroom lessons, interviews with students as well as journal writing by students throughout the intervention.

Promoting Critical Thinking in Mathematics and Science Classrooms: A Modelling Approach for Teacher Education

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Curriculum documents in both Australia and New Zealand advocate an investigative, problem-solving approach in mathematics and science. Such approaches require critical thinking on the part of the learner, yet many teachers have not experienced learning in this way themselves. At Dunedin College of Education the Mathematics and Science Departments collaborated to write a course titled, "Cognitive Processes in Mathematics and Science". The course gave participants the opportunity to engage in investigating and problem solving as learners in a highly structured, supportive environment. One group who took the paper were in-service teachers up-grading their qualifications. Some reactions from this group will be presented, including the impact on their current classroom practices.

Problem Solving and Number Sense Focuses in a Teacher Education Programme

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The adequacy of primary pre-service students' mathematical content knowledge is widely recognised as problematic. At the Auckland College of Education, the development of aspects of students' own mathematics content knowledge is generally addressed only implicitly. However, one area identified as critical to promoting thinking teachers of mathematics, as opposed to procedural bound ones, was number sense. This paper reports on a degree programme that attempts to directly address this aspect of students' content knowledge through problem solving and investigative approaches.

Professional Development: A Process for Becoming an Exemplary Mathematics Teacher in the South African Outcomes-Based Education Environment

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There is research evidence that confirms that teachers' conceptions and teachers' reflections about mathematics and mathematics instruction profoundly affect their teaching and the learning process (Feldt, 1993). With the introduction of the new curriculum in South Africa, it becomes important to review the professional teaching standards. Thus my study's principal aim is to identify and investigate critical features of professional development that will assist teachers in deconstructing a traditional conceptual framework of learning and teaching that they have built up over years of practice in order to implement OBE principles. As I am finding it important to devise guidelines for professional development in line with the

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described OBE environment, both qualitative and quantitative data collection methods will be necessary to confirm the transition. Four case studies (the researcher, a colleague and two Grade 9 and Grade 10 school teachers based at the same school), observations, learners' interviews, video taping, journal keeping, discussions and 'What is Happening In this Classroom (WIHIC) questionnaires (both Class and Personal forms) (Aldridge, 2000) will be used to determine how effective the emerging guidelines for professional development in this study are in assisting teachers to create an OBE environment and implement the implied approach.

Discovering Mathematics in the Mathematical Playground

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Although mathematics is an integral component of our life, most of mathematics is learned in the classroom with little or no connection to everyday life. The Mathematical Playground has been developed in order to bring to evidence that mathematics exists everywhere, and can be taught and learned even in the playground. The Mathematical Playground contains several "math" devices on which one can climb, measure, tie ropes, investigate and discover different mathematical concepts and connections. In addition, it contains stuff that can be found in any regular yard: tables, benches, garbage cans, paved paths, etc.- all of which involve different mathematical aspects. For example, a Teeter-Totter, which can be found in any playground, involves mathematical concepts such as symmetry. The Mathematical Playground enables students and teachers to realise that mathematics can be done everywhere, not only in the classroom at school during math lessons. It enables students and teachers to do and feel mathematics with the whole body and thus internalise math concepts and reduce math anxiety. It makes possible the use of manipulatives and other materials, which cannot be used in the classroom. From our experience with students, preservice and inservice teachers, it seems that the Mathematical Playground not only enables math investigations, brings to evidence different misconceptions and makes the learning of math a wonderful experience, but also "opens eyes" to discover and take advantage of the mathematics around us.

Student Outcome Statements and Teachers' Decisions During the Interactive Stage in a Mathematics Classroom

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This study investigates decisions mathematics teachers make in the process of translating student statements into actions during their interactions with learners. Research has revealed that teachers' content knowledge, beliefs, and goals mainly dictate to their interactive decisions (Fennema & Franke, 1992; Thompson, 1992). In South Africa, the system of education that is currently being phased out was mainly content loaded and summative assessment driven, with little emphasis on the established goals. With the introduction of outcomes based education, the outcomes have been placed in the forefront for the first time. The ultimate success of the system rests mainly on teachers being capable of task-analysis in the outcome statements to decide on appropriate learning activities for their learners, and how to interact with them. In this study involving three mathematics teachers, critical observations will be made to capture their moment-to-moment interactive decisions. Clinical interviews (Truran & Truran, 1998) will be used to establish the rationale for those decisions and the extent to which they are influenced by the outcome statements. This in-depth and prolonged engagement will further lead to the documentation of the teachers' shift with respect to the types of decisions they made as a result of their enhanced understanding of the outcome statements. Whilst this critical information will be contributing to the development of a theory on teaching in context (Schoenfeld, 1998), some guidelines will be drawn to inform intervention strategies aimed at preparing teachers for the implementation of Outcomes Based Education.

Evaluating the Impact of a 'Cascade' Model Utilised for the Professional Development of Teachers from Disadvantaged Schools in the Eastern Cape Province in South Africa

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The Education Department in the Eastern Cape recognises that there is a need for in-service teacher education in mathematics. It is envisaged that, through a cascade model, leader teachers will enhance their teaching skills, and as a result of this primary intervention that second generation teachers from the leader teacher's school will also implement more effective teaching practices. Also, it is envisaged that these teachers will create an atmosphere that is conducive to effective teaching and learning in all the classes in the school. This paper reports on research in progress. The study utilises a grounded approach and combines both qualitative and quantitative methods. Qualitative data will be collected through interviews with the 'leader' teacher, their colleagues and the Education Development officers. Questionnaires will be used to analyse the impact that the cascading model has on the second-generation teacher's classes in terms of the teacher's practices. The research will be written up as a case study of two teachers in two different schools.

Learning Environment Perceptions of Mathematics Students in Canadian and Australian Schools and Colleges

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The actual and preferred learning environments of students taking mathematics at the senior secondary and post secondary levels in British Columbia, Canada and the Australian Capital Territory were compared using the modified form of the *College and University Classroom Environment Inventory* (CUCEI). Students at the senior secondary level from both countries perceived their classrooms almost identically in all scales with the exception of Task Orientation, where Canadian students perceived this scale more favourably. However, male students in Canada perceived more task orientation and equity in their mathematics classes, whereas Australian students perceived more student cohesiveness. In both countries, when students progressed from their senior secondary level to the post-secondary level, they perceived less student cohesiveness but more task orientation and equity. When female students progressed to the post secondary level, they perceived there was less personalisation and individualisation in comparison to what they experienced at the senior secondary level. However, male students perceived more task orientation and equity at the post secondary level and less student cohesiveness. Students in both countries generally indicated a preference for a more favourable learning environment in all areas measured by the seven scales in the CUCEI. Interestingly, female and male students at the post secondary level perceived their classroom environments similarly.

An Investigation of Teachers' Perceptions of their School Environment as a Factor Conducive to the Achievement of Exemplary Results in Matriculation Mathematics in South African Schools

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The recent findings in TIMMS led to an outcry in South Africa in terms of the performance of South African mathematics students. In addition, the National Department of Education emphasised a greater need for research to improve the present matriculation results. The purpose of this study was to investigate the perceptions of teachers of their school environment as one factor that my lead to the achievement of exemplary results in 'average' South African schools – that is, schools which are performing reasonably well in matriculation mathematics. A comparison of the perception of teachers of their school environment in an 'average' school and an underachieving school was made. This was accomplished by administering the school-level-environment questionnaire (SLEQ). The findings from this intervention were used to introduce various activities in low achieving schools to help teachers to improve instruction and learning practices that would impact on matriculation results. This study also investigated factors that are conducive to the achievement of exemplary results in matriculation mathematics in these 'average' South African schools. Again a comparison of teaching and learning strategies used in 'average' schools, and under-achieving schools was made. A number of useful guidelines for teachers constitute the outcomes of the study.

The 'Angle in Geometry' Notion as an Illustrative Example of the Tension Between 'Expert Knowledge' and Students' Sources of Alternative Constructions

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After the teaching of basic concepts in geometry, students, more often than not, demonstrate what are commonly called 'alternative conceptions'. This is hardly surprising since there is ample research evidence that suggests that the learners' conceptions of certain mathematical or scientific concepts are often markedly different from the conceptions of the same scientific or mathematical constructions held by experts. However, even though students do have their own 'alternative constructions', it appears that for basic geometric constructions like angle, the formal meaning of angle as portrayed in numerous school text books is neither clear-cut nor precisely defined to qualify as proper 'expert knowledge'. This, as the author ventures to show, is because of the fact that it is virtually impossible to formalise the notion of 'angle'. This paper aims to focus on the notion of angle in order to illustrate the existing, yet unexplored, tension between expert knowledge and students' sources of alternative conceptions and pre-instructional knowledge of angle. The paper therefore seeks to explore the various meanings of angle that appear in literature with a view to locating sources of the students' alternative constructions in some of these very same descriptions of angle that appear in text. In the process, the author casts doubt on the existence of some commonly held 'Expert' or 'Scientific Community' knowledge of angle. The author argues that, instead of seeking an undisputed formal definition of angle, a multiple perspective of this geometric construction is much more fruitful. The adoption of this stance by teachers would allow for the roots of geometry to be correctly seen as embedded in direct unmitigated human visual experience and intuition, and which students who are new to formal geometry already possess.

An Investigation of the Role and Effects of Journal Writing by Pre-Service Mathematics Teachers in South Africa

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The proposed research seeks to investigate the role and defects of journal writing by pre-service mathematics teachers in order to prepare reflective practitioners in the senior phase of schooling (grades 7–9). The study will also provide pre-service teachers with skills to reflect on their work and to explore the mathematics they teach as well as to encourage them to use journal writing as a teaching and learning strategy. The literature will be reviewed with the intention of determining the features of exemplary journal writing. This will assist in the development of a package of exemplary journal writing skills that will be used to workshop a group of pre-service teachers on journal writing. Before introducing the package at the workshop, a short survey questionnaire will be administered to determine how much pre-service teachers know about journal writing. At the conclusion of the workshop, the pre-service teachers will be given a second questionnaire to determine the extent of their knowledge after the intervention. Three or four pre-service teachers will be interviewed using structured and semi-structured interviews to gain feed back as part of the evaluation of the package.

Development of the Test: Thinking about 3D Shapes

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There was no appropriate, comprehensive group test of 3D Spatial Thinking for primary school students. A test of spatial thinking aroused by quality 3D classroom activities was needed. In the psychology literature there are a number of tests, often for adults or for individual testing, that measure spatial abilities. Some of the abilities and the type of items

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used to test these abilities were adapted in the current test. The analysis of data from testing 265 students has indicated high internal consistency and validity of most items. It has also shown those items which may be eliminated in order to improve the quality of the test and to shorten its length. A key to its success was the introduction to the paper-and-pencil test with concrete materials.

Using Different Strategies to Teach the Central Limit Theorem

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An ongoing research project involves presenting the concepts of sampling variability and the Central Limit Theorem to undergraduate students using three different modes of instruction: traditional chalk and talk, hands-on sampling exercises, and computer-based instruction. One aim of the experiment was to see if recently developed interactive computer-based learning packages could improve student comprehension of this difficult topic. Both computer-based instruction and hands-on sampling groups showed greater short-term improvement than those taught by traditional methods, but the third group had caught up by the final exam.

Promoting Reflective Practice among South African Mathematics Teachers: A Study of the Efficacy of Journal Writing

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The proposed research is aimed at developing, implementing and evaluating a program that will assist teachers to become self-reflective in their teaching practices. My assumption is that journaling is one means by which mathematics and science teachers can improve their instruction and become reflective practitioners. The research is aimed at first piloting with teachers whether journal writing can help them to improve their daily teaching practices. The methods to be used will embrace both quantitative and qualitative through interviewing experts, consulting the literature already existing, making observations of teachers in practice and conducting interviews with them. Initially, the program will be piloted in two schools at the intermediate and the senior phase levels. Secondly, the revised program will be trialed in five schools for the duration of one school term (3 months). People involved in this will be the principals of the two schools and their teachers. Thirdly, questionnaires will be administered and interviews will be held with teachers from the participating schools as a way of evaluating the effectiveness of the program. As a consequence, the researcher hopes that a package will be compiled for the schools and all departmental officials for their usage. This will also help in the formation of new educational policy with regard to teachers' preparation and annual plans.

One Thousand Words? Symbols, Sounds and Language in Mathematics for a Pre-service Student from a Non-English Speaking Background

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When English is a second or even third or fourth language for teachers, or students, it is possible for communication to break down between the students and the teacher and between students. This problem is exacerbated by the abstract nature of mathematics and many teachers and students are unsuccessful merely because they cannot express in the English language the mathematical ideas that occur in classroom situations. This study reports an attempt on the part of the researcher to investigate one possible strategy designed to assist a group of pre-service teachers from non-English speaking backgrounds and with a background of failure or anxiety in learning mathematics to overcome their difficulties. A series of worksheets designed to give practice in six different types of mathematical activities resulted in greater confidence, though gains in deeper understanding and skill appear minimal.

Professional Development of Teachers in South Africa: A Case Study

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The introduction of a transformed curriculum (Curriculum 2005) in South Africa marks a shift from the present school content-driven syllabus to an outcomes-based curriculum. Arguably the most demanding challenge facing teachers is the move away from "teacher-centred" approaches to "learner-centredness" with the introduction of outcomes-based education. The purpose of this paper is to describe an intervention programme of professional development for in-service primary school teachers that focuses on outcomes-based methods within a constructivist framework in mathematics education. A specific component of the research was to explore whether teachers' mathematics beliefs translated into appropriate classroom action. Two case studies (Jan and Lindelwa) reveal that in one case practice was aligned with expressed beliefs and the other not, even though in both cases they have expressed enthusiasm for the intervention.

Attitudes to Mathematics in the Middle School: Is there a common cause?

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Proponents of middle schooling believe that schools exist to cater for the needs of the students through the better understanding of the nature of young adolescence. A structure should be provided that best suits these students in the age group 10-14+ who are in the process of moving from dependency to independence. The motivation for this study arose

from observations that suggested many students when entering Year 9 (late middle school years) seemed alienated and disengaged from the study of mathematics. This case study/ action research explores whether students' attitudes to mathematical learning are a result of issues relating to students' developmental stages. The conditions, strategies and teaching style conducive to students' preferred classroom environment, the consideration of the characteristics of students at this particular age group and the students' developmental stages are part of the research. To that end the work of Lawrence Kohlberg is examined with regard to the applicability of his theories in the context of the Year 9 classroom. Both qualitative and quantitative means of data collection are undertaken within a theoretical framework in which a constructivist theory of learning has been utilised.

A Study of some Alternate Conceptions (in Algebra) that are held by some Practising Mathematics Teachers

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The recent TIMMS study indicated that the level of achievement in mathematics in South Africa is very low. Another indicator, the matriculation examination, had an 18% pass rate in the mathematics standard grade paper. One of our concerns is the understanding of the mathematics content of teachers themselves. This paper reports on some alternate conceptions (in algebra) that are held by some practising mathematics teachers. The research method involves extracting qualitative data from samples of teacher responses to past examination papers. These examinations were written by practising teachers who are enrolled for an FDE (Further Diploma in Education) in mathematics to upgrade their qualifications. This FDE is administered by an Non-Government Organ-isation that is affiliated to the local university.

Dis-Quietening Moments:
Another Look at the Issue of Girls' Silence in the Classroom

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This paper extends what is taken for granted about the nice, quiet and good girl within the available psycho-educational discourse in mathematics education. Working with issues which the conventional discourse fails to engage, it addresses the problems associated with the construction of the classic female designation as a universal category. Its motivating text is towards a theory of the possible for girls in school mathematics. The analysis which ensues relies on an understanding of the girl's formation as a production, one which is constructed by the discourses and practices through which she becomes subjected.

Mathematical Identity: Mathematics Classrooms as Communities of Practice

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This paper investigates Years 9 and 10 students' perceptions of mathematics teaching and learning. Using Lave and Wenger's social theory of learning where both the subjective and social aspects of learning are considered, it is proposed that students need to develop a sense of belonging if they are to feel as a member of the community of practice within mathematics. Using semi-structured interviews, the paper identifies some commonly occurring themes in the responses offered by the students.

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